7.1

- Assuming laminar flow apply equation 7.1a-fs:

$$\frac{\delta}{x} \approx \frac{5}{Re_x^{1/2}} \Rightarrow \frac{\delta}{x} \approx \frac{5}{\sqrt{\frac{Ux}{\nu}}} \Rightarrow \text{solve for } \nu$$

$$\Rightarrow \frac{\delta^2}{x^2} \approx \frac{25}{\frac{Ux}{\nu}} \Rightarrow \frac{\delta^2}{25x^2} \approx \frac{\nu}{Ux} \Rightarrow$$

$$\Rightarrow \nu \approx \frac{\delta^2 U}{25x} \Longrightarrow \begin{array}{l} U = 2 m/s \\ \delta = 0{,}016 m \\ x = 1 m \end{array}$$

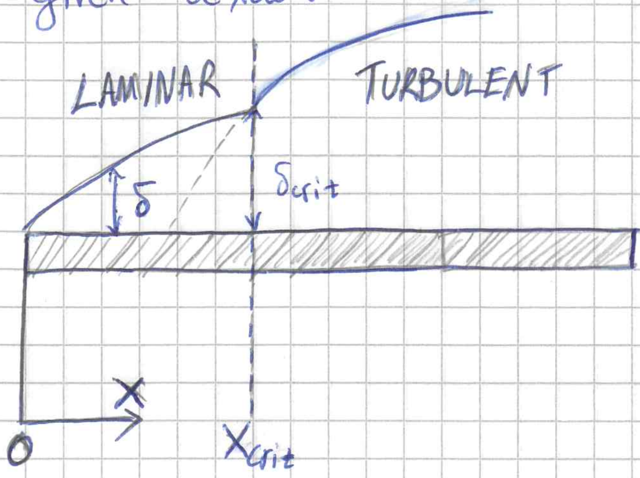
$\Rightarrow \nu \approx 2{,}048 \, 10^{-5} \, m^2/s$ Looking at table A.4 it is probably

$CH_4 \rightarrow \nu = 2{,}01 \, 10^{-5} \, m^2/s$

- Sanity check $Re_x = \frac{Ux}{\nu} = 97656{,}25 < 10^6$

7.2

- A sketch depicting the exercise suggestion is given below:



- At the point X_{crit} where turbulent boundary layer takes over, boundary layer thickness δ_{crit} is the same calculated by laminar or turbulent approximation

- Assume $Re_{x,crit} \simeq 1.2\,10^6$ to find X_{crit}

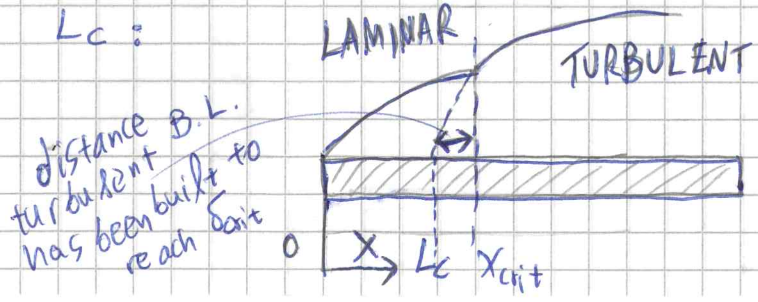
$$Re_{x,crit} \approx 1.2\,10^6 \Rightarrow \frac{\rho U x_{crit}}{\mu} \xRightarrow{\text{air@ }20^\circ C}$$

$$\Rightarrow 1.2\,10^6 \simeq \frac{1.2\;40\;x_{crit}}{1.8\;10^{-5}} \Rightarrow X_{crit} \simeq 0.45\,m$$

- Calculate the boundary layer thickness @ X_{crit} using the laminar approximation (7.1a-fs):

$$\frac{\delta_{crit}}{X_{crit}} \simeq \frac{5}{(Re_{x,crit})^{1/2}} \Rightarrow \delta_{crit} \simeq 0{,}00205\,m$$

- The turbulent boundary layer which will be δ_{crit} at X_{crit} has been building upstream from location L_c:



- Calculate the (X_{turb}) distance the turbulent boundary layer needs to develop to reach δ_{crit} → (7.16-fs)

$$\frac{\delta_{crit}}{X_{turb}} \simeq \frac{0.16}{(Re_{xturb})^{1/7}} \Rightarrow \text{solve for } x_{turb}$$

$$\Rightarrow \quad X_{turb} \simeq \left(\frac{\delta_{crit}}{0{,}16}\right)^{7/6} \left(\frac{\rho U}{\mu}\right)^{1/6} \Rightarrow$$

$$\Rightarrow \quad X_{turb} \simeq 0{,}0731 m$$

- To get the boundary layer thickness at $1.5m = X$ down stream from the plate's edge, implement the turbulent BL approximation (7.16-fs) but starting at position L_c.

This means that the development of turbulent boundary layer is not for 1.5m but

$$X_{eff} = X + X_{turb} - X_{crit} = 1.123 \ m$$

- For verification we can check the Re_{eff}:

$$Re_{eff} = \frac{\rho U . X_{eff}}{\mu} = \frac{1.2\ 40\ 1.123}{1.8\ 10^{-5}} = 2.995\ 10^{6}$$

above critical

- Use (7.16-fs) for X_{eff}:

$$\delta\Big|_{at\ X=1.5m} \simeq \frac{0.16 X_{eff}}{(R_{eff})^{1/7}} \simeq 0{,}0213m$$

is the boundary layer thickness at 1.5m

- If we use directly (7.16-fs) for $X = 1.5m$

$$\delta'\Big|_{at\ X=1.5m} \simeq \frac{0{,}16 X}{Re_x} \simeq 0{,}027m$$

which is 25 % over estimate!

7.3

: Given the profile approximation $u/U \approx 2\eta - \eta^2$, where $\eta = y/\delta$, compute

$$\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right) dy = \delta \int_0^1 (2\eta-\eta^2)(1-2\eta+\eta^2)\, d\eta = \frac{2}{15}\delta$$

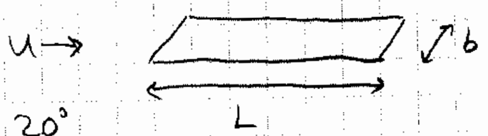
$$\delta^* = \int_0^\delta \left(1-\frac{u}{U}\right) dy = \delta \int_0^1 (1-2\eta+\eta^2)\, d\eta = \frac{1}{3}\delta$$

Hence $H = \delta^*/\theta = (\delta/3)/(2\delta/15) \approx$ **2.5** (compared to *2.59* for Blasius solution)

7.4

Givet: Plan platta, strömmar på bägge sidorna

$L = 1{,}1$ [m]
$b = 0{,}55$ [m]
$U = 6$ [m/s]
$\rho = 891$ [kg/m³]
$\mu = 2{,}9 \cdot 10^{-1}$ [kg/m s]



SAE 30 olja 20°

Sökt: Dragkraften på plattan om:

(a) L parallell med U

(b) b parallell med U

Lösning: Dragkraften orsakas av väggskjuvspänningen τ_w.

