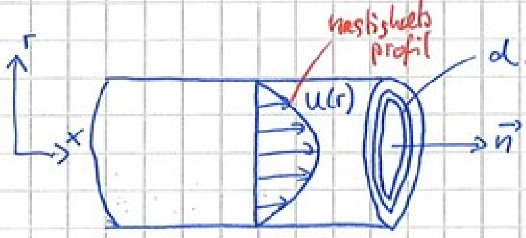
(RÖ3) (3.01) Volymflödet genom ett rör!

3.1



Framsida

$u(r) = C(R^2 - r^2)$

↖ då $r = R$, $u = 0$ (no-slip)

Ekv.(3.7) $Q = \int_A \vec{V} \cdot \vec{n}\, dA$

↖ vi behöver ersätta detta

ett area element: $dA = 2\pi r\, dr$ jag kan säga till er

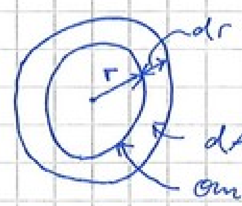
hur kommer man fram till detta?

1) matematiskt: vi letar efter area förändringen med radius

$A = \pi r^2$

vi tar derivaten $\frac{dA}{dr} = 2\pi r \Rightarrow dA = 2\pi r\, dr$

2) geometriskt:



man kan multiplicera omkretsen med dr:

$dA = 2\pi r\, dr$

(3.7) $Q = \int_A \vec{V} \cdot \vec{n}\, dA = \int_0^R C(R^2 - r^2) \cdot 2\pi r\, dr$

hastighet åt samma håll som $\vec{n}$, alltså positivt tecken

$= \int_0^R 2\pi C(R^2 r - r^3)\, dr = 2\pi C\left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R$

$= 2\pi C\left(\frac{R^4}{2} - \frac{R^4}{4}\right) = 2\pi C \frac{R^4}{4} = \frac{\pi C R^4}{2}$

3.03

3.2

Kontinuitets ekv.

$$\frac{d}{dt}\int_{c.v.} \rho \, dV + \int_{c.s} \rho \bar{V} \cdot \hat{n} \, dA = 0$$

$$\frac{d}{dt}\int_{c.v.} \rho \, dV = \frac{d}{dt}\left(\rho \cdot \pi \frac{d^2}{4} \cdot h(t)\right) = \frac{\rho \pi d^2}{4} \frac{dh}{dt}$$

Inkompr.

$$\frac{\pi d^2}{4}\frac{dh}{dt} + \underbrace{\int_{c.s.} \bar{V} \cdot \hat{n} \, dA}_{= Q_1 + Q_2 + Q_3} = 0$$

$$\frac{dh}{dt} = -\frac{4(Q_1 + Q_2 + Q_3)}{\pi d^2}$$

Givet: $Q_3 = -0.01 \, m^3/s$, $V_1 = 3 \, m/s$, $\frac{dh}{dt} = 0$

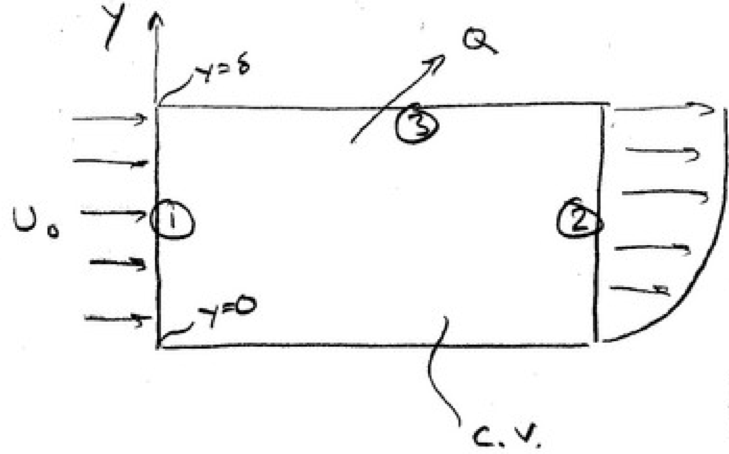
$$\int_{c.s.} \bar{V} \cdot \hat{n} \, dA = \int_{①} -V_1 \, dA + \int_{②} V_2 \, dA + Q_3 =$$

$$-V_1 A_1 + V_2 A_2 + Q_3 = 0$$

$$V_2 = \frac{V_1 A_1 - Q_3}{A_2} = \frac{3 \cdot \frac{\pi \cdot 0.05^2}{4} - (-0.01)}{\frac{\pi \cdot 0.07^2}{4}} \approx 4.13 \, m/s$$

3.04

3.3



$$u(\eta) = U_0\left(\frac{3\eta - \eta^3}{2}\right)$$

$$\eta = \frac{y}{\delta}$$

$$\{dy = \delta d\eta\}$$

Kontinuitets ekv.

$$\frac{d}{dt}\int_{c.v.} \rho \, dV + \int_{c.s} \rho \, \bar{V}_r \cdot \hat{n} \, dA = 0$$

Fix kontrollvolym, inkompr. fluid

$$\left.\begin{aligned} &\frac{d}{dt}\int_{c.v.} \rho \, dV = \int_{c.v.} \frac{\partial \rho}{\partial t} dV = 0 \\ &\int_{c.s.} \rho \bar{V}_r \cdot \hat{n} \, dA = \rho \int_{c.s.} \bar{V} \cdot \hat{n} \, dA \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \int_{c.s.} \bar{V} \cdot \hat{n} \, dA = 0$$

$$\int_{c.s.} \bar{V} \cdot \hat{n} \, dA = -\int_{①} U_0 \, dA + \int_{②} u(\eta) \, dA + Q =$$

$$= -\int_0^{\delta} U_0 \cdot b \cdot dy + \int_0^1 U_0 \frac{3\eta - \eta^3}{2} b \cdot \delta d\eta + Q =$$

$$= -\left[U_0 b y\right]_0^{\delta} + \left[\frac{U_0}{2}\left(\frac{3\eta^2}{2} - \frac{\eta^4}{4}\right) b\delta\right]_0^1 + Q =$$

$$= -U_0 b \delta + \frac{5 U_0 b \delta}{8} + Q = 0$$

$$Q = \frac{3 U_0 b \delta}{8}$$

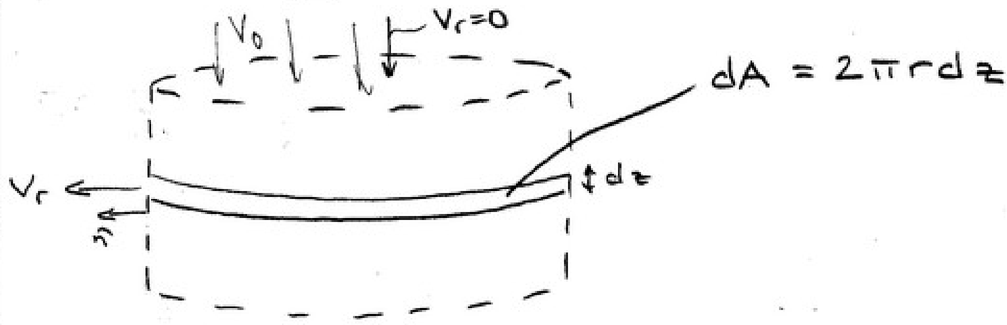
3.05 **Kont. ekv.** **Deformerbar C.V.**

3.4

$$\frac{d}{dt}\int_{c.v.} \rho \, dV + \int_{c.s.} \rho \bar{V}_r \cdot \hat{n} \, dA = 0$$

$$\frac{d}{dt}\int_{c.v.} \rho \, dV = \frac{d}{dt}\left(\rho \pi r^2 h(t)\right) = \rho \pi r^2 \frac{dh}{dt} \qquad \text{(inkompr.)}$$

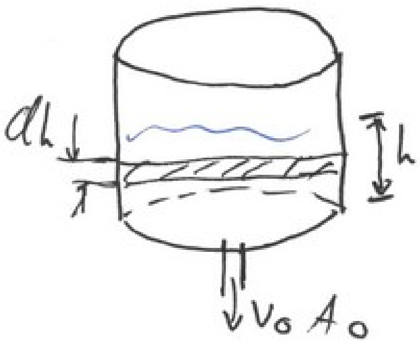
$$\pi r^2 \frac{dh}{dt} + \int_{c.s.} \bar{V}_r \cdot \hat{n} \, dA = 0 \quad , \quad \frac{dh}{dt} = -V_0$$



$$\pi r^2(-V_0) + \int_0^h V(r) \cdot 2\pi r dz = \pi r^2 V_0 + V(r) \cdot 2\pi r \cdot h = 0$$

$$V(r) = \frac{V_0 \pi r^2}{2\pi r h} = \frac{V_0 r}{2h}$$

3.5 3.5



Given: V_0, h_0

Find: $\frac{dh}{dt}$

Assume: 1D, incomp

Solution: Use RTT for conservation of mass (3.20)

$$0 = \frac{d}{dt}\left(\int_{cv} \rho \, dV\right) + \int_{cs} \rho(\vec{V}\cdot\vec{n})\, dA = 0$$

