9.1

At saturation conditions, steam is *not ideal*. Use the Steam Tables:

At 377°C and 1.6 MPa, read $h_1 = 3.205\text{E}6$ J/kg and $s_1 = 7153$ J/kg·K

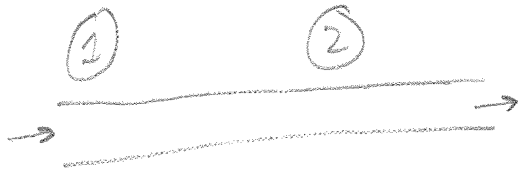
At *saturation* for $s_1 = s_2 = 7153$, read $p_2 = 185$ kPa,

$\mathbf{T_2 = 118°C}$, and $h_2 = 2.527\text{E}6$ J/kg

Then $h + \frac{1}{2}V^2 = 3.205\text{E}6 + \frac{1}{2}(200)^2 = 2.527\text{E}6 + \frac{1}{2}V_2^2$, solve $\mathbf{V_2 \approx 1180\ \frac{m}{s}}$ *Ans.*

This exit flow is **supersonic**, with a Mach number exceeding 2.0. We are assuming with this calculation that a (supersonic) shock wave does not form.

9.2



Givet: CO_2 strömmar i rör med konstant tvärsnitt:

Inlopp:
$T_1 = 477{,}6\,K$
$V_1 = 152{,}4\,m/s$
$P_1 = 689{,}48\,kPa$

En bit nedströms:
$T_2 = 755{,}4\,K$
$V_2 = 304{,}8\,m/s$

Sökt: a) P_2 b) Tillfört värme, q c) Entropiändringen, Δs

d) Massflöde per areaenhet

Lösning: Kontinuitetsekv, 1D $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

a) $A_1 = A_2 \Rightarrow \rho_1 V_1 = \rho_2 V_2$

Antag idealgas: $\left\{ p = \rho R T \quad (9.2) \Rightarrow \rho = \frac{p}{RT} \right\}$

$$\Rightarrow \frac{P_1 V_1}{R T_1} = \frac{P_2 V_2}{\cancel{R} T_2} \Rightarrow P_2 = P_1 \cdot \frac{V_1}{V_2} \cdot \frac{T_2}{T_1} = \underline{\underline{545{,}26\,kPa}}$$

b) Energiekv: (9.20) $h_1 + \frac{V_1^2}{2} + g z_1 = h_2 + \frac{V_2^2}{2} + g z_2 - q + w_v \Rightarrow$

$\Rightarrow$ {ingen höjdskillnad och försumma friktionen} $\Rightarrow$

$$\Rightarrow h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} - q \Rightarrow q = h_2 - h_1 + \frac{1}{2}(V_2^2 - V_1^2)$$

Antag perfekt gas: (c_p, c_v konst.) $\Rightarrow h_2 - h_1 = c_p (T_2 - T_1)$

Appendix A.4 $\Rightarrow \begin{cases} k = 1{,}3 \\ R = 189 \left[\frac{m^2}{s^2 K}\right] \end{cases}$

(9.4) $$c_p = \frac{kR}{k-1} = 819 \left[\frac{m^2}{s^2 K}\right] \Rightarrow q = \ldots = \underline{\underline{262 \left[\frac{kJ}{kg}\right]}}$$

c) Entropiändring: (9.8) $$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = \ldots = \underline{\underline{419{,}8 \left[\frac{J}{kgK}\right]}}$$

d) Massflöde: $\dot{m} = \rho A V \Rightarrow$ $$\frac{\dot{m}}{A} = \rho_1 V_1 = \{p = \rho R T\} = \frac{P_1 V_1}{R T_1} = \underline{\underline{1164 \left(\frac{kg}{s m^2}\right)}}$$

9.3 CO_2 expands isentropically through a duct from p_1 = 125 kPa and T_1 = 100°C to p_2 = 80 kPa and V_2 = 325 m/s. Compute (a) T_2; (b) Ma_2; (c) T_o; (d) p_o; (e) V_1; and (f) Ma_1.

Solution: For CO_2, from Table A.4, take k = 1.30 and R = 189 J/kg·K. Compute the specific heat: $c_p = kR/(k-1) = 1.3(189)/(1.3-1) = 819$ J/kg·K. The results follow in sequence:

(a) $T_2 = T_1(p_2/p_1)^{(k-1)/k} = (373\text{ K})(80/125)^{(1.3-1)/1.3} = \mathbf{336\ K}$ *Ans.* (a)

(b) $a_2 = \sqrt{kRT_2} = \sqrt{(1.3)(189)(336)} = 288\text{ m/s},\ Ma_2 = V_2/a_2 = 325/288 = \mathbf{1.13}$ *Ans.* (b)

(c) $T_{o1} = T_{o2} = T_2\left(1+\frac{k-1}{2}Ma_2^2\right) = (336)\left[1+\frac{0.3}{2}(1.13)^2\right] = \mathbf{401\ K}$ *Ans.* (c)

(d) $p_{o1} = p_{o2} = p_2\left(1+\frac{k-1}{2}Ma_2^2\right)^{1.3/(1.3-1)} = (80)\left[1+\frac{0.3}{2}(1.13)^2\right]^{1.3/0.3} = \mathbf{171\ kPa}$ *Ans.* (d)

(e) $T_{o1} = 401\ K = T_1 + \frac{V_1^2}{2c_p} = 373 + \frac{V_1^2}{2(819)}$, Solve for $\mathbf{V_1 = 214\ m/s}$ *Ans.* (e)

(f) $a_1 = \sqrt{kRT_1} = \sqrt{(1.3)(189)(373)} = 303\text{ m/s},\quad Ma_1 = V_1/a_1 = 214/303 = \mathbf{0.71}$ *Ans.* (f)