Den totala kraften på <u>en</u> sida är:

$$D(x) = b\int_0^x \tau_w(x)\, dx$$

Finns färdigintegrerat för en platta med längden L

$$D(L) = \frac{1}{2}\rho U^2 b L \underbrace{\frac{1{,}328}{Re_L^{\frac{1}{2}}}}_{C_D}$$

OBS! C_D för laminärt

för (a)

$$Re_L = \frac{\rho U L}{\mu} = \frac{891 \cdot 6 \cdot 1{,}1}{0{,}29} = 20\,300 < 10^6 \Rightarrow \text{laminärt}$$

$D(L) = 90{,}4 \; [N]$ per sida

$F_{drag} = 2 \cdot D(L) = \underline{\underline{180{,}8 \; [N]}}$

för ⓑ

$Re_b = \dfrac{891 \cdot 6 \cdot 0{,}55}{0{,}29} = 10\,140 < 10^6 \Rightarrow$ laminärt

$D(L) = 128 \, [N]$

$F_{drag} = 2 \cdot D(L) = \underline{\underline{256 \; [N]}}$

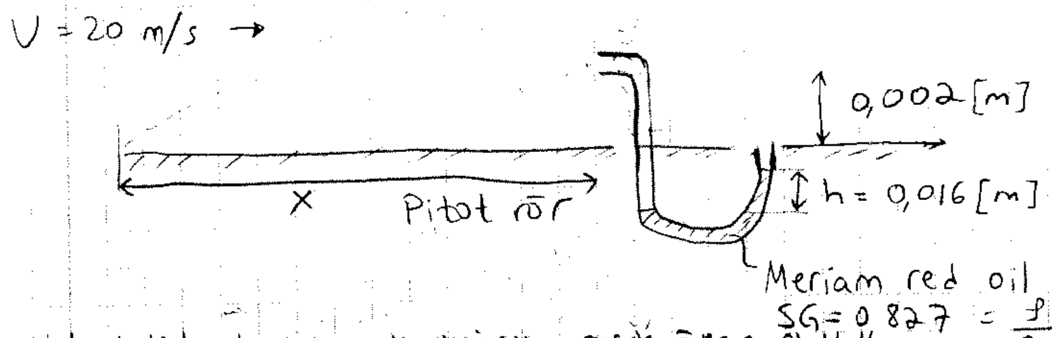
Vilket är 41% mer!

Detta beror på att $\tau_w \sim \dfrac{1}{\sqrt{x}}$ dvs avtar med ökande x → bättre med smal lång platta.

7.5

Givet:

luft 20° } Tabell $\rho = 1,2\ [kg/m^3]$
1 atm } $\nu = 1,5\cdot10^{-5}\ [m^2/s]$



det statiska trycket är konst. i ett gränssk. över en platta!

Sökt: Använd ovanstående information för att hitta x (positionen för Pitot röret). Antag laminär strömning.

Lösning: Antar att vi är i ett gränsskickt
Använder Blausius hastighetsprofil tabell (7.1) som gäller vid laminärt gränsskickt (givet)

I Blausius tabellen finns värden för

$\frac{u}{U}$ som korresponderar till $y\left(\frac{U}{\nu x}\right)^{\frac{1}{2}}$

u vid y=0,002 [m] kan hittas från Pitot röret och Bernoulli ⇒ I tabellen kan vi då också finna korresponderande värde för $y\left(\frac{U}{\nu x}\right)^{\frac{1}{2}}$, där x är enda okända variabeln.

Pitot rör mäter $P_{stagnation} - P_{at}$, $P_{stag} = P_{at} + \frac{1}{2}\rho u^2 \Rightarrow u = \sqrt{\frac{2(P_{st}-P_{at})}{\rho_{luft}}}$
(detsamma som Pitot static, då det statiska trycket är Pat i hela gräns-skickt)

$P_{st} - P_{at}$ = gå runt i röret

luft pelaren som sticker upp försummas (dvs. $0{,}002\cdot\rho_{luft} g$)

$$P_{st} - P_{at} = -\rho_{luft}\, g\,(h-\text{"0"}) - \rho_{olja}\cdot g\,(\text{"0"}-h) = (\rho_{olja} - \rho_{luft})\,gh$$

$$u = \sqrt{\frac{2(\rho_{olja} - \rho_{luft})\,gh}{\rho_{luft}}} = 14{,}68\ m/s$$

Vi vet nu att vi är i ett gräns skickt
$u < U = 20\ [m/s]$

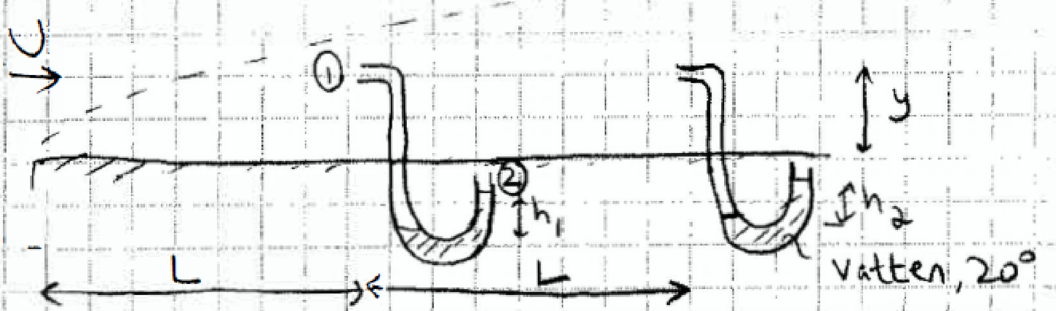
$$\frac{u}{U} = \frac{14,68}{20} = 0,734$$

får nu interpolera i tabellen (Blausius, 7.1)

$$y\left(\frac{U}{\nu x}\right)^{\frac{1}{2}} = 2,423$$

$$\rightarrow \quad x = \left(\frac{y}{2,423}\right)^2 \frac{U}{\nu} = \underline{\underline{0,908\ [m]}}$$