(incomp) (incomp)

outflow

$$= \frac{d}{dt}(V) + V_0 \cdot A_0 = 0 \rightarrow \frac{d}{dt}(V) = -V_0 A_0 \quad \dots [*]$$

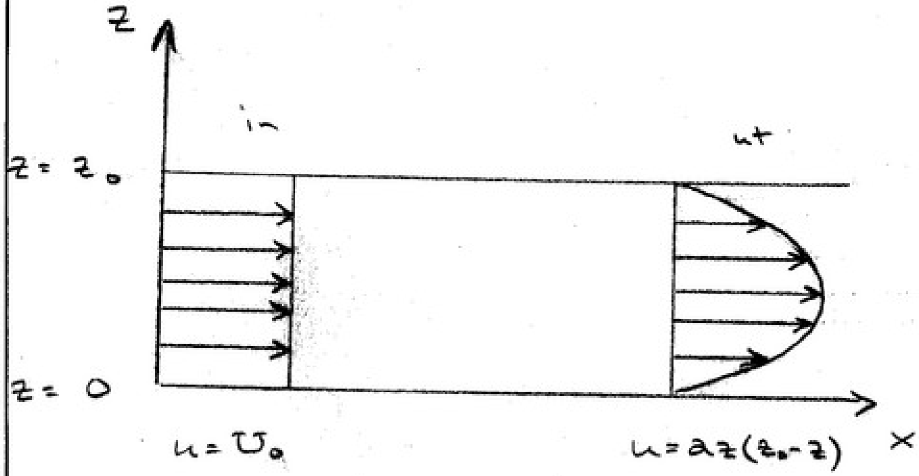
$$V = \frac{\pi d^2}{4} \cdot h$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\pi d^2}{4} \cdot h\right) = -V_0 A_0 \Rightarrow$$

$$\frac{dh}{dt} = -\frac{4 V_0 A_0}{\pi d^2}$$

3.06

3.6



Givet:

$U_0 = 8$ cm/s

$z_0 = 4$ cm

Kontinuitet vid inkompressibel strömning

$$Q_{in} = Q_{ut}$$

Antag djupet i y-led är b.

$$Q_{in} = \int_0^b \int_0^{z_0} U_0 \, dz\, dy = z_0 U_0 b$$

$$Q_{ut} = \int_0^b \int_0^{z_0} az(z_0 - z)\, dz\, dy = ab \int_0^{z_0} z_0 z - z^2 \, dz$$

$$= ab \left[\frac{z_0}{2} z^2 - \frac{z^3}{3} \right]_0^{z_0} = \frac{a z_0^3}{6} b$$

$$Q_{in} = Q_{ut} \Rightarrow z_0 U_0 b = \frac{1}{6} a z_0^3 b$$

$$a = \frac{6 U_0}{z_0^2}$$

Vid utlopp: $u(z) = \frac{6U_0}{z_0^2} z(z_0 - z)$

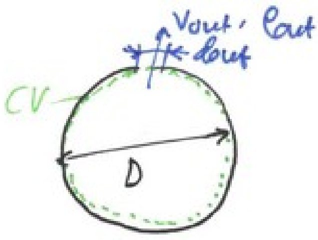
Sök u_{max}:

Hitta z då u är störst: $\frac{du}{dz} = \frac{6U_0}{z_0^2}(z_0 - 2z)$

$\frac{du}{dz} = 0 \Rightarrow z = \frac{1}{2} z_0$ (dvs mitt i kanalen)

$$u_{max} = u\left(\tfrac{1}{2} z_0\right) = \frac{6U_0}{z_0^2} \tfrac{1}{2} z_0 \left(z_0 - \tfrac{1}{2} z_0\right) = \frac{3U_0}{2} = \underline{12 \text{ cm/s}}$$

P3.7 | 3.7



Known:
$V_{out} = 360\ m/s$
$\rho_{out} = 2.5\ kg/m^3$
$d_{out} = 0.005\ m$
$D = 0.35\ m$

Task: Derive an expression for the rate of change of average density $d\rho/dt$ in the tank & calculate the value of $d\rho/dt$ for the given data

Assume: 1D in/out, constant outlet properties, stationary CV ⇒ $\tilde{V}_r \equiv \tilde{V}$, fix CV

Solution: Use RTT for conservation of mass to derive $\frac{d\rho}{dt}$.

$$(3.20)\quad \frac{d}{dt}\left(\int_{CV} \rho\, dV\right) + \int_{CS} \rho\,(\tilde{V}_r \cdot \tilde{n})\, dA = 0, \quad \tilde{V}_r = \tilde{V} \text{ (stationary CV)} =$$

$$= \underbrace{\frac{d}{dt}(\rho V_{CV})}_{\text{fix CV}} + \underbrace{\rho_{out} V_{out} \frac{\pi d_{out}^2}{4}}_{\text{1D in/out}} = 0$$

Volume of the CV given by the sphere volume: $V_{CV} = \frac{1}{6}\pi D^3$ ⇒

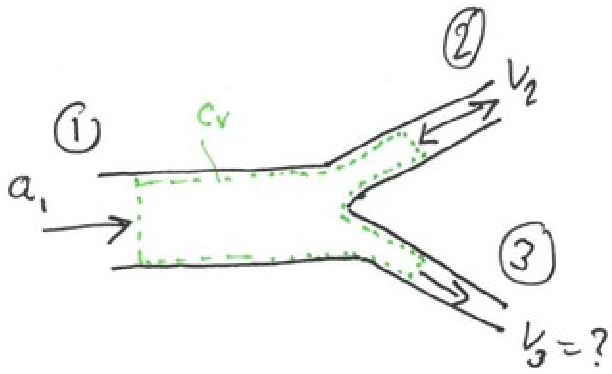
$$\frac{d}{dt}\left(\rho \frac{1}{6}\pi D^3\right) = -\rho_{out} V_{out} \pi \frac{d_{out}^2}{4} \Rightarrow$$

$$\frac{d\rho}{dt} = \frac{\rho_{out} V_{out} \pi \frac{d_{out}^2}{4}}{\frac{1}{6}\pi D^3} = -\frac{3}{2}\,\frac{\rho_{out} V_{out} d_{out}^2}{D^3}$$

b) Calculate the value of $\frac{d\rho}{dt} = \left[V_{out} = 360\ m/s,\ \rho_{out} = 2.5\ kg/m^3,\ d_{out} = 0.005\ m \,\middle|\, D = 0.35\ m \right] = -0.79\ \frac{kg}{m^3 \cdot s}$

P 3.10

3.8



Known:

$Q_1 = 0{,}5\ m^3/s$

$V_2 = 12\ m/s$

$d_2 = 0{,}18\ m$

$d_3 = 0{,}13\ m$

$\rho = 680\ kg/m^3$ (Table A3)

Task: a) Calculate the velocity a (3)

b) is the flow going in or out?

Assume: Steady-state $\left(\frac{d}{dt} = 0\right)$, stationary CV $(\tilde{V}_r \equiv \tilde{V})$, 1D in/out, incompressible $(\rho = \text{constant})$

Solution: Use RTT for conservation of mass and calculate V_2

$$(3.20)\quad \frac{d}{dt}\left(\int_{cv} \rho\, dv\right) + \int_{cs} \rho (\tilde{V}_r \cdot \tilde{n})\, dA = 0 \Rightarrow \int_{cs} \rho (\tilde{V} \cdot \tilde{n})\, dA = 0$$

$\frac{d}{dt} = 0$, Steady state $\qquad \tilde{V}_r = \tilde{V}$, stationary

1D in/out gives: $\sum (\rho_i A_i V_i)_{out} - \sum (\rho_i A_i V_i)_{in} = 0$ (incomp.)

