5.1 Given: prototyp conditions: $\left.\begin{array}{l}T_p = 10°C = 283K \\ P_p = 83\ kPa\end{array}\right\} \rho, \mu$

model condition: $T_m = 20°C = 293\ K$

$P_m = 1atm = 101.325\ kPa$

$V_m = 70\ m/s$

Scale 1:7

Asked: V_p

Dynamical similarity: $Re_p = Re_m$ $C_{D_p} = C_{D_m}$

$$Re = \frac{\rho V L}{\mu} \quad (1.24)$$

μ for the condition can be found from eq 1.27

$$\frac{\mu}{\mu_0} = \frac{(T/T_0)^{3/2}(T_0+S)}{T+S}$$

$T_0 = 273K$, $\mu_0 = \underbrace{\mu_{@T=273}}_{\text{found in A:2 air!}} = 1.71 \cdot \underbrace{10^{-5}}_{kg/ms}$ $S = 110.4K$

for prototype:

$$\frac{\mu_p}{\mu_0} = \frac{(T_p/T_0)^{3/2}(T_0+S)}{T_p+S} \Rightarrow \mu_p \approx 1.76 \cdot 10^{-5}\ kg/ms$$

ink!!!

for model $\Rightarrow \mu_m \approx 1.81 \cdot 10^{-5}\ kg/ms$

ρ from perfect gas law

$$\rho = \frac{P}{RT} \quad (1.10) \qquad R = 287\ m^2/s^2K$$ from Table A4 – dry air.

for prototype: $\rho_p = \frac{P_p}{RT_p} = 1.022\ kg/m^3$

for model $\rho_m = \frac{P_m}{RT_m} = 1.205\ kg/m^3$

$Re_p = Re_m$

$$\frac{\rho_p V_p L_p}{\mu_p} = \frac{\rho_m V_m L_m}{\mu_m}$$

L_p and L_m unknown

but $\frac{L_m}{L_p} = \frac{1}{7}$

$$V_p = \frac{\mu_p}{\mu_m} \frac{\rho_m}{\rho_p} \frac{L_m}{L_p} V_m = 11.5\,m/s$$

5.2

Given: model: Water @ 20°C $\Rightarrow \rho_m = 998\ kg/m^3$, $\mu_m = 0.001\ kg/ms$ Table A1

$V_m = 2\ m/s$ $\quad d_m = 8cm = 0.08m$ $\quad F_{D_m} = 5N$

balloon: sea-level standard air $\Rightarrow \rho_b = 1.2255\ kg/m^3$ $\mu_b = 1.78 \cdot 10$ table A6.

$d_b = 1.5\ m$

Asked: V_b and F_{Db}

$$Re_m = \frac{\rho_m V_m d_m}{\mu_m} \quad (1.24) \quad Re_m = 1.6 \cdot 10^5$$

$$\underbrace{\quad}_{\text{known}}$$

$$\underset{\substack{\parallel \\ Re_m}}{Re_b} = \frac{\rho_b \overset{\text{asked}}{V_b} d_b}{\mu_b} \Rightarrow V_b = \frac{Re_m \mu_b}{\underbrace{\rho_b d_b}_{\text{known}}} \approx 1.55\ m/s$$

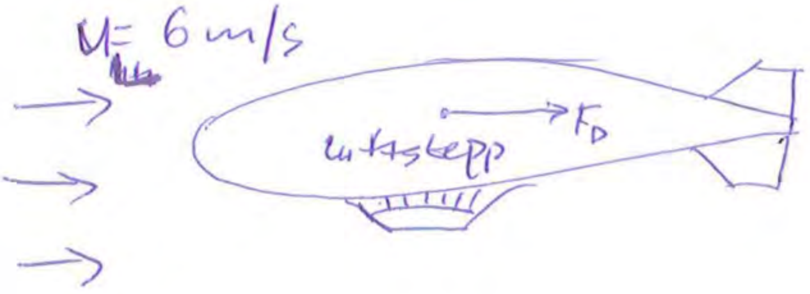
since dynamically similar!