7.6



<u>Givet</u>: Laminärt gränsskickt, Luft 20°C [A2] → $\rho = 1,2\ kg/m^3$

$\nu = 1,8 \cdot 10^{-5}\ \left[\frac{Ns}{m^2}\right]$

$y = 2\ mm$
$U = 15\ m/s$
$L = 0,5\ m$

<u>Sökt</u>: h_1 och h_2

<u>Lösning</u>: Ur manometern fås $\quad \underset{(P_1 - P_2)}{P_{stagnation} - P_{at}} = \rho_{vatten}\, g\, h$

(P, statiska trycket är konst. i ett gränsskickt $P = P_{at}$)

Pitot röret mäter $P_{stag} - P_{at}$ (OBS! Inte Bernoulli mellan ① och ② som i Pitot static tube (se teorin))

$$= P_{stagnation} = P_{tot} = P_{at} + \frac{1}{2}\rho_{luft}\, u^2$$

$$P_{st} - P_{at} = \frac{1}{2} \rho_{luft} u^2$$

$$\rho_{vatten} g h = \frac{1}{2} \rho_{luft} u^2$$

$$h = \frac{\frac{1}{2} \rho_{luft} u^2}{\rho_{vatten} \cdot g}$$

$\Rightarrow$ Både h & u okända $\Rightarrow$ u kan fås ur Blausius då $y\left(\frac{U}{\nu x}\right)^{\frac{1}{2}}$ är känd för bägge punkterna

första pitot:

$\eta = y\left(\frac{U}{\nu x}\right)^{\frac{1}{2}}$ korresp. till $\frac{u_1}{U} = 0,861$ $u_1 = 0,861 \cdot 15 = 12,25$ [m/s]

efter interpolering

andra pitot:

$\eta = 2 \Rightarrow \frac{u_2}{U} = 0,63 \Rightarrow u_2 = 9,45$ [m/s]

$\Rightarrow h_1 = 9,2$ [mm] $\rightarrow$ pga högre hastighet vid det första röret!

$h_2 = 5,5$ [mm]

7.7 For laminar flow over any <u>one</u> wall of size *a* by *L*, we estimate

$$\frac{F_{\text{one wall}}}{(1/2)\rho U^2 aL} \approx \frac{1.328}{\sqrt{(\rho UL/\mu)}}, \quad \text{or} \quad F_{\text{one wall}} \approx 0.664\,(\rho\mu L)^{1/2} U^{3/2} a$$

Thus, for **4** walls and $\mathbf{N^2}$ boxes, $\quad F_{total} \approx \mathbf{2.656N^2(\rho\mu L)^{1/2}U^{3/2}a}$ *Ans.* (a)

The pressure drop across the array is thus

$$\Delta p_{array} = \frac{F_{total}}{(Na)^2} \approx \frac{\mathbf{2.656}}{\mathbf{a}}\mathbf{(\rho\mu L)^{1/2}\,U^{3/2}} \quad \textit{Ans. (b)}$$

This is *completely* different from the predicted Δp for laminar flow through a square duct, as in Section 6.8

$$\Delta p_{duct} = f\frac{L}{D_h}\frac{\rho}{2}U^2 = \left(\frac{56.91\mu}{\rho Ua}\right)\left(\frac{L}{a}\right)\frac{\rho}{2}U^2 \approx \frac{28.5\mu LU}{a^2} \quad (?)$$

This has almost <u>no</u> relation to *Answer* (b) above, being the Δp for a <u>long square duct filled with boundary layer</u>. Answer (b) is for a very <u>short</u> duct with thin wall boundary layers.

7.8 For sea-level air, take ρ = 1.205 kg/m³ and μ = 1.78E−5 kg/m·s. The analytical formulas for array drag and pressure drop are given above. Hence

$$F_{array} = 2.656N^2(\rho\mu L)^{1/2}U^{3/2}a = 2.656(20)^2[1.205(1.78E{-}5)(0.25)]^{1/2}(12)^{3/2}(0.04)$$

$$\text{or:}\quad \mathbf{F \approx 4.09\ N} \quad (Re_L = 203000,\text{OK, laminar}) \quad \textit{Ans. (a)}$$

$$\Delta p_{array} = \frac{F}{(Na)^2} = \frac{4.09}{[20(0.04)]^2} \approx \mathbf{6.4\ Pa} \quad \textit{Ans. (b)}$$

This is a far cry from the (much lower) estimate would have by assuming the array is a bunch of <u>long square ducts</u> as in Sect. 6.8 (as shown in Prob. 7.7):

$$\Delta p_{long\ duct} \approx \frac{28.5\mu LU}{a^2} = \frac{28.5(1.78E{-}5)(0.25)(12)}{(0.04)^2} \approx \mathbf{0.95\ Pa} \quad \text{(not accurate)} \quad \textit{Ans.}$$

7.9

Givet:

$$\frac{u}{U} = \sin \frac{\pi y}{2\delta}$$

Sökt: Upprepa von Karmans integral metod för laminär strömning med den givna hastighetsprofilen, räkna ut:

ⓐ impuls förlust tjockleken, θ, (momentum thickness)
ⓑ förträngnings tjockleken, δ^* (displacement thickness)
ⓒ $\frac{\delta}{x}$
ⓓ skjuvspännings koefficienten, C_f, (skin friction)

Lösning: Von Karman:

$$\begin{cases} \tau_w = \rho U^2 \dfrac{d\theta}{dx} & (7.5) \\ \theta = \displaystyle\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy & (7.3) \end{cases}$$

ⓐ $$\theta = \int_0^\delta \sin\left(\frac{\pi y}{2\delta}\right)\left(1 - \sin\left(\frac{\pi y}{2\delta}\right)\right) dy = \int_0^\delta \left(\sin\left(\frac{\pi y}{2\delta}\right) - \sin^2\left(\frac{\pi y}{2\delta}\right)\right) \cdot dy$$

$$= \{\text{från } \beta\} = \left[-\frac{2\delta}{\pi}\cos\frac{\pi y}{2\delta} - \left(\frac{y}{2} - \frac{\delta}{2\pi}\sin\left(\frac{\pi y}{\delta}\right)\right)\right]_0^\delta$$

$$\int \sin x = \frac{-\cos x}{x'}$$

$$= \left[-0 - \frac{\delta}{2} + 0\right] - \left[-\frac{2\delta}{\pi} - 0 + 0\right]$$