Volume flow defined as: $Q = V \cdot A$ $\Rightarrow$

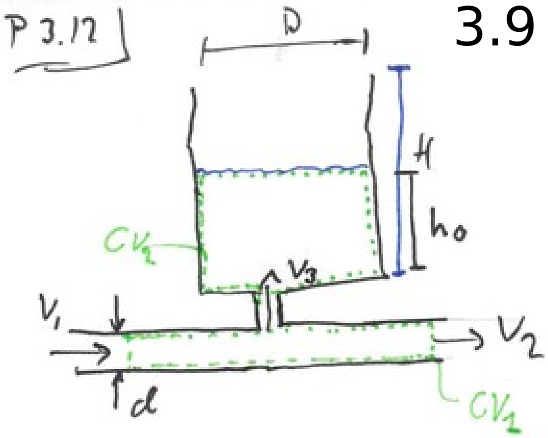
$$-Q_1 + Q_2 + Q_3 = 0 \Rightarrow Q_3 = Q_1 - V_2 A_2 = Q_1 - V_2 \pi \frac{d_2^2}{4} = +0{,}19\ldots \frac{m^3}{s}$$

(assume outflow)

$\therefore Q_2 = 0{,}19\ldots \frac{m^3}{s}$. the positive sign show that Q_2 is flowing out of CV. (b)

$$V_3 = \frac{Q_3}{A_3} = \frac{4 Q_3}{\pi d_3^2} = 14{,}7\ m/s \quad (a)$$

P 3.12



Known:

$V_1 = 2.5\,m/s$ | $d = 0.12\,m$ | $H = 1\,m$
$V_2 = 1.9\,m/s$ | $D = 0.75\,m$ | $h_0 = 0.3\,m$ (at t=0)

Task: Calculate the time to fill the tank

Assume: CV1: Steady-state, 1D in/out, incomp, stationary
CV2: Unsteady, ——— " ———.

Solution: Use RTT for conservation of mass for CV_1 to calculate the flow into CV_2 (the tank). → Use RTT for mass conservation of CV_2 to find the time.

RTT CV_1 (3.20) $\frac{d}{dt}\left(\int_{CV} \rho\, dV\right) + \int_{CS} \rho(\tilde{V}\cdot\tilde{n})\,dA = 0 \Rightarrow$ [1D in/out] ⇒

(incomp; $d/dt = 0$, SS; $\tilde{V}_r = \tilde{V}$)

$$-Q_1 + Q_2 + Q_3 = 0 \Rightarrow Q_3 = Q_1 - Q_2 = \left[Q = V \cdot A = V\frac{\pi d^2}{4}\right] \Rightarrow$$

$$Q_3 = \frac{\pi d^2}{4}(V_1 - V_2) = 6.785\ldots \cdot 10^{-3}\ \frac{m^3}{s}$$, Flow into the tank

RTT CV_2 (3.20): $\frac{d}{dt}\left(\int_{CV} \rho\, dV\right) + \int_{CS} \rho(\tilde{V}\cdot\tilde{n})\,dA = 0 =$ (incomp.)

$$= \frac{d}{dt}(V) - Q_3 = 0 \Rightarrow \frac{d}{dt}(V) = Q_3 \qquad [A]$$

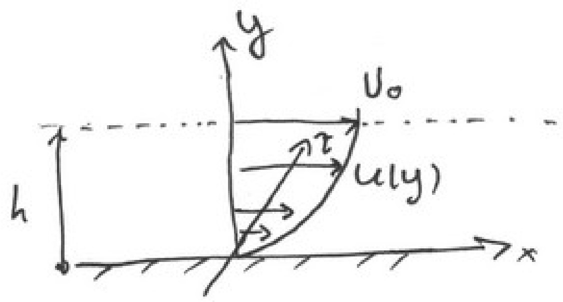
(1D in/out, negative since Q_3 is in)

The volume V is: $V = \frac{\pi D^2}{4}\cdot h$ [Where $h = h_0$ at $t = 0$ (B.C)] in [A]

$$\frac{d}{dt}\left(\frac{\pi D^2}{4}h\right) = Q_3 \Rightarrow \frac{dh}{dt} = \frac{4Q_3}{\pi D^2}$$ → Integrate this to find t

$$\int_{h_0}^{H} dh = \int_0^t \frac{4Q_3}{\pi D^2}\,dt \Rightarrow H - h_0 = \frac{4Q_3}{\pi D^2}[t - 0] \Rightarrow t = \frac{(H-h_0)\pi D^2}{4Q_3} = 45.6\,s$$

P3.26 3.10



Laminar

$u(y) = U_0\left(\frac{2y}{h} - \frac{y^2}{h^2}\right)$

$h = 13\,mm = 0.013\,m$

$Q/b = 5\,L/min/m$ (per unit width) $= 5 \cdot \frac{10^{-3}}{60}\ \frac{m^3}{m \cdot s}$

b = width of channel

Task: Find U_0, in mm/s

Assume: Steady-state, incompressible

Solution: Use the definition of volumetric flow rate (3.7)

$$(3.7): \quad Q = \int(\tilde{V}\cdot n)\,dA = \int_0^b\int_0^h u(y)\,dy\,dz = b\int_0^h u(y)\,dy = Q \Rightarrow$$

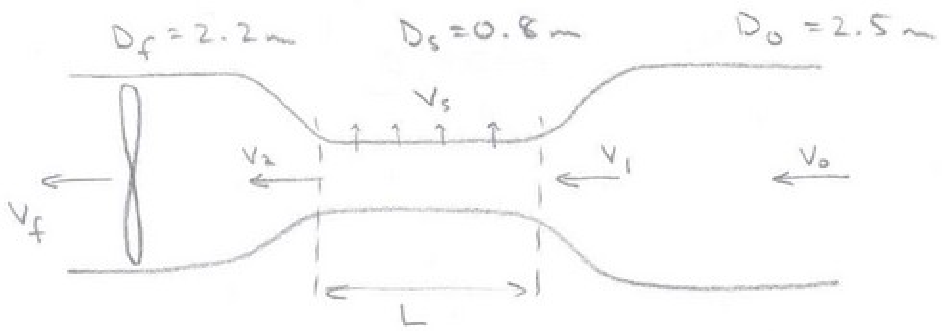
$$\frac{Q}{b} = \int_0^h u(y)\,dy = \int_0^h U_0\left(\frac{2y}{h} - \frac{y^2}{h^2}\right)dy = \frac{U_0}{h}\left[y^2 - \frac{y^3}{3h}\right]_0^h$$

$$\frac{Q}{b} = \frac{2}{3}U_0 h \Rightarrow \text{Solve for } U_0$$

$$U_0 = \frac{3}{2h}\cdot\left(\frac{Q}{b}\right) = \frac{3}{2\cdot 0.013}\cdot\left(5\cdot\frac{10^{-3}}{60}\right) = 9{,}6\ mm/s$$

3.11

3.11



Antal hål : 1200/m^2

$D_{hål} = 5$mm

$V_{hål} = 8$m/s

Antag inkompressibelt

a) sök V_0

Kontinuitet ger: $Q_0 = Q_1$

$$A_0 V_0 = A_1 V_1$$

$$\frac{\pi D_0^2}{4} V_0 = \frac{\pi D_s^2}{4} V_1$$

$$V_0 = \frac{D_s^2}{D_0^2} V_1 \approx \underline{3.58 \text{ m/s}}$$

b) sök V_2

Kontinuitet ger: $Q_1 = Q_2 + Q_{hål}$

$$A_1 V_1 = A_2 V_2 + A_{hål} V_{hål}$$

antal hål $= \pi D_s L \cdot 1200 \approx 12064$ st

$$\Rightarrow \frac{\pi D_s^2}{4} V_1 = \frac{\pi D_s^2}{4} V_2 + \frac{\pi D_{hål}^2}{4} \cdot 12064 \cdot V_{hål}$$

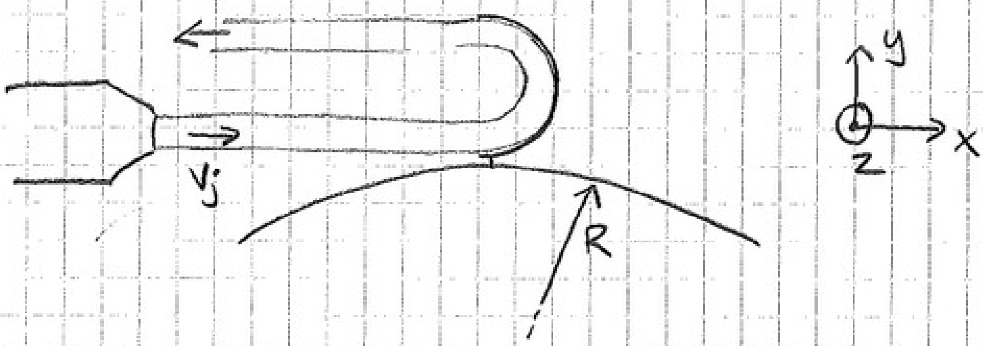
$$V_2 = \frac{D_s^2 V_1 - 12064 D_{hål}^2 V_{hål}}{D_s^2} \approx \underline{31.2 \text{ m/s}}$$

c) sök V_f, kontinuitet ger: $V_f = \frac{D_s^2}{D_f^2} V_2$

$$V_f \approx 4.13 \text{ m/s}$$

3.13 3.12 (1)

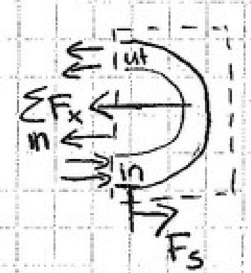
Strålens area = A_j , densitet = ρ



Deluppgift 1:

> Härled ett uttryck för effekten P som strålen överför till hjulet som funktion av indatan ovan.