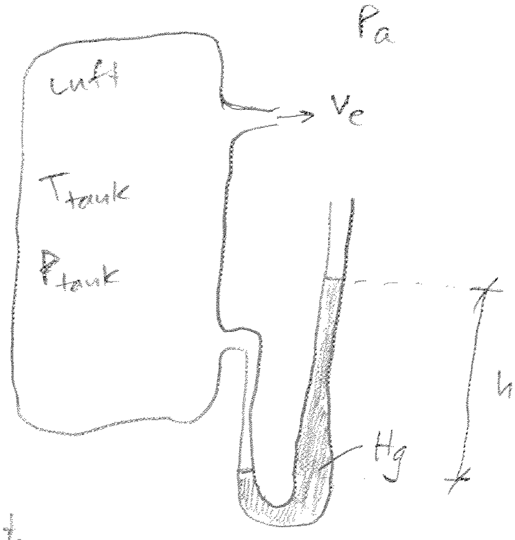
9.4

Givet:

$T_{tank} = 30°C = 303\ K$

$h = 0{,}3\ m$

$V_e = 235\ m/s$



Sökt: a) P_{tank}

b) P_a

c) Ma_e, Machtalet i utloppet.

Lösning: (9,23) $c_p T_0 = c_p T_e + \frac{1}{2} V_e^2$

T_0: Stagnationstemp. / Totaltemp

stillastående gas $\Rightarrow T = T_0 \Rightarrow T_{tank} = T_0$

$$\Rightarrow T_e = T_0 - \frac{V_e^2}{2c_p} = \left\{ c_{p,luft} = \frac{kR}{k-1} = \frac{1{,}4 \cdot 287}{0{,}4} = 1004{,}5 \left[\frac{m^2}{s^2 K}\right] \right\} = 275{,}5\ K$$

(9,16) $a_e = \sqrt{kRT_e} = \sqrt{1{,}4 \cdot 287 \cdot 275{,}5} = 332{,}7\ m/s$ (ljudhastigheten i utloppet!)

c) $Ma_e = \frac{V_e}{a_e} = \underline{\underline{0{,}706}}$ (9,26)

(9,28a) $\frac{P_0}{P_e} = \frac{P_{tank}}{P_a} = \left(1 + \frac{k-1}{2} Ma_e^2\right)^{\frac{k}{k-1}}$

$\Rightarrow P_{tank} = P_a \left(1 + 0{,}2\, Ma_e^2\right)^{3,5} = P_a \cdot 1{,}395$ (1)

Manometern ger: $P_{tank} = \rho_{Hg} \cdot g \cdot h + P_a$ (2) (försummar luftpelaren!)

a) (1) & (2) $\Rightarrow P_{tank} = \rho_{Hg} g h + \frac{P_{tank}}{1{,}395} \Rightarrow \ldots \Rightarrow P_{tank} = \underline{\underline{140{,}8\ kPa}}$

b) $P_a = \ldots = \underline{\underline{101{,}0\ kPa}}$

9.5 First evaluate the unblocked test conditions:

$$T = 293°K,\ a = \sqrt{kRT} = \sqrt{1.4(287)(293)} = 343\ \frac{m}{s},\quad \therefore\ V = (1.1)(343) = \mathbf{377\ \frac{m}{s}}$$

$$\text{Also,}\quad \frac{A}{A^*} = \frac{[1+0.2(1.1)^2]^3}{1.728(1.1)} = \underset{\text{(unblocked)}}{1.007925},\quad \text{or}\quad A^* = 1.0/1.007925 \approx \mathbf{0.99214\ m^2}$$

If A is blocked by 0.004 m^2, then $A_{new} = 1.0 - 0.004 = 0.996\ m^2$, and now

$$\frac{A_{new}}{A^*} = \frac{0.996}{0.99214} = 1.00389,\quad \text{solve Eq. (9.45) for } Ma(\text{blocked}) \approx \mathbf{1.0696}$$

Same $T_o = 364$ K, new $T = 296$ K, new $a = 345$ m/s, new $\mathbf{V} = Ma(a) \approx \mathbf{369\ \frac{m}{s}}$ *Ans.*

Thus a 0.4% decrease in test section area has caused a 2.1% decrease in test velocity.

9.6 Assume $p_e = 1$ atm. For a control volume surrounding the plate, we deduce that

$$F = 135\,N = \rho_e V_e^2 A_e = k p_e Ma_e^2 A_e = 1.4(101350)(0.002\ m^2)Ma_e^2,$$

$$\text{or}\quad \mathbf{Ma_e \approx 0.69}\quad \textit{Ans. (b)}$$

$$T_e = 293/[1+0.2(0.69)^2] \approx 268\ K,\quad a_e = \sqrt{1.4(287)(268)} = 328\ \frac{m}{s},\ \text{thus}$$

$$V_e = a_e Ma_e = (328)(0.69) \approx \mathbf{226\ m/s}\quad \textit{Ans. (a)}$$

$$\text{Finally,}\quad p_{tank} = p_o = 101350[1+0.2(0.69)^2]^{3.5} \approx \mathbf{139000\ Pa}\quad \textit{Ans. (c)}$$

9.7

Givet:

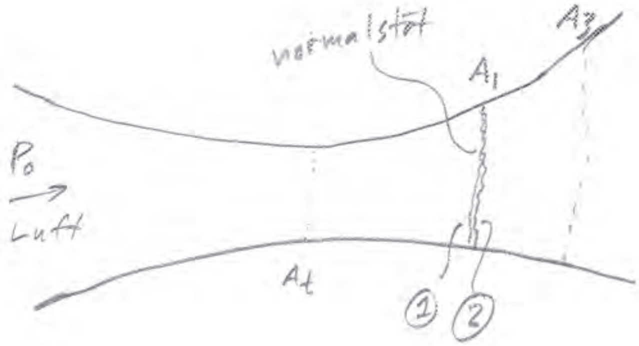
$P_0 = 450$ kPa (stagnationstryck/ total tryck)

$A_t = 12\ cm^2$

$A_1 = 20\ cm^2$

$A_3 = 30\ cm^2$

Stöt vid A_1!