(b) $\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) \cdot dy$

$$= \int_0^\delta \left(1 - \sin\left(\frac{\pi y}{2\delta}\right)\right) \cdot dy = \left[y + \frac{2\delta}{\pi} \cos \frac{\pi y}{2\delta} \right]_0^\delta$$

$$= [\delta + 0] - \left[0 + \frac{2\delta}{\pi}\right] = \delta\left(1 - \frac{2}{\pi}\right) = \underline{\underline{0{,}3634\ \delta}}$$

(c) $\frac{\delta}{x} = ?$

$$\tau_w = \rho U^2 \frac{\partial \theta}{\partial x}$$

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu \frac{\partial}{\partial y} \left(\underbrace{U \sin \frac{\pi y}{2\delta}}_{\text{givna } u} \right)\Bigg|_{y=0}$$

$$= \mu \left[U \frac{\pi}{2\delta} \cos\left(\frac{\pi y}{2\delta}\right) \right]_{y=0} = \mu U \frac{\pi}{2\delta} \cdot 1$$

enl. andra uttrycket för τ_w (Karman's):

$$\tau_w = \rho U^2 \frac{\partial \theta}{\partial x} \underset{\downarrow}{=} \rho U^2 \left(\frac{4-\pi}{2\pi}\right) \frac{\partial \delta}{\partial x}$$

mha (a) $\theta = \delta\left(\frac{2}{\pi} - \frac{1}{2}\right) = \left(\frac{4-\pi}{2\pi}\right)\delta$

Sätter uttrycken för τ_w = varann $\Rightarrow$ $\mu U \frac{\pi}{2\delta} = \rho U^2 \left(\frac{4-\pi}{2\pi}\right) \frac{\partial \delta}{\partial x}$

$$\frac{\mu \pi}{2\delta} = \rho U \left(\frac{4-\pi}{2\pi}\right) \frac{\partial \delta}{\partial x} \Rightarrow \{\text{separabel diff. ekv.}\}$$

$$\frac{\mu \pi}{2} \frac{1}{\rho U} \left(\frac{2\pi}{4-\pi}\right) \cdot dx = \delta \; d\delta \qquad \{\text{integrera}\}$$

$$\frac{\mu \pi^2}{\rho U (4-\pi)} x + C = \frac{\delta^2}{2} + D \qquad \left\{ \begin{array}{l} D - C = C_1 \\ \delta(x=0) = 0 \\ 0 = 0 + C_1 \\ C_1 = 0 \end{array} \right\}$$

$\left\{\text{dividera med } x^2\right\}$ $$\underbrace{\frac{2\cdot\mu\pi^2}{(4-\pi)\rho U x}} = \left(\frac{\delta}{x}\right)^2$$

$$\left\{Re_x = \frac{\rho U x}{\mu}\right\} \Rightarrow \frac{2\cdot\pi^2}{(4-\pi)\quad Re_x}$$

$$\Rightarrow \frac{\delta}{x} = \frac{\sqrt{\frac{2\pi^2}{4-\pi}}}{\sqrt{Re_x}} = \underline{\underline{\frac{4{,}795}{\sqrt{Re_x}}}}$$

(d) $C_f = ?$

$$C_f = \frac{2\tau_w}{\rho U^2} \left\{\tau_w = \rho U^2 \frac{\partial\theta}{\partial x}\right\} = \frac{2\cdot\cancel{\rho U^2}\left(\frac{4-\pi}{2\pi}\right)\frac{\partial\delta}{\partial x}}{\cancel{\rho U^2}}$$

$$= \left(\frac{4-\pi}{\pi}\right)\frac{\partial\delta}{\partial x}$$

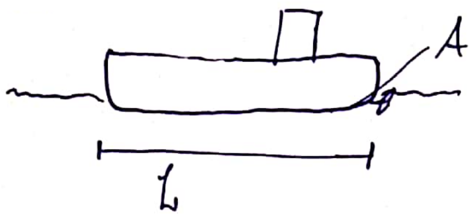
$$\left\{ \frac{\delta}{x} = \sqrt{\frac{2\pi^2}{4-\pi}} \cdot \frac{1}{\sqrt{Re_x}} = \sqrt{\frac{2\pi^2}{4-\pi}} \cdot \sqrt{\frac{\nu}{Ux}} \Rightarrow \delta = \sqrt{x}\cdot\sqrt{\frac{2\pi^2\nu}{(4-\pi)U}} \right.$$

$$C_f = \left(\frac{4-\pi}{\pi}\right)\frac{1}{2}\frac{1}{\sqrt{x}}\sqrt{\frac{2\pi^2\nu}{(4-\pi)U}} = \sqrt{\frac{(4-\pi)\cdot 2\nu}{U}}\cdot\frac{1}{2}\frac{1}{\sqrt{x}}$$

$$\left\{Re = \frac{Ux}{\nu}\right\} = \sqrt{\frac{(4-\pi)}{2\cdot Re_x}} = \underline{\underline{\frac{0{,}655}{Re_x^{0{,}5}}}}$$

sinus profilen stämmer bra överens med Blasius exakta profil $\Rightarrow C_f = \frac{0{,}664}{Re_x^{0{,}5}}$

7.10



Known:

$L = 125\,m$

$A = 3500\,m^2$ (wetted area)

$P = 1.1 \cdot 10^6\,W$

Sea water @ 20°C → $\begin{cases} \rho = 1025\,kg/m^3 \\ \mu = 0{,}00107\,\frac{kg}{m \cdot s} \end{cases}$

Task: Estimate the ship speed in kn

Assume: Nigligable effects from air, turbulent flow, smooth wall ($\varepsilon = 0$!)

Solution: Use the definition of power and drag coeff.

$$P = F \cdot V \quad [A] \quad \left[P = \frac{dE}{dt},\ E = F \cdot L \Rightarrow P = F \cdot \frac{L}{t} = F \cdot V\right]$$

$$(7.666) \quad C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} \rightarrow F = \frac{1}{2} C_D \rho V^2 A \quad [B] \quad C_D?$$

$$(7.45) \quad C_D = \frac{0{,}031}{Re_L^{1/7}} = \frac{0{,}031}{\left(\frac{\rho V L}{\mu}\right)^{1/7}} \quad \text{With } [A] \text{ \& } [B]$$

$$P = \frac{1}{2} \cdot \frac{0{,}031}{\left(\frac{\rho V L}{\mu}\right)^{1/7}} \cdot \rho V^2 A \cdot V \Rightarrow$$

$$V^{20/7} = \frac{2P}{0{,}031 A} \cdot \left(\frac{L}{\mu \rho^6}\right)^{1/7} \Leftrightarrow$$

$$V = \left(\frac{2P}{0{,}031 A}\right)^{7/20} \left(\frac{L}{\mu \rho^6}\right)^{1/20} = 7{,}2\,m/s$$

Check Re

$$Re = \frac{\rho V L}{\mu} = 8 \cdot 10^8$$

Most likely turbulent!