Bestäm kontrollvolym!



- Kontrollvolymen är rörlig och rör sig med turbinhjulet. $V_{cv} = \Omega R$
- Vi antar alltså att den translaterar linjärt (= ingen acceleration)
- In- och utlopp kan antas 1-dimensionella

Notera; Summan av krafterna på kontrollvolymen (här ΣF_x) måste utöva en motriktad kraft på omgivningen (här F_s),

dvs: $F_s = -\Sigma F_x$

Effekten kan bestämmas genom:

$$P = F_s \cdot \Omega R \qquad [A]$$

Utgå från impulssatsen:

$$\frac{d}{dt}(m\mathbb{V})_{syst} = \Sigma \mathbb{F} = \frac{d}{dt}\left(\int_{cv} \mathbb{V}\rho \, d\mathcal{V}\right) + \int_{cs} \mathbb{V}\rho(\mathbb{V}_r \cdot n)\, dA \quad (3.35)$$

Hur kan vi förenkla?

<u>Vätske</u>stråle → antag inkompressibelt → ρ konstant

Dessutom kan vi anta att flödet är stationärt, inte heller ändrar kontrollvolymen sig i storlek

$\Rightarrow \frac{d}{dt}\left(\int_{cv} \mathbf{V}\rho \, d\mathcal{V}\right) = 0$ dvs;

$\Sigma \mathbf{F} = \int_{cs} \mathbf{V}\rho(\mathbf{V}_r \cdot \mathbf{n}) dA$

Vi har bara flöde över två ytor ("in" och "ut"):

$\Sigma \mathbf{F} = \int_{in} \mathbf{V}_{in} \rho \left(\mathbf{V}_{r,in} \cdot \mathbf{n}_{in}\right) dA + \int_{ut} \mathbf{V}_{ut} \rho \left(\mathbf{V}_{r,ut} \cdot \mathbf{n}_{ut}\right) dA$

Notera att detta är en vektor-relation som med vektorerna utskrivna ser ut som;

$$\begin{bmatrix} \Sigma F_x \\ \Sigma F_y \\ \Sigma F_z \end{bmatrix} = \int_{in} \begin{bmatrix} V_{in,x} \\ 0 \\ 0 \end{bmatrix} \rho \begin{bmatrix} V_{r,in,x} \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} dA + \int_{ut} \begin{bmatrix} V_{ut,x} \\ 0 \\ 0 \end{bmatrix} \rho \begin{bmatrix} V_{r,ut,x} \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} dA \quad [B]$$

Ovan har jag använt mig av det faktum att vi inte har några hastigheter, vare sig av strålen eller kontrollvolymen, i y- eller z-riktningen.

Normalvektorn till kontrollvolymens yta är definierad som positiv ut från kontrollvolymen vilket här innebär att

$$\mathbf{n}_{in} = \mathbf{n}_{ut} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

För att räkna ut ΣF_x behöver vi uttrycka $V_{in,x}$, $V_{r,in,x}$, $V_{ut,x}$ och $V_{r,ut,x}$ i kända variabler. Vi vet att

$V_{in,x} = V_j$

Relativhastigheter ges av:

$\mathbf{V}_r = \mathbf{V} - \mathbf{V}_{cv}$ (3.14)

Här är $V_{cv,x}$ (kontrollvolymens hastighet i x-riktning) $= \Omega R$

$\rightarrow$

$V_{r,in,x} = V_{in,x} - V_{cv,x} = V_j - \Omega R$

3.12 (3)

För att uttrycka ut-hastigheterna använder vi oss av kontinuitetsekvationen:

$$0 = \frac{d}{dt}\left(\int_{cv} \rho \, dV\right) + \int_{cs} \rho (V_r \cdot n)\, dA \qquad (3.20)$$

Inkompressibelt + att kontrollvolymen inte ändrar sig i storlek:

$$0 = \int_{cs} \rho (V_r \cdot n)\, dA$$

Utskrivet för våra två ytor:

$$0 = \int_{in} \rho \begin{bmatrix} V_{r,in,x} \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} dA + \int_{ut} \rho \begin{bmatrix} V_{r,ut,x} \\ \\ \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} dA$$

i x-led med $V_{r,in,x} = V_j - \Omega R$ och $A_{in} = A_{ut} = A_j$:

$$-\rho (V_j - \Omega R) A_j - \rho V_{r,ut,x} A_j = 0 \Rightarrow$$

$$V_{r,ut,x} = -(V_j - \Omega R) = (\Omega R - V_j)$$

mha ekv 3.14 får vi:

$$V_{r,ut,x} = V_{ut,x} - V_{cv}$$

$$V_{ut,x} = V_{r,ut,x} + V_{cv} = -(V_j - \Omega R) + \Omega R = -V_j + 2\Omega R$$

Insättning i [B] ger (endast x-led utskrivet):

$$\Sigma F_x = V_j \rho (V_j - \Omega R)\cdot(-1) A_j + (-V_j + 2\Omega R)\rho(\Omega R - V_j)\cdot(-1) A_j$$

$$= -V_j \rho A_j (V_j - \Omega R) + (-V_j + 2\Omega R)\rho A_j (V_j - \Omega R)$$

$$= \rho A_j (V_j - \Omega R)(-V_j - V_j + 2\Omega R)$$

$$= -2\rho A_j (V_j - \Omega R)^2$$

Som förklarat ovan; $F_s = -\Sigma F_x = 2\rho A_j (V_j - \Omega R)^2$

Insättning i [A] ger:

$$P = F_s \cdot \Omega R = \underline{\underline{2\rho A_j \Omega R (V_j - \Omega R)^2}}$$

3.12 (4) Deluppgift 2:

Vid vilken vinkelhastighet överförs maximal effekt?

$P(\Omega)$ och då $\frac{dP}{d\Omega} = 0$ och $\frac{d^2P}{d\Omega^2} < 0$ fås P_{max}

Förenkla P för derivering:

$$P = 2\rho A_j \Omega R (V_j - \Omega R)^2 =$$

$$2\rho A_j R (V_j^2 \Omega - 2V_j R \Omega^2 + R^2 \Omega^3)$$

$$\frac{dP}{d\Omega} = 2\rho A_j R (V_j^2 - 4V_j R \Omega + 3R^2 \Omega^2) = 0 \Rightarrow$$

$$V_j^2 - 4V_j R \Omega + 3R^2 \Omega^2 = 0$$

$$\Omega^2 - \frac{4V_j}{3R}\Omega + \frac{V_j^2}{3R^2} = 0$$

$$\Omega = \frac{2V_j}{3R} \pm \sqrt{\frac{4}{9}\left(\frac{V_j}{R}\right)^2 - \frac{1}{3}\left(\frac{V_j}{R}\right)^2} = \frac{2V_j}{3R} \pm \frac{1V_j}{3R}$$

$\Omega_1 = \frac{V_j}{R}$ (minimal effekt, stråle och turbinhjul rör sig lika snabbt)

$\Omega_2 = \frac{1}{3}\frac{V_j}{R}$

Dubbelkolla att Ω_2 är ett maximum

$$\frac{d^2P}{d\Omega^2} = 2\rho A_j R (-4V_j R \Omega + 6R^2 \Omega)$$

Ω_2 ger $\frac{d^2P}{d\Omega^2} < 0$

$$P_{max} = 2\rho A_j \frac{1}{3}\frac{V_j}{R} R \left(V_j - \frac{1}{3}\frac{V_j}{R} R\right)^2 = \ldots = \underline{\underline{\frac{8}{27}\rho A_j V_j^3}}$$