Sökt: a) Statiska trycket efter stöten, P_2

b) P_3 c) A_3^* d) Ma_3

Lösning: * Stöt i divergenta delen av dysan ger att det

a) måste vara supersoniskt strömning före stöten.

* $A_t = A^*$, trängsta sektionen har chokad strömning ($Ma = 1$)

* Isentropisk strömning (P_0, ρ_0, T_0, A^* konst.) före och efter stöten, men genom stöten sker irreversibel process.

(9,44) $\frac{A}{A^*} = \frac{1}{Ma}\left(\frac{1 + \frac{1}{2}(k-1)Ma^2}{\frac{1}{2}(k+1)}\right)^{\frac{1}{2}\left(\frac{k+1}{k-1}\right)}$ ← obs! fel i boken

$\frac{A_1}{A^*} = 1{,}67$ Måste passningsräkna med (9.44)! Alternativ när $k = 1{,}4$:

(9,48c) $Ma_1 = 1 + 1{,}2\left(\frac{A}{A^*} - 1\right)^{1/2} = 1{,}98$ (supersoniskt!)

(9,28a) $P_1 = P_0\left(1 + \frac{k-1}{2}Ma_1^2\right)^{\frac{-k}{k-1}} = 58{,}9$ kPa (statiskt tryck före stöten!)

(9,55) $\frac{P_2}{P_1} = \frac{1}{k+1}\left(2k\,Ma_1^2 - (k-1)\right) \Rightarrow P_2 = 260{,}9$ kPa

c) (9,59) $\frac{A_2^*}{A_1^*} = \frac{Ma_2}{Ma_1}\left[\frac{2 + (k-1)Ma_1^2}{2 + (k-1)Ma_2^2}\right]^{\frac{1}{2}\left(\frac{k+1}{k-1}\right)}$ ← obs! fel i boken!

$\Rightarrow A_2^* = 16{,}5\ cm^2 = A_3^*$ (Efter stöten gäller ny "minsta area"!)

$$(9{,}57)\quad Ma_2 = \sqrt{\frac{(k-1)\,Ma_1^2 + 2}{2k\,Ma_1^2 - (k-1)}} = 0{,}58$$

Isentrop omräkning efter stöten. Passningsräkna med (9.44) eller när k=1,4:

d) $$\frac{A_3}{A_2^*} = 1{,}82$$

$$(9.48a)\quad Ma_3 = \frac{1 + 0{,}27\left(\frac{A_3}{A_2^*}\right)^{-2}}{1{,}728\left(\frac{A_3}{A_2^*}\right)} = 0{,}34 \qquad \text{(subsoniskt!)}$$

$$(9.28a)\quad \frac{p_{03}}{p_3} = \left(1 + \frac{k-1}{2}Ma_3^2\right)^{\frac{k}{k-1}}$$

Obs! $\begin{cases} p_{03} \neq p_{01} \\ p_{03} = p_{02} \end{cases}$

$$(9{,}58)\quad \frac{p_{02}}{p_{01}} = \left[\frac{(k+1)\,Ma_1^2}{2 + (k-1)\,Ma_1^2}\right]^{\frac{k}{k-1}} \left[\frac{k+1}{2k\,Ma_1^2 - (k-1)}\right]^{\frac{1}{k-1}}$$

$$\Rightarrow \quad p_{02} = 327{,}5 \text{ kPa} = p_{03}$$

b) $$p_3 = \frac{p_{03}}{\left(1 + \frac{k-1}{2}Ma_3^2\right)^{k/k-1}} = \underline{\underline{302{,}2 \text{ kPa}}}$$

9.8 If a shock forms, the throat is **sonic**, $A^* = 10\ \text{cm}^2$. Now

$$p_1^* = 0.5283p_{o1} = 0.5283(500) \approx \mathbf{264\ kPa} = p_{o2} \quad \text{also}$$

$$\text{Then}\quad \frac{p_{o2}}{p_{o1}} = \frac{264}{500} = 0.5283: \quad \text{Table B.2, read } Ma_1 \approx 2.43$$

$$\text{So}\quad A_1/A_1^* = \frac{[1+0.2(2.43)^2]^{3.0}}{1.728(2.43)} \approx 2.47, \quad \text{or} \quad A_1(\text{at shock}) = 2.47(10) \approx \mathbf{24.7\ cm^2} \quad \textit{Ans.}$$

9.9

- Investigate the design condition which corresponds to supersonic isentropic flow for the given area ratio : $\frac{A_e}{A_t} = \frac{0{,}0046}{0{,}0009} = 5.11$

- Find the respective design Mach number by utilizing Eq. 9.44-fs.* for $\gamma = 1{,}4$ (air)

  * Make a guess and solve it repeatedly (e.g. excel)

  If done it converges to $Ma_{e_{design}} \simeq 3.2$

- Calculate the design pressure ratio :
  ↳ Assume isentropic flow with no shocks within throat section
  ↳ Combine Eq. 9.26-fs → $\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2$
  and
  Eq. 9.28a-fs → $\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\gamma/\gamma-1}$ ⇒

$$\xRightarrow{\gamma=1{,}4} \frac{p_0}{p} = \left(1 + 0{,}2 M^2\right)^{3.5} \quad (1) \xRightarrow[p=p_e]{M=M_e} \frac{p_0}{p_e} = 49{,}4 \Rightarrow$$

$$\xRightarrow{p_0 = 120\,kPa} p_{e_{design}} = 10.1\ kPa$$

- Finally for the design condition the throat is sonic thereby the mass flow is given by 9.46a-fs :

$$\dot{m} = \frac{p_0 A^*}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} \xRightarrow{\gamma=1{,}4}$$