Appendix C: $knot = kn = \frac{m/s}{0.5144} \Rightarrow 7{,}2\,m/s = \underline{14\,kn}$

7.11

Known: Appendix G

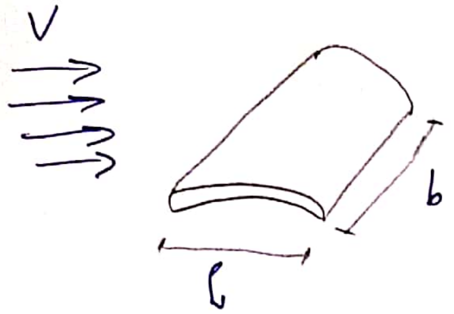
$V = 28kn = 28 \cdot 0.5144\ m/s = 14,4\ m/s$

$b = 4m$

$L = 0.5m$

Seawater @ 20°C Table A.3 $\begin{cases} \rho = 1025\ kg/m^3 \\ \mu = 0.00107\ kg/ms \end{cases}$

$Re_{trans} = 5 \cdot 10^5$



Task: Calculate the (F_D) drag for a) Smooth wall, b) rough wall, $\varepsilon = 0,3 \cdot 10^{-3}m$

Solution: Check Re and use C_D for flat plate theory.

(1.24) $Re = \frac{\rho V L}{\mu} = 6.9 \cdot 10^6 > Re_{trans} \rightarrow$ Turbulent.

both sides!

(7.66) $C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A_p} = [A_p = L \cdot b] \Rightarrow F_D = C_D \frac{\rho}{2} V^2 L \cdot b \cdot 2$ [A]

For smooth plate use (7.45)

$C_D = \frac{0,031}{Re_L^{1/7}}$ with [A]

$F_D = \frac{0,031}{Re_L^{1/7}} \frac{\rho}{2} V^2 L b 2 = 1390\ N$

b) For rough wall, with $\frac{L}{\varepsilon} = 1667$ and $Re = 6.9 \cdot 10^6$ in Fig 7.6

$\Rightarrow$ Fully rough region $\rightarrow$ Use Eq. (7.48b)

(7.48b) $C_D = \left[1,89 + 1.62 \log_{10}\left(\frac{L}{\varepsilon}\right)\right]^{-2,5}$ with [A]

$F_D = \left[1.89 + 1.62 \log_{10}\left(\frac{L}{\varepsilon}\right)\right]^{-2,5} \frac{\rho}{2} V^2 L b \cdot 2 = 3154$

I.e that small roughness increased the drag 2,3 times!

7.12

For air at 20°C, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8\text{E}{-}5$ kg/m·s. Assume a *smooth* beach and use the log-law velocity profile, Eq. (7.34), given u = 10 m/s at y = 80 m:

$$\frac{u}{u^*} = \frac{10\text{ m/s}}{u^*} \approx \frac{1}{\kappa}\ln\left(\frac{yu^*}{\nu}\right) + B = \frac{1}{0.41}\ln\left(\frac{80u^*}{1.5\text{E}{-}5}\right) + 5.0, \quad \text{solve } u^* \approx 0.254\text{ m/s}$$

$$\text{Hence} \quad \tau_{surface} = \rho u^{*2} = (1.2)(0.254)^2 \approx \mathbf{0.0772\ Pa} \quad \textit{Ans.}$$

The log-law should be valid as long as we stay above y such that $yu^*/\nu > 50$:

$$\text{(a)}\ \ y = 1.7\text{ m:} \quad \frac{u}{0.254} \approx \frac{1}{0.41}\ln\left[\frac{1.7(0.254)}{1.5\text{E}{-}5}\right] + 5, \quad \text{solve } u_{1.7\,m} \approx \mathbf{7.6\ \frac{m}{s}} \quad \textit{Ans.}\text{ (a)}$$

$$\text{(b)}\ \ y = 17\text{ cm:} \quad \frac{u}{0.254} \approx \frac{1}{0.41}\ln\left[\frac{0.17(0.254)}{1.5\text{E}{-}5}\right] + 5, \quad \text{solve } u_{17\,cm} \approx \mathbf{6.2\ \frac{m}{s}} \quad \textit{Ans.}\text{ (b)}$$

The (b) part seems <u>very</u> close to the surface, but $yu^*/\nu \approx 2800 > 50$, so the log-law is OK.

7.13

For seawater at 20°C, take $\rho = 1025$ kg/m^3 and $\mu = 0.00107$ kg/m·s. Convert 15 kn = 7.72 m/s. Evaluate $\text{Re}_L = (1025)(7.72)(150)/(0.00107) \approx 1.11\text{E}9$ (turbulent). Then

$$F = \frac{\text{Power}}{U} = \frac{5.22\text{E}6\text{ W}}{7.72} = 6.76\text{E}5\text{ N}, \quad C_D = \frac{2F}{\rho U^2 A} = \frac{2(6.76\text{E}5)}{1025(7.72)^2(5000)} \approx 0.00443$$

Fig. 7.6 or Eq. (7.48b):

$$\frac{L}{\varepsilon} \approx 16800, \quad \varepsilon_{barnacles} = \frac{150}{16800} \approx \mathbf{0.0089\ m} \quad \textit{Ans.}\text{ (a)}$$

If the surface were smooth, we could use Eq. (7.45) to predict a higher ship speed:

$$P = FU = \left[C_D \frac{\rho U^2}{2} A\right] U = \left\{\frac{0.031}{[1025U(150)/.00107]^{1/7}}\right\}\left(\frac{1025}{2}\right) U^2 (5000) U,$$

$$\text{or:} \quad P = 5.22\text{E}6\text{ watts} = 5428U^{20/7}, \quad \text{solve for } U = 11.1\text{ m/s} \approx \mathbf{22\ knots} \quad \textit{Ans.}\text{ (b)}$$

7.14

- For air @ $20^\circ C$ from table A.3

  $\rho = 1.2\ Kg/m^3$ $\mu = 1.8\,10^{-5}\ Kg/ms$

- Convert given velocity to SI:

  $80\ Km/h = 22.\overline{22}\ m/s$

- Check Reynolds number for this velocity approximating the cable as a cylinder (use diameter)

  $$Re_D = \frac{1.2\ (22.22)\,0.6}{1.8\ 10^{-5}} = 888\ 888$$

- Assume turbulent flow and 2D approximation ($L \gg D$)

- For cylinder (2D) take C_D from table 7.2

  $C_D = 0.3$ (or figure 7.16_a)

- Implement (7.66b fs) for the exposed surface:

  $$F_{drag} = C_D\, \rho \tfrac{1}{2}\, U^2 D L = 0.3\ \frac{1.2}{2}(22.22)^2\ 90 \cdot 0.6$$

  $\Rightarrow \quad F_{drag} = 4800\ N$

7.15

For sea-level air, take ρ = 1.225 kg/m^3 and μ = 1.78E–5 kg/m·s. Convert 90 mi/h = 40.2 m/s. We cannot compute Re without knowing the side length a, so we assume that Re > 1E4 and that Table 7.2 is valid. The worst case drag is when the square cylinder has its *flat* face forward, $C_D \approx 2.1$. Then the drag force is

$$F = C_D \frac{\rho}{2} U^2 aL = 2.1\left(\frac{1.225}{2}\right)(40.2)^2 a(52) \stackrel{?}{=} 90000 \text{ N}, \quad \text{solve } \mathbf{a \approx 0.83\ m} \quad \textit{Ans.}$$

$$\text{Check } Re_a = (1.225)(40.2)(0.83)/(1.78E{-}5) \approx 2.3E6 > 1E4, \text{ OK.}$$

7.16

For the standard altitude (Table A-6), read ρ = 1.112 kg/m^3 at 1000 m altitude and ρ = 1.0067 kg/m^3 at 2000 meters. Viscosity is not a factor in Table 7.3, where we read $C_D \approx 1.2$ for a low-porosity chute. If acceleration is negligible,

$$W = C_D \frac{\rho}{2} U^2 \frac{\pi}{4} D^2, \quad or: \quad 90(9.81)\ N = 1.2\left(\frac{\rho}{2}\right) U^2 \frac{\pi}{4}(8.5)^2, \quad or: \quad U^2 = \frac{25.93}{\rho}$$

$$\text{Thus} \quad U_{1000\ m} = \sqrt{\frac{25.93}{1.1120}} = \mathbf{4.83\ \frac{m}{s}} \quad \text{and} \quad U_{2000\ m} = \sqrt{\frac{25.93}{1.0067}} = \mathbf{5.08\ \frac{m}{s}}$$

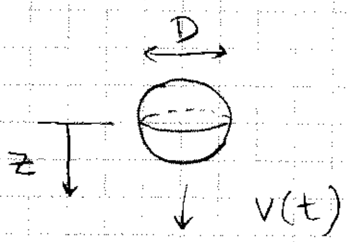
Thus the change in velocity is very small (an average deceleration of only –0.001 m/s^2) so we can reasonably estimate the time-to-fall using the average fall velocity:

$$\Delta t_{fall} = \frac{\Delta z}{V_{avg}} = \frac{2000 - 1000}{(4.83 + 5.08)/2} \approx \mathbf{202\ s} \quad \textit{Ans.}$$

7.17

Givet:

En sfär



ρ_s = sfärens densitet
ρ = fluidens densitet
μ = fluidens viskositet
D = sfärens diameter

Anta att C_D är konstant $= C_{D0}$
(sfärens form motstånds koefficient)
→ befinner oss på "platån" (se $C_D(Re)$ grafen i boken för sfär)

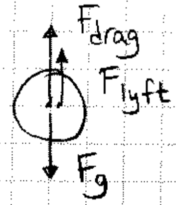
Sökt: Härled en diff. ekv. för fallhastigheten $v(t)$ och visa att

$$v(t) = \left[\frac{4 \cdot g \cdot D\,(s-1)}{3\,C_{D0}}\right]^{1/2} \tanh(Ct)$$

där $$C = \left[\frac{3g\,C_{D0}\,(s-1)}{4s^2 D}\right]^{1/2}$$ då $S = \frac{\rho_s}{\rho}$

Lösning:

Ställ upp Newtons andra lag för sfären



$$\begin{cases} \sum F_x = m \cdot a = \rho_s \cdot \underbrace{\frac{\pi D^3}{6}}_{\text{Volym}} \cdot a \\ a = \frac{dV}{dt} \\ \sum F = F_g - F_d - F_l \end{cases}$$

$$\begin{cases} F_g = m \cdot g = \rho_s \underbrace{\frac{\pi D^3}{6}}_{\text{Volym}} \cdot g \\ F_d = C_{D0} \frac{\rho V^2(t)}{2} \cdot \underbrace{\frac{\pi D^2}{4}}_{\text{Area}} \\ F_l = \rho \underbrace{\frac{\pi D^3}{6}}_{\text{Volym}} \cdot g \quad \text{(undanträngd fluid)} \end{cases}$$

$\Rightarrow$

$$\rho_s \frac{\pi D^3}{6} \frac{dV}{dt} = \frac{\pi D^3}{6} \cdot g(\rho_s - \rho) - C_{Do}\, \rho \frac{V^2(t)}{2} \frac{\pi D^2}{4}$$

dela med $\frac{\pi D^3}{6}$

$$\Rightarrow \rho_s \frac{dV}{dt} = g(\rho_s - \rho) - \frac{C_{Do}\, \rho\, V^2(t) \cdot 6}{D \cdot 8}$$

$$\frac{dV}{dt} = g \cdot \frac{(\rho_s - \rho)}{\rho_s} - \frac{C_{Do}\, \rho\, V^2(t) \cdot 3}{D \rho_s \quad 4}$$

$$\left\{ \frac{\rho_s}{\rho} = S \right\} \Rightarrow \frac{dV}{dt} = \underbrace{\left(1 - \frac{1}{S}\right)}_{\frac{S-1}{S}} \cdot g - \frac{1}{S} \frac{C_{Do} \cdot 3}{4D} \cdot V^2(t)$$