3.15 Vi söker D [N/m], dragkraften på plattan

3.13 (1) RTT för $m\bar{V}$

$$\frac{d}{dt}(m\bar{V})_{sys} = \sum \bar{F} = \frac{d}{dt}\int_{c.v.} \bar{V} \rho \, dV + \int_{c.s} \bar{V} \rho (\bar{V}_r \cdot \hat{n}) dA$$

$$\begin{cases} \sum F_x = -D \cdot b \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases}$$

Fix c.v. : $\bar{V}_r = \bar{V}$

Inkompr. : $\frac{\partial \rho}{\partial t} = 0$

Stationärt : $\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = \frac{\partial w}{\partial t} = 0$

$$-Db = \int_{c.s.} u \rho (\bar{V} \cdot \hat{n}) dA =$$

$$= \int_{①} U_0 \rho (-U_0) dA + \int_{③} u \rho u \, dA = \{dA = b\,dy\} =$$

$$= \int_0^h (-\rho U_0^2) b\,dy + \int_0^\delta \rho (u(y))^2 b\,dy \qquad (*)$$

För att finna h, RTT för m

$$\left(\frac{dm}{dt}\right)_{sys} = 0 = \frac{d}{dt}\int_{c.v.} \rho \, dV + \int_{c.s.} \rho (\bar{V}_r \cdot \hat{n}) dA$$

Inkompr., fix c.v., stationärt :

$$0 = \int_{c.s.} \rho \bar{V} \cdot \hat{n} \, dA = \int_{①} \rho (-U_0) \, dA + \int_{③} \rho u(y) \, dA \Rightarrow$$

$$\Rightarrow \int_0^h \rho U_0 b\,dy = \int_0^\delta \rho u(y) b\,dy \qquad (**)$$

3.15 forts.

3.13 (2)

(**) in i (*):

$$-U_0\int_0^h \rho U_0 b\,dy + \int_0^\delta \rho (u(y))^2 b\,dy = -Db$$

$$-U_0\int_0^h \rho U_0 b\,dy = \int_0^\delta \rho u(y) b\,dy$$

$$\Rightarrow \int_0^\delta (-\rho U_0 u(y))\,dy + \int_0^\delta \rho u(y) u(y)\,dy = -D$$

$$\rho \int_0^\delta u(y)[u(y)-U_0]\,dy = -D$$

$$D = \rho \int_0^\delta u(y)[U_0 - u(y)]\,dy$$

Momentum integral theory (Ch.7)

$$D = \rho \int_0^\delta U_0^2 \sin\left(\frac{\pi y}{2\delta}\right) - U_0^2 \sin^2\left(\frac{\pi y}{2\delta}\right) dy =$$

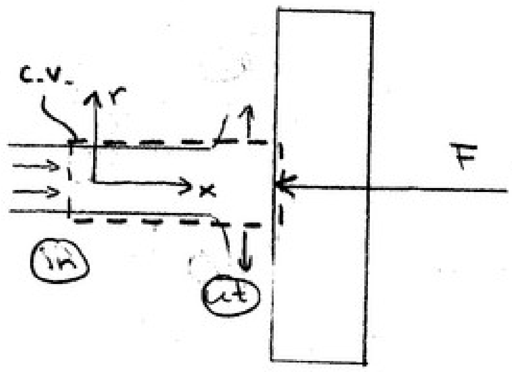
$$= \rho U_0^2 \left[-\frac{2\delta}{\pi}\cos\left(\frac{\pi y}{2\delta}\right) - \left(\frac{y}{2} - \frac{\sin\left(\frac{\pi y}{\delta}\right)}{\left(\frac{2\pi}{\delta}\right)}\right)\right]_0^\delta =$$

$$= \rho U_0^2 \left[-\frac{\delta}{2} + \frac{2\delta}{\pi}\right] = \rho U_0^2 \delta \left(\frac{2}{\pi} - \frac{1}{2}\right) =$$

$$= 998 \cdot 3^2 \cdot 2 \cdot 10^{-3} \left(\frac{2}{\pi} - \frac{1}{2}\right) \approx 2.45\ N/m$$

3.17

3.14



Sök F genom kraftbalans:

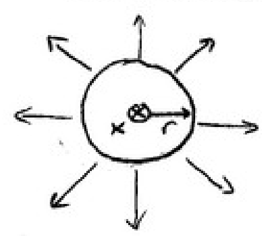
Impulssatsen: Reynolds transport teorem för $\begin{cases} B = m\bar{V} \\ \beta = \bar{V} \end{cases}$

$$\sum \bar{F} = \frac{d}{dt}(m\bar{V})_{sys} = \underbrace{\frac{d}{dt}\int_{cv} \bar{V}\rho \, d\mathcal{V}}_{=0} + \int_{c.s} \bar{V}\rho(\bar{V}_r \cdot \hat{n})dA$$

antar stationärt
fix c.v.
inkompressibelt

$$\sum \bar{F} = \int_{in} V_j \hat{x} \rho(-V_j)dA + \underbrace{\int_{ut} V_{ut} \hat{r} V_{ut} dA}_{=0}$$

utloppet ger krafter som tar ut varandra

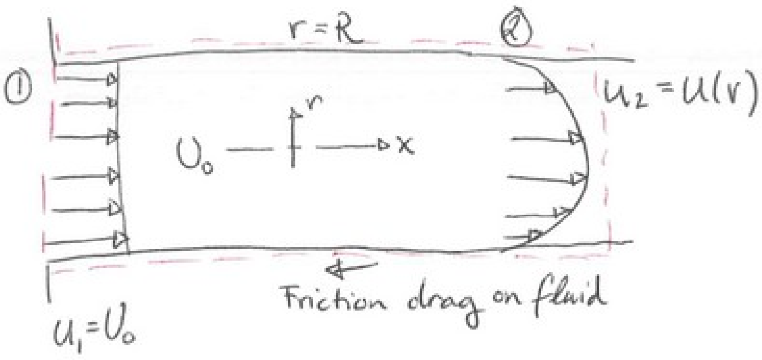


$$\sum \bar{F} = -F\hat{x} = \int_{in} -\rho V_j^2 \hat{x} dA$$

$$F = \frac{\pi D_j^2}{4} \rho V_j^2 = \frac{\pi \cdot 0.10^2}{4} \cdot 998 \cdot 8^2 \approx \underline{502N}$$

3.15

3.15 (1)



Laminar:
$u_2 = U_{max}\left(1 - \frac{r^2}{R^2}\right)$

Turbulent:
$u_2 = U_{max}\left(1 - \frac{r}{R}\right)^{1/7}$

Find the wall drag force, $F(p_1, p_2, \rho, U_0, R)$ for

a) laminar flow

b) turbulent flow

Reynolds transport theorem for linear momentum:

$$\sum F = \underbrace{\frac{d}{dt}\int_{CV} \bar{V}\rho\, dV}_{=0 \text{ steady state, incompressible}} + \int_{CS} \bar{V}\rho(\bar{V}_r \cdot \bar{n})\, dA$$

Only forces in X-direction so we will look at x-component only

$$\sum F_x = \underbrace{(P_1 - P_2)}_{\text{the pressure diff. driving the flow}} \overbrace{\pi R^2}^{\text{cross-section area}} - F_{drag}$$

Two control surfaces: (again only looking at x-component)

(note fix CV, no $\bar{V}_r$)

$$\int_{CS} V_x \rho (V_x \cdot n)\, dA = \int_{①} U_0 \rho(-U_0)\, dA + \int_{②} u_2 \rho u_2\, dA$$

(incompressible can be moved out of integral)

$$\Rightarrow \sum F_y = -\int_{①} U_0^2 \rho\, dA + \int_{②} u_2^2 \rho\, dA = [\text{1D inlet}] = -\pi R^2 \rho U_0^2 + \rho \int_{②} u_2^2\, dA$$

$$\Rightarrow (P_1 - P_2)\pi R^2 - F_{drag} = -\pi R^2 \rho U_0^2 + \rho \int_{②} u_2^2\, dA$$

$$\Rightarrow F_{drag} = \pi R^2 \left(P_1 - P_2 + \rho U_0^2 - \rho \int_{②} u_2^2\, dA\right)$$