$R = 287\ m^2/s^2K$
$T_0 = 300\,K$
$p_0 = 120\ kPa$
$A^* = A_t$

$$\Rightarrow \dot{m} = 0{,}91\ kg/s$$

- Now that the design condition is defined the conditions in the nozzle for different back pressure can be estimated. Figure 9.12 can be used as an aiding tool.

a) - For $P_e = 400\ KPa$ there is chance that the conditions correspond to above subsonic isentropic conditions [curve C]

- To check the above calculate the case of curve C.
- For this case $\frac{A_e}{A_t}$ remains same but the subsonic solution of equation 9.44 its applies.

By iteration $\rightarrow$ $Ma_e' \simeq 0,115$

- The above Ma_e corresponds to exit pressure P_e' (use equation 1) : $P_e' = 495,39\ KPa$

- Since the P_e is $400\ KPa < P_e'$ the conditions corresponds to curve D. That is a normal shock is experienced downstream of the throat. The throat is chocked and $P_b = P_e = 400\,KPa$ while $\dot{m} = \dot{m}_{max} = 0,91\ kg/s$

b) This case is definitely below subsonic isentropic condition but might even be below, when oblique shocks form outside exit plane [curve G]
To check the above calculate case of curve F when the normal shock forms right before the exit plane.

- In that case $Ma_e \simeq 3,2$ just upstream before the normal shock. and $P_e = P_{e,design} = 10.1$ kPa
- To calculate the conditions after the normal shock use Eq. 9.55 fs $\frac{P_2}{P_1} = \frac{1}{} + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)$ $\overset{M_1 = M_e}{\Rightarrow}$ $\gamma = 1.4$ $P_2 = P_e$

$\Rightarrow P_2 = 118,97$ kPa

- Since the above pressure is below $P_b = 120$ kPa the conditions correspond to curve E.
- That is normal shock is upsteam of exit plane
- The exit flow is subsonic with back pressure equal to exit pressure: $P_e = P_b = 120$ kPa
- The throat remains chocked with $\dot{m} = \dot{m}_{max} = 0,91$ kg/s

c) - For $P_b = 9$ kPa, this pressure is below the design back pressure ($P_{e,design} = 10.1$ kPa)
- This corresponds to curve I.
- The nozzle is chocked and it cannot respond
- The mass flow is chocked: $\dot{m} = \dot{m}_{max} = 0,91$ kg/s
- The exit flow expands in a complex series of supersonic wave motions until it reaches the low back pressure

9.10 Let us answer the second question first, to see where 0.12 kg/s stands:

$$\dot{m}_{max} = \frac{0.6847 p_o A^*}{\sqrt{RT_o}} = \frac{0.6847(120000)(0.0005)}{\sqrt{287(300)}}$$

$$\approx \mathbf{0.140\ kg/s} \quad \textit{Ans.}\ \text{(b)} \ \ (\text{if } p_{atm} < 63\text{ kPa})$$

So the given mass flow is about 86% of maximum and $p_{atm} > 63$ kPa. We could just go at it, guess the exit pressure and iterating, or we could express it more elegantly:

$$\dot{m} = \rho AV = \frac{\rho_o}{(1+0.2Ma^2)^{2.5}} A\,Ma\sqrt{kR}\sqrt{\frac{T_o}{1+0.2Ma^2}} = \frac{\text{Const Ma}}{(1+0.2Ma^2)^3},$$

where Const ≈ 0.2419 in SI units. If $\dot{m} = 0.12$ kg/s, we thus solve for Ma:

$$Ma \approx 0.496(1+0.2Ma^2)^3 \quad \text{to obtain } Ma \approx 0.62,\ p_{atm} \approx \mathbf{92.6\ kPa} \quad \textit{Ans.}$$

9.11

- Investigating 9.12 it is obvious that supersonic flow inside the diverging section occurs for back pressure equal or below levels that correspond to curve F

- That is a normal shock occurs just at the beginning of exit plane

- Assume that the given mass flow is the maximum/design it is possible to calculate the throat area from Eq 9.46a fs: $\dot{m} = \frac{P_o A^*}{\sqrt{T_o}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} \Rightarrow$

$R = 287\ m^2/s^2K$
$\gamma = 1.4$

$\Rightarrow$ $A^* = A_t = 0{,}001\ m^2$

$T_o = 600K$
$P_o = 500kPa$

- Then the exit Mach numbe before the shock is equal to the $Ma_{e,design}$ which can be calculated from Eq. 9.44-fs by repititive guesses. This gives

$Ma_{e\,design} \simeq 3.2$

- From $Ma_{e,design}$ and isentropic flow until the shock calculate $P_{e,design}$:

↳ combine Eq. 9.26-fs $\rightarrow \frac{T_o}{T} = 1 + \frac{\gamma-1}{2} M^2$

and

Eq. 9.28a-fs $\rightarrow \frac{P_o}{P_{e,design}} = \left(\frac{T_o}{T}\right)^{\gamma/\gamma-1}$ $\Rightarrow$

$\gamma = 1.4$ $\Rightarrow$ $\frac{P_o}{P_{e,design}} = \left(1 + 0{,}2\ M^2\right)^{3.5}$ $\xRightarrow[P_o = 500kPa]{M = M_{e,design}}$ $P_{e,design} = 10{,}1\ kPa$