Sätt $\alpha = \frac{(S-1)}{S} \cdot g$ & $\beta = \frac{3\, C_{Do}}{S \cdot 4D}$

$$\Rightarrow \frac{dV}{dt} = \alpha - \beta V^2(t) \qquad \{\text{separabel diff. ekv.}\}$$

$$\int_0^V \frac{1}{\alpha - \beta V^2(t)} \cdot dV = \int_0^t dt$$

Beta ger:

$$t = \frac{1}{\sqrt{\alpha\beta}} \operatorname{arctanh}\left(\sqrt{\frac{\beta}{\alpha}} \cdot V\right) \Rightarrow V = ?$$

gångrar med $\sqrt{\alpha\beta}$

$$\sqrt{\alpha\beta} \cdot t = \operatorname{arctanh}\left(\sqrt{\frac{\beta}{\alpha}} \cdot V\right)$$

tar tanh på bägge led:

$$\tanh\left(\sqrt{\alpha\beta}\, t\right) = \sqrt{\frac{\beta}{\alpha}} \cdot V \Rightarrow \underline{V(t) = \sqrt{\frac{\alpha}{\beta}} \cdot \tanh\left(\sqrt{\alpha\beta} \cdot t\right)}$$

Sätter in värdena på α & β:

$$v(t) = \sqrt{\frac{\frac{(\cancel{s}-1)}{\cancel{s}}\cdot g}{\frac{3C_{D0}}{\cancel{s}\,4D}}}\cdot \tanh\left(\sqrt{\frac{(s-1)}{s}g\cdot\frac{3C_{D0}}{s\cdot 4D}}\,t\right)$$

$$= \sqrt{\frac{(s-1)\cdot g\cdot 4D}{3C_{D0}}}\cdot \tanh\left(\sqrt{\frac{3\cdot g\,(s-1)\,C_{D0}}{s^2\,4D}}\cdot t\right)$$

V.S.V.

7.18 For sea-level air, take $\rho = 1.225\ \text{kg/m}^3$ and $\mu = 1.78\text{E}{-5}$ kg/m·s. Assume a laminar drag coefficient $C_D \approx 0.47$ from Table 7.3. Convert $\Omega = 400$ rpm $\times\ 2\pi/60 = 41.9$ rad/s. Each ball moves at a centerline velocity

$$V_b = \Omega r_b = (41.9)(0.28 + 0.0735/2) \approx 13.3\ \text{m/s}$$

$$\text{Check}\quad \text{Re} = 1.225(13.3)(0.0735)/(1.78\text{E}{-5}) \approx 67000; \quad \text{Table 7.3: } C_D \approx 0.47$$

Then the drag force on each baseball is approximately

$$F_b = C_D\frac{\rho}{2}V_b^2\frac{\pi}{4}D^2 = 0.47\left(\frac{1.225}{2}\right)(13.3)^2\frac{\pi}{4}(0.0735)^2 \approx \mathit{0.215}\ \text{N}$$

Make a similar approximate estimate for the drag of each rod:

$$V_r = \Omega r_{avg} = 41.9(0.14) \approx 5.86\ \frac{\text{m}}{\text{s}}, \quad \text{Re} = \frac{1.225(5.86)(0.007)}{1.78\text{E}{-5}} \approx 2800, \quad C_D \approx 1.2$$

$$F_{rod} \approx C_D\left(\frac{\rho}{2}\right)V_r^2 DL = 1.2\left(\frac{1.225}{2}\right)(5.86)^2(0.007)(0.28) \approx \mathit{0.0495}\ \text{N}$$

Then, with two balls and two rods, the total driving power required is

$$P = 2F_bV_b + 2F_rV_r = 2(0.215)(13.3) + 2(0.0495)(5.86) = 5.71 + 0.58 \approx \mathbf{6.3\ W} \quad \textit{Ans.}$$

7.19 For sea-level air take $\rho = 1.225\ \text{kg/m}^3$. From Table 7.3 for a parachute, read $C_{Dp} \approx 1.2$. The force balance during deceleration is, with $V_o = 50$ m/s,

$$\sum F = -F_{roll} - F_{drag} = -5000 - \frac{1.225}{2}\left(0.4 + 1.2\frac{\pi}{4}D_p^2\right)V^2 = (ma)_{car} = 1500\frac{dV}{dt}$$

Note that, if drag = 0, the car slows down linearly and stops in 50(1500)/(5000) = **15 s**, not fast enough—so we definitely need the drag to cut it down to 8 seconds. The first-order differential equation above has the form

$$\frac{dV}{dt} = -b - aV^2, \quad \text{where } a = \frac{1.225}{2}\left(\frac{0.4 + 1.2\pi D_p^2/4}{1500}\right) \quad \text{and} \quad b = \frac{5000}{1500}$$

Separate the variables and integrate, with $V = V_o = 50$ m/s at $t = 0$:

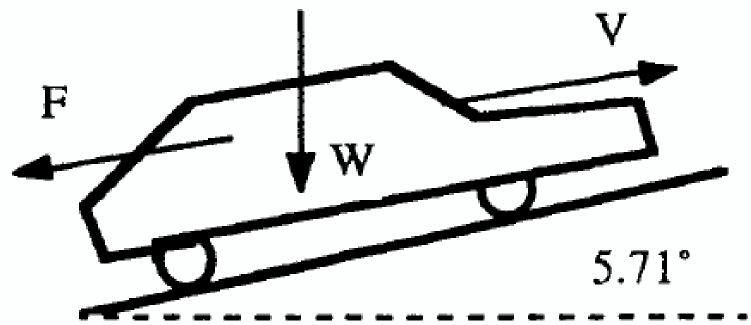
$$\int_{V_o}^{0} \frac{dV}{b + aV^2} = -\int_0^t dt, \quad \text{Solve: } t = \frac{1}{\sqrt{ab}}\tan^{-1}\left(V_o\sqrt{\frac{a}{b}}\right) = 8\ s?$$

The unknown is D_p, which lies within *a*! Iteration is needed—an ideal job for EES! Well, anyway, you will find that $D_p = 3$ m is too small ($t \approx 9.33$ s) and $D_p = 4$ m is too large ($t \approx 7.86$ s). We may interpolate (or EES will quickly report):

$$\mathbf{D_{parachute(t=8\ s)} \approx 3.9\ m} \quad \textit{Ans.}$$

*ESS= Engineering Equation Solver

7.20



For sea-level air, take $\rho = 1.225$ kg/m^3 and $\mu = 1.78\text{E}{-5}$ kg/m·s. If x denotes *uphill*, the equation of motion is

$$m\frac{dV}{dt} = -W\sin\theta - F_{rolling} - C_D A\frac{\rho}{2}V^2, \quad \text{separate the variables and integrate:}$$

$$V = V_f \tan\left[\tan^{-1}\left(\frac{V_o}{V_f}\right) - t\frac{W\sin\theta + F_r}{mV_f}\right], \quad \text{where } V_f = \sqrt{\frac{W\sin\theta + F_r}{C_D A\rho/2}}$$