3.15 (2)

a) *laminar flow:*

We will use the momentum flux correction (see lecture 5)
(note that this is a very small part of the course and not even in the FS)

$$\rho \int u_2^2 \, dA = \zeta V_{ave}^2 \rho A = \zeta V_{ave} \dot{m}$$

From continuity $Q_1 = Q_2 \Rightarrow V_{ave} = U_{2,ave} = U_0$

$$\Rightarrow \rho \int_{②} u_2^2 \, dA = \zeta U_0^2 \rho \pi R^2 = \frac{4 U_0^2 \rho \pi R^2}{3}$$

OBS! find in book or lecture slides.

where the momentum flux correction $\zeta = \frac{4}{3}$ for laminar flow.

$$\Rightarrow F_{drag} = \pi R^2 \left(P_1 - P_2 - \frac{1}{3} \rho U_0^2\right)$$

b) *turbulent flow*

$\zeta = 1.02$ for turbulent flow with $1/7$ as exponent of velocity profile

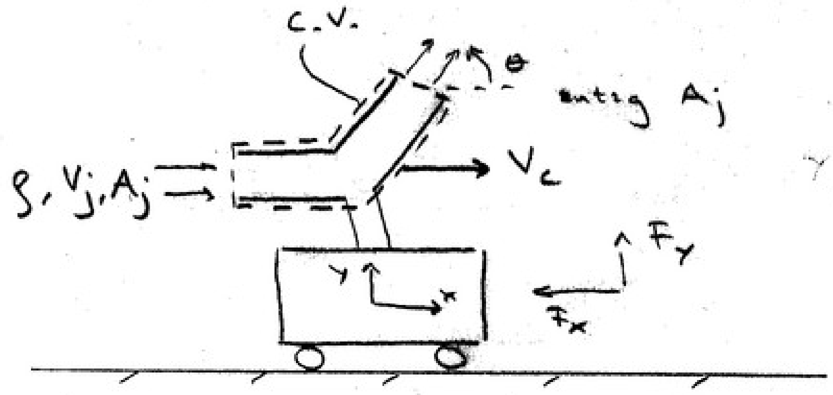
$$\rho \int_{②} u_2^2 \, dA = \zeta U_0^2 \rho \pi R^2 = 1.02 \, U_0^2 \rho \pi R^2$$

$$\Rightarrow F_{drag} = \pi R^2 (P_1 - P_2 - 0.02 \rho U_0^2)$$

We could solve this by integration but we would get complicated integrals and a function dependent on U_{max} which was not asked for. Using the momentum flux correction is very convenient as we only need to know the average velocity and not the maximum.

3.18

3.16 (1)



Låt koord. systemet följa vagnen!

$\Rightarrow \bar{V} = \bar{V}_r$ i impulssatsen!

REYNOLDS TRANSPORT TEOREM $\begin{cases} B = m\bar{V} \\ \beta = \bar{V} \end{cases}$

$\Rightarrow$ IMPULSSATSEN!

$$\sum \bar{F} = \underbrace{\frac{d}{dt} \int_{C.V.} \rho \bar{V} \, dV}_{=0} + \int_{C.S.} \rho \bar{V} (\bar{V}_r \cdot \hat{n}) \, dA$$

$=0$ ty inkompr. stationärt farlig men ej deformerbar C.V.

$\hat{n}_{in} = -\hat{x}$

$\hat{n}_{ut} = \cos\theta \, \hat{x} + \sin\theta \, \hat{y}$

$|V_{r,in}| = V_j - V_c$

Kontinuitet ger $V_{ut} = V_{in} = V_j$

$|V_{r,ut}| = V_j - V_c$

$\bar{V}_{r,in} = (V_j - V_c) \hat{x}$

$\bar{V}_{r,ut} = (V_j - V_c) \cos\theta \, \hat{x} + (V_j - V_c) \sin\theta \, \hat{y}$

3.18
forts.

3.16 (2)

$$\sum \bar{F} = -F_x\hat{x} + F_y\hat{y} = \int_{(in)} \rho \bar{V}(\bar{V}_r\cdot\hat{n})dA + \int_{(ut)} \rho \bar{V}(\bar{V}_r\cdot\hat{n})dA$$

$\bar{V} = \bar{V}_r$ ty koord. sys. följer vagnen!

$$-F_x\hat{x} + F_y\hat{y} = \int_{(in)} \rho \underbrace{(V_j - V_c)\hat{x}}_{\bar{V}_r} \underbrace{(V_j - V_c)\hat{x}}_{\bar{V}_r} \cdot \underbrace{(-\hat{x})}_{\hat{n}} dA +$$

$$+ \int_{(ut)} \rho \underbrace{(V_j - V_c)(\cos\theta\hat{x} + \sin\theta\hat{y})}_{\bar{V}_r} \underbrace{(V_j - V_c)(\cos\theta\hat{x} + \sin\theta\hat{y})}_{\bar{V}_r} \cdot \underbrace{(\cos\theta\hat{x} + \sin\theta\hat{y})}_{\hat{n}} dA$$

$$= A_j \rho (V_j - V_c)(V_c - V_j)\hat{x} + A_j \rho (V_j - V_c)^2 (\cos\theta\hat{x} + \sin\theta\hat{y})$$

$$= \rho A_j (V_j - V_c)^2 [(\cos\theta - 1)\hat{x} + \sin\theta\hat{y}]$$

a)
$$\begin{cases} F_x = \rho A_j (V_j - V_c)^2 (1 - \cos\theta) \\ F_y = \rho A_j (V_j - V_c)^2 \sin\theta \\ \bar{F} = -F_x\hat{x} + F_y\hat{y} \end{cases}$$

b) Effekt till vagnen:

$$P = \underbrace{V_c \cdot F_x}_{\text{endast hastighet i x-riktning!}} = \rho A_j V_c (V_j - V_c)^2 (1 - \cos\theta)$$

3.18 forts.

3.16 (3)

c) Sök $|\overline{F}|_{max}$

$$|\overline{F}| = \rho A_j (V_j - V_c)^2 \underbrace{\sqrt{(\cos\theta - 1)^2 + \sin^2\theta}}_{= 2-2\cos\theta} = \sqrt{(-F_x)^2 + F_y^2}$$

$$|\overline{F}| = \rho A_j (V_j - V_c)^2 \sqrt{2 - 2\cos\theta}$$

$|F|$ är maximalt då $V_c = 0$

ty det ger max av $(V_j - V_c)^2$

d) Sök P_{max}

$$P = \rho A_j V_c (V_j - V_c)^2 (1 - \cos\theta)$$

$$= \rho A_j (1-\cos\theta) \; V_c (V_j^2 - 2V_j V_c + V_c^2)$$

$$= C_1 \cdot (V_c^3 - 2V_j V_c^2 + V_j^2 V_c)$$

$$\frac{dP}{dV_c} = 3C_1 V_c^2 - 4C_1 V_j V_c + C_1 V_j^2$$

P_{max} fås då $\frac{dP}{dV_c} = 0$

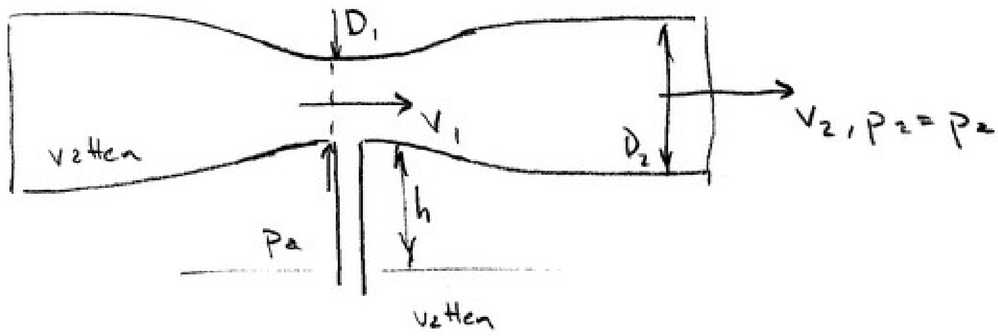
$$\frac{dP}{dV_c} = 0 \Rightarrow V_c' = \frac{2V_j}{3} \pm \sqrt{\frac{4}{9}V_j^2 - \frac{1}{3}V_j^2} = V_j\left(\frac{2}{3} \pm \frac{1}{3}\right)$$

$V_c = V_j \Rightarrow P = 0$

$V_c = \frac{1}{3} V_j \Rightarrow$ maximal effekt P_{max}

3.21

3.17



Hydrostatiska spänningstillståndet:

$$p_1 - p_a = -\rho g h \Rightarrow p_1 = p_a - \rho g h$$

Bernoullis utvidgade ekv:

$$p_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2 + \rho w_s + \Delta p_f$$