$$C_{Dm} = \underbrace{\frac{F_{Dm}}{\frac{1}{2}\rho_m V_m^2 \frac{d_m^2 \pi}{4}}}_{\text{known}} \quad (5.26) \quad C_{Dm} = 0.498$$

$$\underset{\substack{\parallel \\ C_{Dm}}}{C_{Db}} = \frac{\overset{\text{asked}}{F_{Db}}}{\frac{1}{2}\rho_b V_b^2 \frac{1}{4} d_b^2 \pi} \Rightarrow F_{Db} = \frac{1}{8} C_{Dm} \rho_b V_b^2 d_b^2 \pi = 1.29N$$

since dynamically similar

5.3



$T = 20°C$

Vi har skalmodell i vatten (20°C) i skal 1:30 (dvs 30 ggr mindre).

Sökt:

a) Vad skall hastigheten för modellen vara? (i vatten)

b) Vad är luftmotståndet F_D i det verkliga luftskeppet om modellens luftmotstånd $F_{D,m} = 2700$ N?

c) Vilken effekt krävs för att driva luftskeppet?

Lösning

Vi använder oss av dynamisk likformighet!

Dvs då $\boxed{Re_L = Re_m}$ och $\boxed{C_{DL} = C_{Dm}}$

a)

$$Re_L = \frac{\rho_{luft} \cdot u_L \cdot L_L}{\mu_{luft}} = \frac{\rho_{vatten}\, u_M \cdot L_M}{\mu_{vatten}} = Re_m$$

$$u_M = \frac{\rho_{luft}}{\rho_{vatten}} \cdot \frac{\mu_{vatten}}{\mu_{luft}} \cdot \underbrace{\frac{L_L}{L_M}}_{30} u_L = \begin{cases} \rho_{luft} = 1.2 \text{ kg/m}^3 \\ \mu_{luft} = 1.8 \cdot 10^{-5} \text{ kg/ms} \\ \rho_{vatten} = 998 \text{ kg/m}^3 \\ \mu_{vatten} = 0.001 \text{ kg/ms} \end{cases}$$

$u_m = 12$ m/s ✓

5

b) Vi söker kraften F_D på verkliga luftskeppen

$$C_{DL} = C_{DM} \rightarrow \frac{F_{DL}}{\frac{1}{2}\rho_{luft} U_L^2 L_L^2} = \frac{F_{DM}}{\frac{1}{2}\rho_{vatten} U_M^2 L_M^2}$$

$$F_{DL} = F_{DM} \cdot \frac{\rho_{luft}}{\rho_{vatten}} \cdot \frac{U_L^2}{U_M^2} \cdot \Big(\underbrace{\frac{L_L}{L_M}}_{30}\Big)^2 = 730 \text{ N}$$

c)

$$P_{DL} = F_{DL} \cdot U_L = 4380 \, [W]$$

⑥

5.4

Given: $\tau_w = f(U, \delta, u', \rho, \frac{dP}{dx})$, (ρ, U, δ) repeating

Asked: Find the dimensionless function

$n = 6$ (all variables) $\quad j = 3$ (repeating variables)

$k = n - j = 3$ 3 expected pi-groups.

τ_w	U [m/s]	δ [m]	u' [m/s]	ρ [kg/m³]	$\frac{dP}{dx}$ [Pa/m]
$\{ML^{-1}T^{-2}\}$	$\{LT^{-1}\}$	$\{L\}$	$\{LT^{-1}\}$	$\{ML^{-3}\}$	$\{ML^{-2}T^{-2}\}$

$[Pa] = [N/m^2] = \left[\frac{kg\,m/s^2}{m^2}\right] = \frac{kg}{m s^2}$

Form Π-groups:

$$\Pi_1 = \underbrace{\tau_w}_{\text{non-repeating}} \underbrace{\rho^a U^b \delta^c}_{\text{repeating}} = \{ML^{-3}\}^a \{LT^{-1}\}^b \{L\}^c \{ML^{-1}T^{-2}\} = M^0 L^0 T^0$$

Solve the equation system:

$$\left.\begin{aligned} M&: a + 0 + 0 + 1 = 0 \\ L&: -3a + b + c - 1 = 0 \\ T&: -b - 2 = 0 \end{aligned}\right\}$$