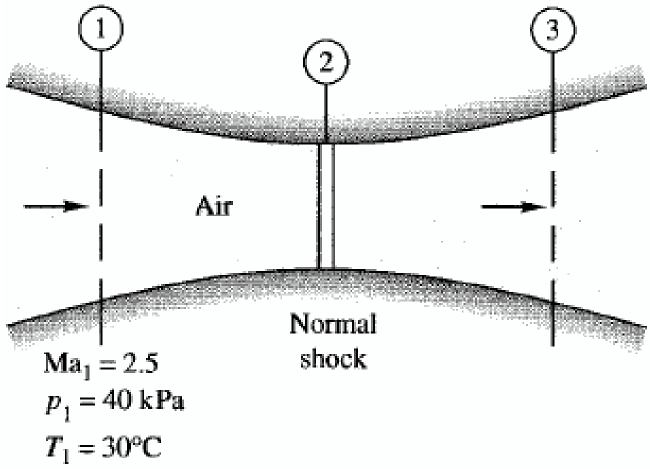
- The conditions after the normal shock are calculated by using Eq 9.55 - fs

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right) \qquad \xRightarrow{\gamma = 1.4} \quad P_1 = P_{e,design} \quad M_1 = M_{e,design}$$

$$\Rightarrow \quad P_2 \simeq 119 \text{ kPa}$$

- Thereby for back pressure up to 119 KPa supersonic flow may be maintained inside the diverging section.

9.12



We have enough information at section 1 to compute the mass flow:

$$a_1 = \sqrt{1.4(287)(30+273)} \approx 349 \text{ m/s}, \quad V_1 = 2.5(349) = 872 \frac{\text{m}}{\text{s}}, \quad \rho_1 = \frac{p_1}{RT_1} = 0.46 \frac{\text{kg}}{\text{m}^3}$$

$$\text{Then} \quad \dot{m} = \rho_e A_e V_e = 0.46(0.0024)(872) \approx \mathbf{0.96\ kg/s} \quad \textit{Ans.} \text{ (a)}$$

Now move isentropically from 1 to 2 upstream of the shock and thence across to 3:

$$Ma_1 = 2.5, \quad \therefore \quad \frac{A_1}{A_1^*} = 2.64, \quad A_1^* = \frac{24}{2.64} = 9.1 \text{ cm}^2, \quad \text{and} \quad \frac{A_2}{A_1^*} = \frac{18}{9.1} = 1.98$$

Read $Ma_{2,\text{upstream}} \approx \mathbf{2.18}$, $p_{o1} = p_{o2} = 40[1+0.2(2.5)^2]^{3.5} \approx 683$ kPa, across the shock, $\frac{A_3^*}{A_2^*} = 1.57$, $A_3^* = 14.3 \text{ cm}^2$, $\frac{A_3}{A_3^*} = 2.24\Big|_{\text{sub}}$, $\mathbf{Ma_3 \approx 0.27}$ *Ans.* (b)

Finally, go back and get the stagnation pressure ratio across the shock:

$$\text{at } Ma_2 \approx 2.18, \quad \frac{p_{o3}}{p_{o2}} \approx 0.637, \quad \therefore \quad p_{o3} = 0.637(683) \approx \mathbf{435\ kPa} \quad \textit{Ans.} \text{ (c)}$$

9.13

The two "scratches" cause *Mach waves* which are directly related to Mach No.:

$\mu_1 = 30°$, $Ma_1 = \csc 30° = \mathbf{2.0}$, $\beta = 50°$, Eq. 9.86 yields $\theta \approx \mathbf{18.13°}$ *Ans.* (a)

$$\text{Then} \quad Ma_{2n} = 0.690 = Ma_2 \sin(50 - 18.13°),$$

$$Ma_2 = 1.307, \ \phi = \sin^{-1}\left(\frac{1}{1.307}\right) \approx \mathbf{49.9°} \quad \textit{Ans.} \text{ (b)}$$

9.14

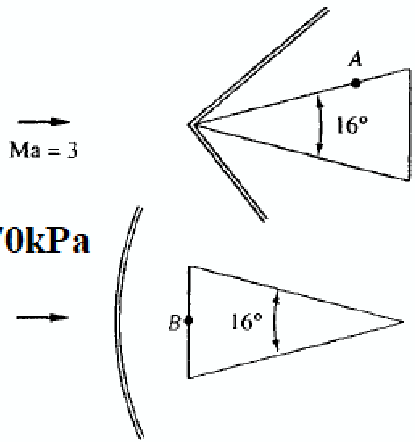
For $Ma = 3,\ \theta = 8°$, Eq. 9.86:

$$\beta = 25.61°,$$

$$p_A/p_1 = \frac{2.8(3\sin 25.61°)^2 - 0.4}{2.4} = 1.80,$$

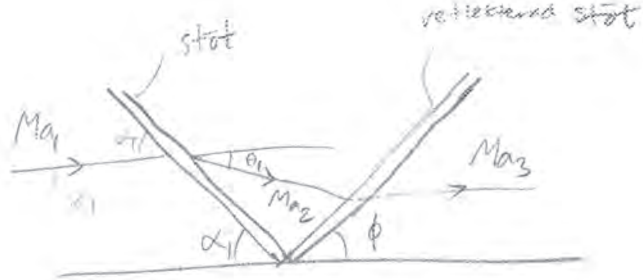
$$\therefore\ \ p_A \approx \textbf{124 kPa}\ \ \textit{Ans.}\ \text{(a)}$$

p1=70kPa



(b) A <u>normal shock</u> forms, and $p_B = p_{o2}$ inside the shock. Given $p_{o1} = p_1/0.0272 =$ **2573.529 kPa**
Table B.2, $Ma = 3$: $p_{o2}/p_{o1} = 0.3283$, hence $p_{o2} = 0.3283(367) =$ **845 kPa** *Ans.* (b)

9.15



Givet:

$Ma_1 = 2,5$

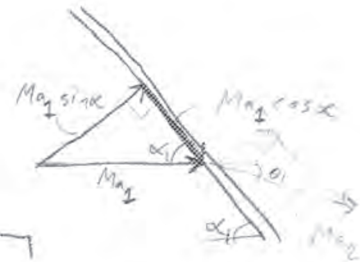
$p_1 = 100$ kPa

$\alpha = 40°$

Luft

Sökt:
a/ Ma_3
b/ p_3
c/ ϕ

Lösning: a/ Räknar på normalkomponenten av hastigheten över en sned stöt. Tangentiella komponenten bevaras.