Utan förluster: $\begin{cases} w_s = 0 \\ \Delta p_f = 0 \end{cases}$

Vet att: $z_1 = z_2$

$p_1 = p_a - \rho g h$

$p_2 = p_a$

$$\Rightarrow p_a - \rho g h + \frac{1}{2}\rho V_1^2 = p_a + \frac{1}{2}\rho V_2^2$$

Använd kontinuitet för att uttrycka V_2 i termer av V_1:

{1-dim, stationär strömning}

$$Q_1 = Q_2 \Leftrightarrow V_1 A_1 = V_2 A_2$$

$$V_2 = \frac{\pi D_1^2}{4} \Big/ \frac{\pi D_2^2}{4} \cdot V_1 = \left(\frac{D_1}{D_2}\right)^2 V_1$$

$$\Rightarrow p_a - \rho g h + \frac{1}{2}\rho V_1^2 = p_a + \frac{1}{2}\rho \left(\frac{D_1}{D_2}\right)^4 V_1^2$$

$$\frac{1}{2}\rho\left[1 - \left(\frac{D_1}{D_2}\right)^4\right] V_1^2 = \rho g h$$

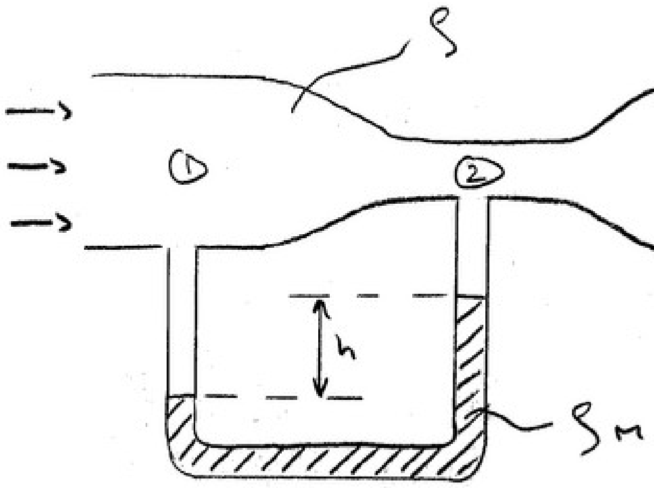
$$V_1 = \sqrt{\frac{2gh}{1-(D_1/D_2)^4}}$$

3.27

3.18

VENTURIMETER:

VISA ATT: $Q = \frac{A_2}{\sqrt{1-(D_2/D_1)^4}}\sqrt{\frac{2gh(\rho_m-\rho)}{\rho}}$



Bernoullis ekv. (stationärt, inkompr, förlustfritt)

$$p_1 + \rho\frac{V_1^2}{2} = p_2 + \rho\frac{V_2^2}{2}$$

Hydrostatik ger: $p_1 - p_2 = (\rho_m - \rho)gh$

Kontinuitet ger: $Q_1 = Q_2 = Q$

$$\Rightarrow A_1V_1 = A_2V_2 \Rightarrow V_1 = \frac{D_2^2}{D_1^2}V_2$$

Bernoulli:

$$\frac{1}{2}\rho\left(V_1^2 - V_2^2\right) + p_1 - p_2 = 0$$

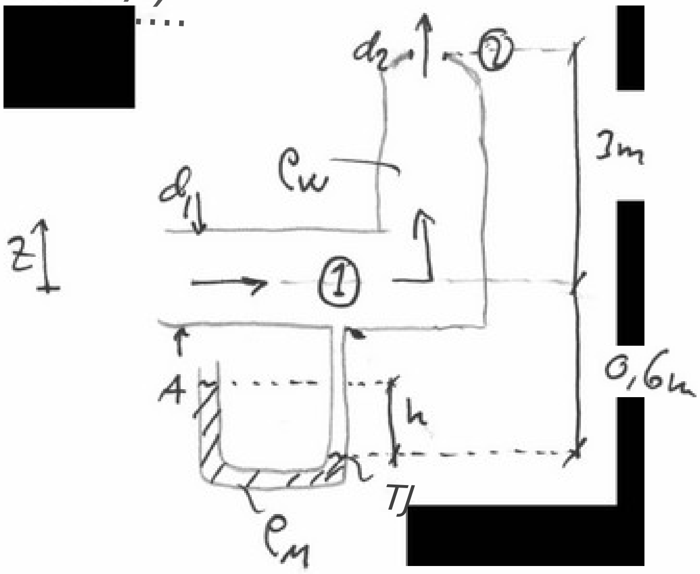
$$\frac{1}{2}\rho\left[\left(\frac{D_2^2}{D_1^2}\frac{Q}{A_2}\right)^2 - \left(\frac{Q}{A_2}\right)^2\right] + (\rho_m - \rho)gh = 0$$

$$\left(\frac{Q}{A_2}\right)^2\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] = \frac{2gh(\rho_m-\rho)}{\rho}$$

$$Q = \frac{A_2}{\sqrt{1-(D_2/D_1)^4}}\sqrt{\frac{2gh(\rho_m-\rho)}{\rho}}$$

V.S.V.!

3.19



Known:

Continuity through the pipe gives V_2 since $Q_2 \equiv Q_1 \Rightarrow V_2 A_2 = V_1 A_1 \Rightarrow$

$$V_2 = V_1 \frac{A_1}{A_2} = V_1 \frac{d_1^2}{d_2^2} \qquad \left(A = \frac{\pi d^2}{4}\right) \qquad \text{this in [A]}$$

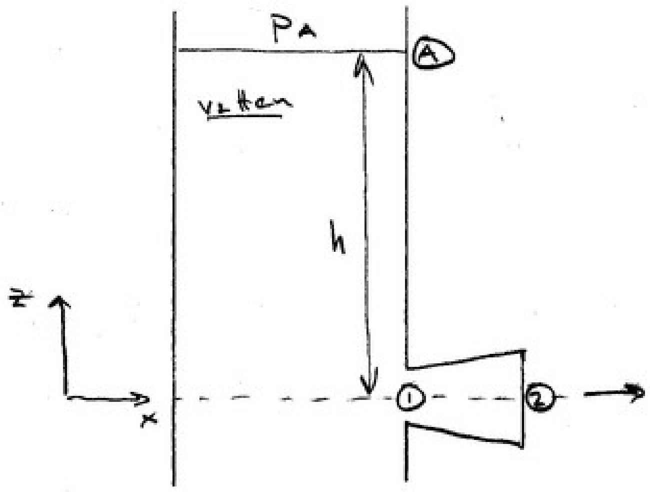
$$P_1 - P_2 = \frac{\rho_w V_1^2}{2}\left[\left(\frac{d_1}{d_2}\right)^4 - 1\right] + \rho_w g(\underbrace{z_2 - z_1}_{3m}) = 34,11 \text{ kPa}$$

$$\ldots - \rho_M g(\underbrace{z_B - z_A}_{-h}) \Rightarrow$$

0,2

3.24

3.20



× Försumma förluster

× Sök h sådant att $p_1 = p_v$ (ångbildningstryck för vatten)

× $p_v = 4.242$ kPa vid T=30°C (tabell A.5 s 812)

→ fri stråle

× $D_2 = 5$cm

× $D_1 = 8$cm

Bernoullis ekv. med inlopp Ⓐ och utlopp ①

$$p_A + \rho g h = p_v + \frac{1}{2}\rho V_1^2 \qquad (1)$$

Bernoullis ekv. ① → ② ($z_1 = z_2$)

$$p_v + \frac{1}{2}\rho V_1^2 = p_A + \frac{1}{2}\rho V_2^2 \qquad (2)$$

Kontinuitets ekv. mellan ① och ②

(från 3.21) : $V_2 = \left(\frac{D_1}{D_2}\right)^2 V_1 \qquad (3)$

sätt in (3) i (2) och lös för $\frac{1}{2}\rho V_1^2$:

$$p_v + \frac{1}{2}\rho V_1^2 = p_A + \frac{1}{2}\rho\left(\frac{D_1}{D_2}\right)^4 V_1^2$$

$$\Rightarrow \frac{1}{2}\rho V_1^2 = \frac{p_A - p_v}{1 - \left(\frac{D_1}{D_2}\right)^4}$$