$a = -1$

$c = 1 + 3a - b = 1 - 3 + 2 = 0$

$b = -2$

$$\Pi_1 = \frac{\tau_w}{\rho U^2}$$

$$\Pi_2 = u' \rho^a U^b \delta^c = \{LT^{-1}\}\{ML^{-3}\}^a \{LT^{-1}\}^b \{L\}^c = M^0 L^0 T^0$$

$$\left.\begin{aligned} M&: \quad a = 0 \\ L&: \quad 1 - 3a + b + c = 0 \\ T&: \quad -1 - b = 0 \end{aligned}\right\} \quad \begin{aligned} &a = 0 \\ &c = -1 + 3a - b = -1 + 0 + 1 = 0 \\ &b = -1 \end{aligned}$$

$$\Pi_2 = \frac{u'}{U}$$

$$\Pi_3 = \frac{dp}{dx} \rho^a U^b \delta^c = \{ML^{-2}T^{-2}\}\{ML^{-3}\}^a\{LT^{-1}\}^b\{L\}^c = M^0L^0T^0$$

$$\left.\begin{aligned} M&: \quad 1 + a = 0 \\ L&: \quad -2 - 3a + b + c = 0 \\ T&: \quad -2 - b = 0 \end{aligned}\right\} \quad \begin{aligned} &a = -1 \\ &c = 2 + 3a - b = 2 - 3 + 2 = 1 \\ &b = -2 \end{aligned}$$

$$\Pi_3 = \frac{dp}{dx} \frac{\delta}{\rho U^2}$$

dimensionless function:

$$\Pi_1 = f(\Pi_2, \Pi_3)$$

$$\underline{\frac{\tau_w}{\rho U^2} = f\left(\frac{u'}{U}, \frac{dp}{dx}\frac{\delta}{\rho U^2}\right)}$$

5.5 Given: $\delta = f(\rho, D, L, E, V, \mu)$

Asked: Dimensionless form

δ [m]	ρ [kg/m³]	D [m]	L [m]	E [$Pa = \frac{N}{m^2} = \frac{kg}{ms^2}$]	V [m/s]	μ [kg/ms]
$\{L\}$	$\{ML^{-3}\}$	$\{L\}$	$\{L\}$	$\{ML^{-1}T^{-2}\}$	$\{LT^{-1}\}$	$\{ML^{-1}T^{-1}\}$

n = 7 number of variables ($\delta \rho D L E V \mu$)

j = 3 number of dimensions (MLT)

k = 7 − 3 = 4 number of pi groups

Choose repeating variables so that all dimensions are covered:

$\rho, D, V \Rightarrow \{\underset{M}{ML^{-3}}\}\{\underset{L}{L}\}\{\underset{T}{LT^{-1}}\}$

$$\Pi_1 = \underset{\text{non repeat}}{\delta}\, \underbrace{\rho^a D^b V^c}_{\text{repeating}} = \{L\}\{ML^{-3}\}^a\{L\}^b\{LT^{-1}\}^c = M^0L^0T^0$$

$$\left.\begin{aligned} M&: 0 + a + 0 + 0 = 0 \\ L&: 1 - 3a + b + c = 0 \\ T&: 0 + 0 + 0 - c = 0 \end{aligned}\right\} \quad \begin{aligned} a &= 0 \\ b &= -1 + 3a - c = -1 + 0 - 0 = -1 \\ c &= 0 \end{aligned}$$

$$\Pi_1 = \frac{\delta}{D}$$

$$\Pi_2 = L\rho^a D^b V^c = \{L\}\{ML^{-3}\}^a\{L\}^b\{LT^{-1}\}^c = M^0L^0T^0$$

$$\left.\begin{aligned} M&: a = 0 \\ T&: c = 0 \\ L&: 1 + b = 0 \Rightarrow b = -1 \end{aligned}\right\} \quad \Pi_2 = \frac{L}{D}$$

$$\Pi_3 = E\rho^a D^b V^c \doteq \{ML^{-1}T^{-2}\}\{ML^{-3}\}^a\{L\}^b\{LT^{-1}\}^c = M^0L^0T^0$$