Använd samma ekvationer som för en normalstöt:

$$Ma_{2n,ut} = \sqrt{\frac{(k-1)Ma_{1n,in}^2 + 2}{2kMa_{1n,in}^2 - (k-1)}} \quad (9.57)$$

$$\left\{\begin{matrix} Ma_{1n,in} = Ma_1 \sin\alpha \\ k = 1,4 \end{matrix}\right\} \Rightarrow \quad Ma_{2n,ut} = 0,666$$

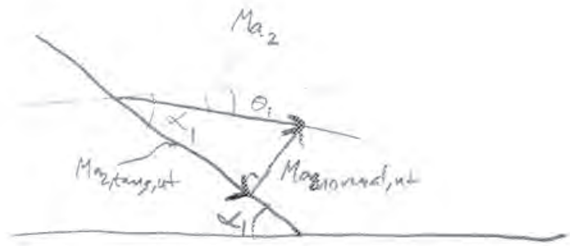
$$Ma_{1t,in} = Ma_{2t,ut}$$

Vi kan hitta deflektionsvinkeln θ_1 mha lite trigonometri, eller så använder vi en färdig formel från boken:

$$(9.86) \quad \tan\theta_1 = \frac{2 \cdot \cot\alpha \left(Ma_1^2 \sin^2\alpha - 1\right)}{Ma_1^2 \left(k + \cos(2\alpha)\right) + 2} \Rightarrow \left\{\begin{matrix} Ma_1 = 2,5 \\ \alpha = 40° \\ k = 1,4 \end{matrix}\right\} \Rightarrow$$

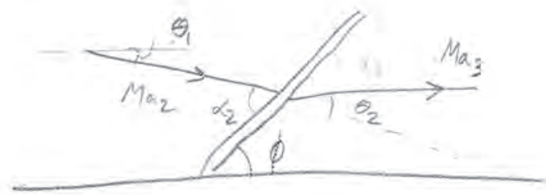
$$\Rightarrow \theta_1 = 17,68°$$

$$\frac{Ma_{2n,ut}}{Ma_2} = \sin(\alpha_1 - \theta_1) \quad (9.92)$$



$$\Rightarrow \quad Ma_2 = \frac{0,666}{\sin(40 - 17,68)} = 1,754$$

Vid den andra / reflekterade stöten:



Eftersom strömningen var parallell med plattan innan första stöten, och blir parallell efter andra, så måste $\theta_2 = \theta_1$

$$(9.86) \Rightarrow \quad \tan\theta_2 = \frac{2\cot\alpha_2\left(Ma_2^2 \sin^2\alpha_2 - 1\right)}{Ma_2^2\left(k + \cos(2\alpha_2)\right) + 2}$$

$$\Rightarrow \left\{\begin{matrix}\text{Iterera fram}\\ \text{eller lös}\end{matrix}\right\} \quad \alpha_2 = 60,49°$$

$$Ma_{2n,in} = Ma_2 \sin\alpha_2 = 1,526$$

Räkna på normalkomponenten genom andra stöten:

$$Ma_{3n,ut} = \sqrt{\frac{(k-1)\,Ma_{2n,in}^2 + 2}{2k\,Ma_{2n,in}^2 - (k-1)}} \Rightarrow \left\{\begin{matrix}k = 1,4\\ Ma_{2n,in} = 1,526\end{matrix}\right\}$$

$$\Rightarrow Ma_{3n,ut} = 0,692 \quad \Rightarrow \quad Ma_3 = \frac{Ma_{3n,ut}}{\sin(\alpha_2 - \theta_2)} \Rightarrow \left\{\begin{matrix}\alpha_2 = 60,49°\\ \theta_2 = \theta_1 = 17,68°\\ Ma_{3n,ut} = 0,692\end{matrix}\right\}$$

$$\Rightarrow \underline{\underline{Ma_3 = 1,02}}$$

b) P_3 fås genom att räkna på normalkomponenterna genom båda stötarna:

$$\frac{P_2}{P_1} = \frac{1}{k+1}\left(2k\,Ma^2_{1n,in} - (k-1)\right) \qquad (9.55)$$

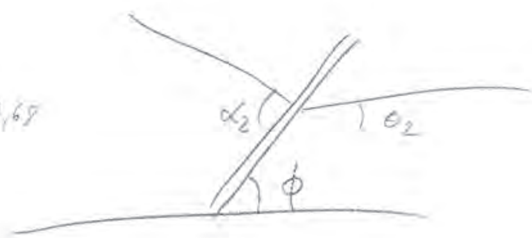
$(Ma_1 \cdot \sin\alpha)$

$$\Rightarrow \quad P_2 = \frac{100\cdot 10^3}{2{,}4}\left(2\cdot 1.4\cdot (2{,}5\cdot \sin 40^\circ)^2 - 0{,}4\right) = 284{,}6 \text{ kPa}$$

$$P_3 = \frac{P_2}{k+1}\left(2\cdot k\cdot Ma^2_{2n,in} - (k-1)\right) = \underline{\underline{725{,}8 \text{ kPa}}}$$