For the particular data of this problem, we evaluate

$$V_f = \sqrt{\frac{9810\sin 5.71° + 70}{0.7(1.225/2)}} \approx 49.4\frac{m}{s}, \quad \frac{W\sin\theta + F_r}{mV_f} = \frac{9810\sin 5.71° + 70}{1000(49.4)} \approx 0.0212$$

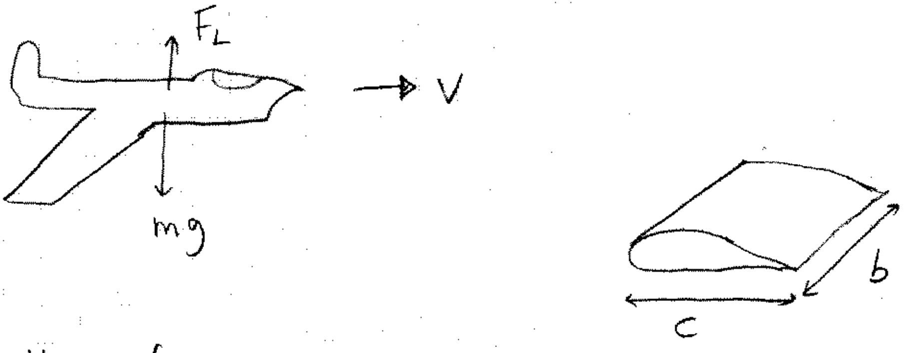
$$\text{also } \tan^{-1}\left(\frac{25}{49.4}\right) = 0.469 \text{ radians. So, finally, } \quad V \approx 49.4\tan[0.469 - 0.0212t]$$

The car stops at V = 0, or $t_{final} = 0.469/0.0212 \approx$ **22.1** s. The distance to stop is computed by the same formula as in Prob. 7.98:

$$\Delta x_{max} = \frac{m}{\rho C_D A}\ln\left[1 + \left(\frac{V_o}{V_f}\right)^2\right] = \frac{1000}{1.225(0.7)}\ln\left[1 + \left(\frac{25}{49.4}\right)^2\right] \approx \mathbf{266\ m} \quad \textit{Ans.}$$

7.21

Givet:



$V = 112$ m/s

$A_p = b \cdot c = 160\ m^2$ (för bägge vingarna)

korda $\bar{c} = 4$ m (medel)

Planet flyger på 3000 m. höjd $\Rightarrow$ Tabell A6
$\rho = 0{,}9092\ kg/m^3$

$F_g = 180$ kN

Använd vingdata från Fig. 7.25 & 7.26 (NACA 0009, no flap)
(oändligt lång vinge)

Sökt: kraften som krävs för att driva planet framåt med konstant hastighet

Lösning: $\Sigma F_x = F_{driv} - F_D = 0 \rightarrow F_{driv} = F_D$ (dragkraften)

$$F_D = C_D \rho \frac{1}{2} V^2 A_p$$

$C_{D\,plan} = ?$ → Bara A_p = vingarean given
$A_{plan} = ?$ Approximerar $C_{D_{plan}} \approx C_{D\,vingarna}$ → en enda lång vinge!!!

$\Rightarrow$ Kan inte räkna ut C_D direkt, $C_D(\alpha)$
anfallsvinkeln.

$\alpha = ?$

Kan fås ur kraftbalans i y-led (då F_g är given)

$\Sigma F_y = 0$

$F_g = F_L$ (gravitationskraft = Lyftkraft)

$$mg = \frac{1}{2} \rho C_L A_p V^2 \Rightarrow C_L = \frac{2mg}{A_p V^2 \rho} = \frac{2 \cdot 180 \cdot 10^3}{160 \cdot 112 \cdot 0{,}9} = 0{,}198$$

Avläs i Figur 7.25. Vilken vinkel (α) ger $C_L = 0{,}198$

$\Rightarrow \alpha = 1{,}8°$

Figur 7.26 ger C_D för en standardvinge mot C_L för oändligt långa vingar

$C_L = 0{,}2 \Rightarrow C_{D\infty} = 0{,}006$ (oändligt lång vinge)

$\Rightarrow$ Kompensera för att vingarna inte är oändligt långa:

(ekv. 7.71) $C_D \approx C_{D,\infty} + \dfrac{C_L^2}{\pi AR}$; $AR = \dfrac{b}{c} = \dfrac{A_p}{c^2}$ (ekv. 7.68)

$$C_D \approx 0{,}006 + \frac{0{,}198^2 \cdot 4^2}{\pi\ 160} = 0{,}00725$$

$$F_D = C_D \cdot \rho \frac{1}{2} V^2 A_p = \ldots = 6{,}6 \text{ kN}$$

$(P = F_D \cdot V = 6600 \cdot 112 = 739\,200 = 991 \text{ hp})$

7.22

For air at sea level, $\rho \approx 1.225\ \mathrm{kg/m^3}$. From Fig. 7.24 with the flap, $C_{L,max} \approx$ 1.75 at $\alpha \approx 6°$. Compute the stall velocity:

$$V_{stall} = \sqrt{\frac{2W}{\rho C_{L,\max} A_p}} = \sqrt{\frac{2(180000\ \mathrm{N})}{(1.225\ \mathrm{kg/m^3})(1.75)(160\ \mathrm{m^2})}} = 32.4\ \frac{\mathrm{m}}{\mathrm{s}}$$

$$\text{Then}\quad V_{landing} = 1.2V_{stall} = \mathbf{38.9\ \frac{m}{s}} \quad \textit{Ans.}\ \text{(a)}$$

$$C_L = \frac{C_{L,\max}}{(V_{land}/V_{stall})^2} = \frac{1.75}{(1.2)^2} = 1.22$$

For take-off at the same speed of 38.9 m/s, we need a drag estimate. From Fig. 7.25 ***with*** a split flap, $C_{D\infty} \approx 0.04$. We don't have a theory for induced drag with a split flap, so we just go along with the usual finite wing theory, Eq. (7.71). The aspect ratio is $b/c = (40\ \mathrm{m})/(4\ \mathrm{m}) = 10$.

$$C_D = C_{D\infty} + \frac{C_L^2}{\pi AR} = 0.04 + \frac{(1.22)^2}{\pi(10)} = 0.087,$$

$$F_{drag} = (0.087)\left(\frac{1.225}{2}\right)(38.9)^2(160) = 12900\ \mathrm{N}$$

Power required = FV = (12900 N)(38.9 m/s) = 501000 W = **672 hp** *Ans.* (b)

version1