$$\left.\begin{aligned} M&: 1+a+0+0=0 \\ L&: -1-3a+b+c=0 \\ T&: -2+0+0-b=0 \end{aligned}\right\} \quad \begin{aligned} &a=-1 \\ &c=1+3a-b=1-3+2=0 \\ &b=-2 \end{aligned}$$

$$\Pi_3 = \frac{E}{\rho V^2}$$

$$\Pi_4 = \mu\rho^a D^b V^c = \{ML^{-1}T^{-1}\}\{ML^{-3}\}^a\{L\}^b\{LT^{-1}\}^c = M^0L^0T^0$$

$$\left.\begin{aligned} M&: 1+a+0+0=0 \\ L&: -1-3a+b+c=0 \\ T&: -1+0+0-c\doteq 0 \end{aligned}\right\} \quad \begin{aligned} &a=-1 \\ &b=1+3a-c=1-3+1=-1 \\ &c=-1 \end{aligned}$$

$$\Pi_4 : \frac{\mu}{\rho V D}$$

$$\Pi_1 = f(\Pi_2, \Pi_3, \Pi_4)$$

$$\frac{\delta}{D} = f\left(\frac{L}{D}, \frac{E}{\rho V^2}, \frac{\mu}{\rho V D}\right)$$

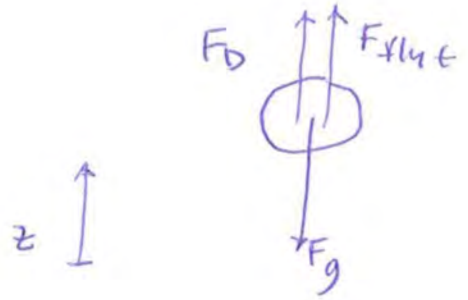
5.6

~~En stål st~~

En stålkula släpps i bensin!

Givet:

$\rho_{stål} = 7800\ kg/m^3$



$\begin{cases} T = 20°C \\ D = 0.025\ m \\ \text{Bensin} \end{cases}$

Sökt:

Vad blir terminalhastigheten?

Lösning:

(A) Frilägg och applicera Newton's andra lag.

(B) Få uttryck för u.

(C) Iterera fram svaret.

(A) Newton's andra:

$$\sum F_z = \{\text{terminalhast}\} = 0 = -F_g + F_{Flyt} + F_D$$

$$\rightarrow \boxed{F_D = F_g - F_{flyt}}$$

$$F_D = \frac{1}{2} C_D \rho_{bensin} u^2 A_{stål} \qquad F_g = \rho_{stål}\, g\, V_{stål}$$

$$F_{flyt} = \rho_{bensin}\, g\, V_{stål}$$

$$\rightarrow \frac{1}{2} C_D \rho_{bensin} u^2 \underbrace{A_{stål}}_{\frac{\pi D_{stål}^2}{4}} = (\rho_{stål} - \rho_{bensin})\, g\, \underbrace{V_{stål}}_{\frac{\pi D_{stål}^3}{6}}$$

③ Sätt in värden!

RÖ9

$\Rightarrow c_D u^2 = 3,42 \quad \rightarrow u = \sqrt{\frac{3,42}{c_D}}$

→ Två okända → Vi itererar:

① Välj c_D

② Lös $u = \sqrt{\frac{3,42}{c_D}}$

③ Kontrollera c_D i figur 5.3a (s. 326)

④ Om c_D är samma → OK

Annars börja om!

$c_D = 0.6 \rightarrow u = \sqrt{\frac{3,42}{0,6}} = 2,387 \frac{m}{s}$

$Re = \frac{\rho_{bensin} \cdot u \cdot D}{\mu_{bensin}} = 139\,000$

→ Fig 5.3a → $c_D \approx 0.4$ Börja om!

$c_D = 0,4 \rightarrow u = 2,92$ m/s → $Re \approx 170\,000$

→ $c_D = 0,4$ → OK!

$\Rightarrow \boxed{u = 2,92 \text{ m/s}}$

(9)