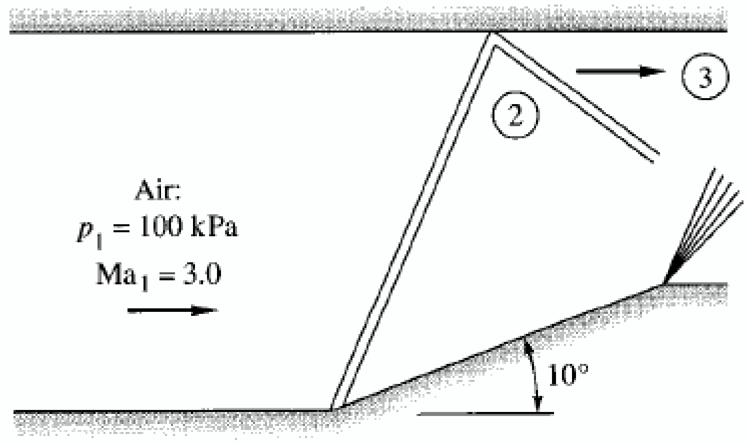
$Ma_2 \cdot \sin\alpha_2$

c) $\phi = \alpha_2 - \theta_2 = 60{,}49 - 17{,}68$



$\rightarrow \quad \underline{\underline{\phi = 42{,}8^\circ}}$

9.16



Given $\theta = 10°$, find state 2:

$Ma_1 = 3.0, \quad \theta_1 = 10°,$

Eq. 9.86 predicts $\beta_1 \approx 27.38°,$

$Ma_{1n} = 1.380, \quad Ma_{2n} = 0.748,$

$$\therefore\ Ma_2 = 2.505, \quad \theta_2 = \theta_1 = 10°, \quad \beta_2 = 31.80°, \quad Ma_{2n} = 1.32, \quad Ma_{3n} = 0.776,$$

$$\therefore\ Ma_3 = \mathbf{2.09} \quad \mathit{Ans.}$$

Meanwhile, $p_2/p_1 = 2.054$, or $p_2 = 205.4$ kPa,

and $p_3/p_2 = 1.866$, $p_3 = 1.866(205.4) \approx$ **383 kPa** *Ans.*

9.17

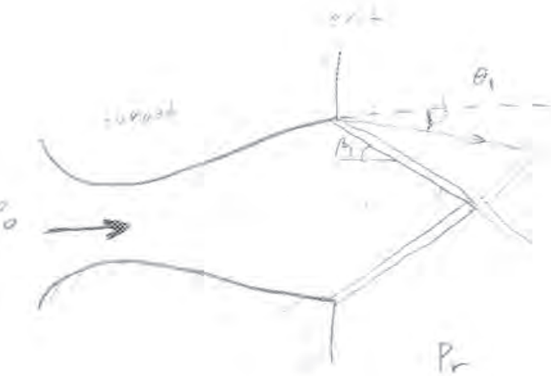
<u>Givet:</u>

$p_0 = 350$ kPa

$\frac{A_e}{A_t} = 3{,}0$

$\theta_1 = 4°$

(Figur 9,12 "G")



<u>Sökt:</u> Mottrycket utanför munstycket, p_r

<u>Lösning:</u> Räknar isentropiskt från $t \to e$ för att finna Mach-talet vid utloppet.

$$\frac{A_e}{A^*} = 3 \qquad (B.1) \text{ eller } (9{,}45) \Rightarrow \begin{cases} Ma_{sub} \approx 0{,}2 \\ Ma_{sup} \approx 2{,}637 \end{cases}$$

Välj supersonisk lösning!

$Ma_e = 2{,}637$

Avlänknings ~~Deflektionsvinkeln~~ θ_1 fås ur:

$$(9{,}86) \qquad \tan\theta_1 = \frac{2\cot\beta_1\left(Ma_e^2 \sin^2\beta_1 - 1\right)}{Ma_e^2\left(k + \cos 2\beta_1\right) + 2}$$

eller fig(9,23)! $\begin{bmatrix} \theta_1 = 4° \\ Ma_e = 2{,}637 \\ k = 1{,}4 \end{bmatrix}$

Iterering ger: $\beta_1 = 25{,}26°$

$$(9{,}82) \qquad Ma_{e,Normal} = Ma_e \cdot \sin\beta_1 = 1{,}1253$$

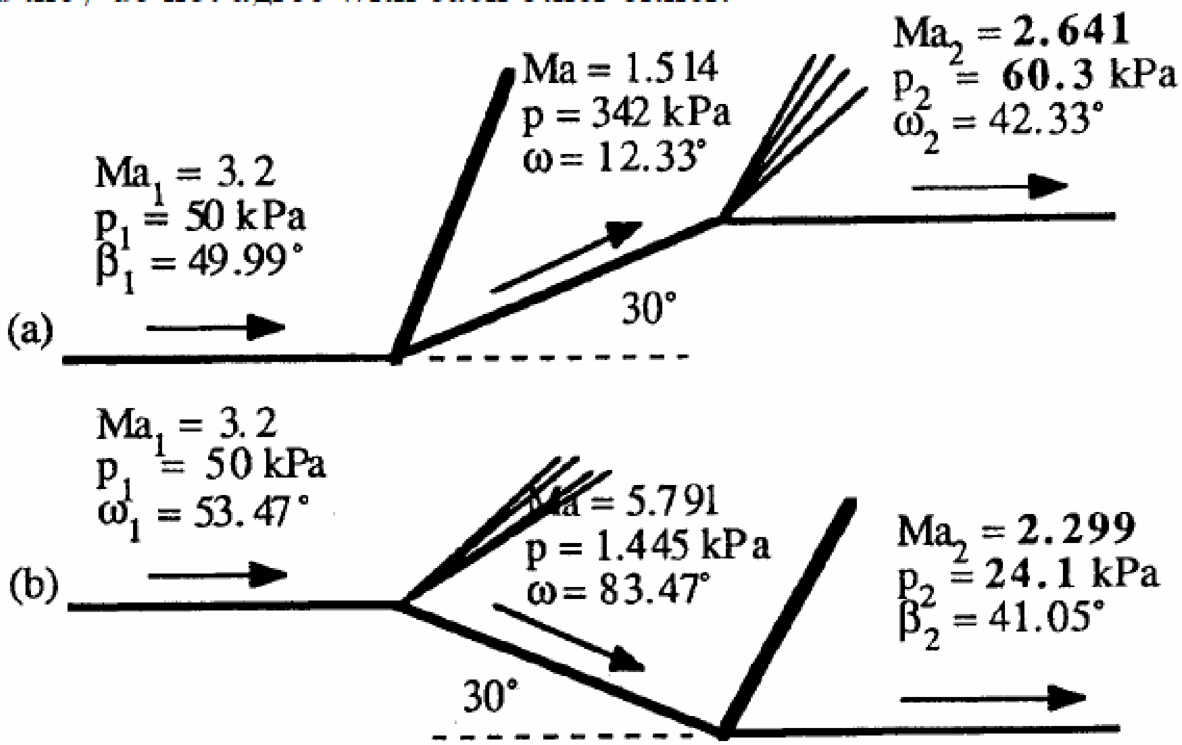
(9,83a) eller (9,55) ger trycket: $\frac{p_r}{p_e} = \frac{1}{k+1}\left(2k\,Ma_{normal}^2 - (k-1)\right)$ (9,55)

$\left\{ \begin{matrix} 9{,}34 \\ \text{luft!} \end{matrix} \right\} \Rightarrow p_e = \frac{p_0}{\left(1 + 0{,}2\,Ma_e^2\right)^{3{,}5}} = \ldots = 16{,}6 \text{ kPa} \Longrightarrow p_r = \frac{p_e}{k+1}\left(2k \cdot Ma_{e,normal}^2 - (k-1)\right) = \ldots =$

$= \underline{\underline{21{,}7 \text{ kPa}}}$

9.18

The solution is given in the form of the two sketches below. A shock wave with a 30° turn is a hugely non-isentropic flow, so the final conditions are nowhere near the original and they do not agree with each other either.



9.19

This is a real 'quickie' compared to what we have been doing for the past few problems. Isentropic expansion to a new pressure specifies the downstream Mach number:

$$p_o = p_1\left[1+0.2\,\mathrm{Ma}_1^2\right]^{3.5} = 100[1+0.2(2)^2]^{3.5} = 782\ \text{kPa}$$

$$p_2/p_o = \frac{50}{782} = 0.0639, \quad \text{read } \mathrm{Ma}_2 \approx 2.44, \quad \text{read } \omega_2 \approx 37.79°,$$

$$\text{while } \omega_1 \approx 26.38°, \quad \therefore\ \Delta\theta = 37.79 - 26.38 \approx \mathbf{11.4°} \quad \textit{Ans.}$$

9.20

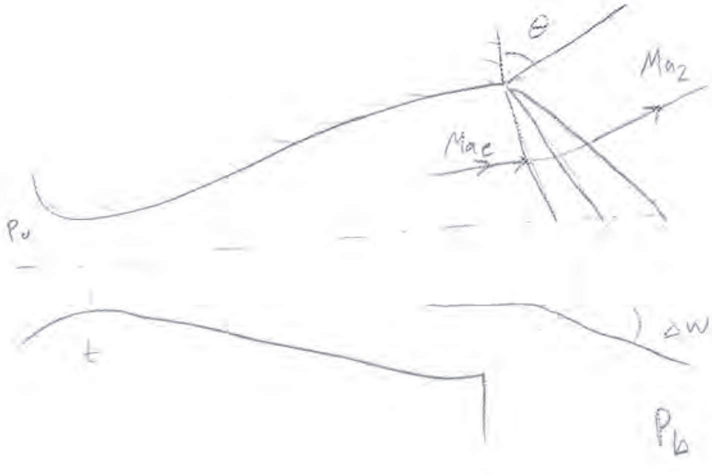
<u>Givet:</u>

$P_0 = 500$ kPa

$\frac{A_e}{A_t} = 4.0$

$P_b = 10$ kPa

(figur 9.12 "I")



<u>Sökt:</u>

a/ Mach-talet efter Prandtl-Meyer expansionen, Ma_2

b/ Vinkeln på jetstrålen precis efter utloppet, θ

<u>Lösning:</u>

a/ $\left\{ B.1 \ \& \ \frac{A_e}{A^*} = 4 \right\} \Rightarrow Ma_e = 2{,}94$

Strömningen expanderar isentropt från Ma_e till Ma_2

Prandtl-Meyer expansionsvinkeln ges av: (9.99)

$$\omega(Ma) = K^{1/2} \cdot \arctan\left(\frac{Ma^2 - 1}{K}\right)^{1/2} - \arctan\left(Ma^2 - 1\right)^{1/2}$$

där $K = \frac{k+1}{k-1}$

men för Mach-talet räcker det med isentrop exp:

$$\frac{P_0}{P_b} = \left(1 + 0{,}2 Ma_2^2\right)^{3{,}5} = \frac{500}{10} \Rightarrow \underline{Ma_2 = 3{,}208}$$

luft $k = 1{,}4$

b/ $\Delta w = w(Ma_2) - w(Ma_e)$

$$(B.5) \Rightarrow \begin{cases} w(Ma_e) = w(2{,}94) = 48{,}58° \\ w(Ma_2) = w(3{,}208) = 53{,}61° \end{cases}$$

$\Rightarrow \Delta w = 5{,}03°$

$\Rightarrow \underline{\theta = 90° - \Delta w = 85°}$