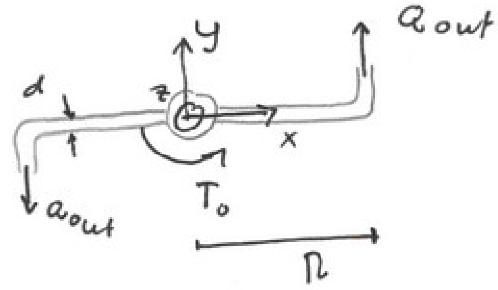
sätt in i (1) och lös för h:

$$p_A + \rho g h = p_v + \frac{p_A - p_v}{1 - \left(\frac{D_1}{D_2}\right)^4}$$

$$\rho g h = (p_A - p_v)\left[\frac{1}{1 - (D_1/D_2)^4} - 1\right] = \frac{p_A - p_v}{\left(\frac{D_2}{D_1}\right)^4 - 1}$$

$$h = \frac{p_A - p_v}{\rho g\left[(D_2/D_1)^4 - 1\right]} \approx 1.76\text{m}$$

P 3.149 | 3.21



Known:

$Q_{in} = \frac{15}{1000 \cdot 60} \frac{m^3}{s}$

$R = 0.15 m$

$d = 0.006 m$

$\rho = 998 \, kg/m^3$

Task: a) Find T_o so that the sprinkler is fixed (not moving)

b) Find the runaway speed (rotation if no retarding torque)

Assume: Steady-state, incomp., fix CV (a)

Solution: Use RTT for conservation of angular momentum (3.59)

$$\sum \tilde{M}_o = \frac{d}{dt}\left(\int_{CV} (\tilde{r} \times \tilde{V}) \rho \, dV\right) + \int_{CS} (r \times \tilde{V}) \rho (\tilde{V} \cdot \tilde{n}) \, dA =$$

(0, SS $\rightarrow \frac{d}{dt} = 0$)

$$= \int_{in} (\tilde{r} \times \tilde{V})_{in} \rho (\tilde{V} \cdot \tilde{n})_{in} \, dA + 2 \int_{out} (\tilde{r} \times \tilde{V})_{out} \rho (\tilde{V} \cdot \tilde{n})_{out} \, dA =$$

($\tilde{r}_{in} = 0$)

$$= 2 \int_{out} \left(\begin{bmatrix} R \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ V_{out} \\ 0 \end{bmatrix}\right) \rho \left(\begin{bmatrix} 0 \\ V_{out} \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) dA = \ldots = 2 \int_{out} \rho V_{out}^2 R \, dA$$

$$= 2 \rho V_{out}^2 R \cdot A$$

$$V_{out} = \frac{Q_{in}}{2A} = \frac{2 Q_{in}}{\pi d^2}$$

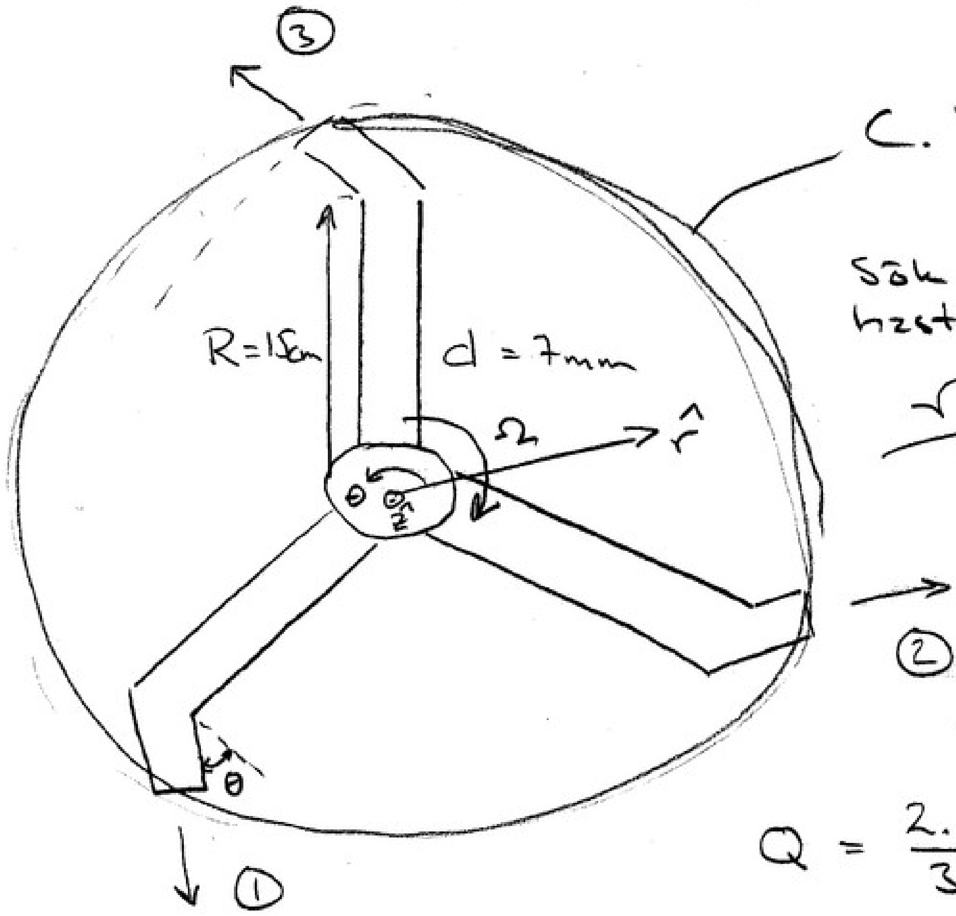
$$\Rightarrow \sum M_o = T_o = 2 \rho \left(\frac{2 Q_{in}}{\pi d^2}\right)^2 R \cdot \frac{\pi d^2}{4} = 0.165 \, Nm$$

b) Rotational speed if $T_o = 0$, i.e. no breaking torque.

$$\omega = \frac{V}{R} = \frac{2 Q_{in}}{\pi d^2 R} = 29.47\ldots \, rad/s = 29.47\ldots \frac{60}{2\pi} \, rpm = 281 \, rpm$$

3.19

3.22 (1)



Sök rot. hastigheten Ω

$$Q = \frac{2.7}{3} = 0.9\, m^3/h = 2.5\cdot 10^{-4}\, m^3/s$$

Impulsmomentsatsen:

RTT med $\begin{cases} B = \bar{H}_o = \int_{sys} \bar{r}\times\bar{V}\, dm \\ \beta = \frac{d\bar{H}_o}{dm} = \bar{r}\times\bar{V} \end{cases}$

$$\frac{d\bar{H}_o}{dt} = \sum \bar{M}_o = \frac{d}{dt}\int_{c.v.} (r\times\bar{V})\,\rho\, d\mathcal{V} + \int_{c.s.} (r\times\bar{V})\,\rho(\bar{V}\cdot\hat{n})\, dA$$

gäller för icke-deformerbar c.v.

$$\frac{d}{dt}\int_{c.v.} (\bar{r}\times\bar{V})\,\rho\, d\mathcal{V} = 0$$ ty stationärt och inkompressibelt

$$\sum \bar{M}_o = \bar{0}$$ (inga yttre krafter)

$$\Rightarrow \int_{①} (\bar{r}\times\bar{V})\rho(\bar{V}\cdot\hat{n})\, dA + \int_{②} (\bar{r}\times\bar{V})\rho(\bar{V}\cdot\hat{n})\, dA + \int_{③} \ldots\ldots = 0$$

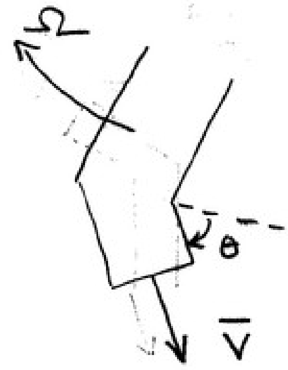
3.19
forts.

3.22 (2)

Beräkna integral över utlopp ①, ② och ③

$$\hat{n} = \sin\theta\,\hat{r} + \cos\theta\,\hat{\theta}$$

$$\bar{V} = \frac{Q}{A}\sin\theta\,\hat{r} + \left(\frac{Q}{A}\cos\theta - \Omega R\right)\hat{\theta}$$



$$\bar{r}\times\bar{V} = R\left(\frac{Q}{A}\cos\theta - \Omega R\right)\hat{z}$$

$$\int_{①②③} (\bar{r}\times\bar{V})\,\rho\,\bar{V}\cdot\hat{n}\,dA = (\bar{r}\times\bar{V})(\dot{m}_1+\dot{m}_2+\dot{m}_3) = 0$$

$$\Rightarrow \quad \bar{r}\times\bar{V} = \bar{0}$$

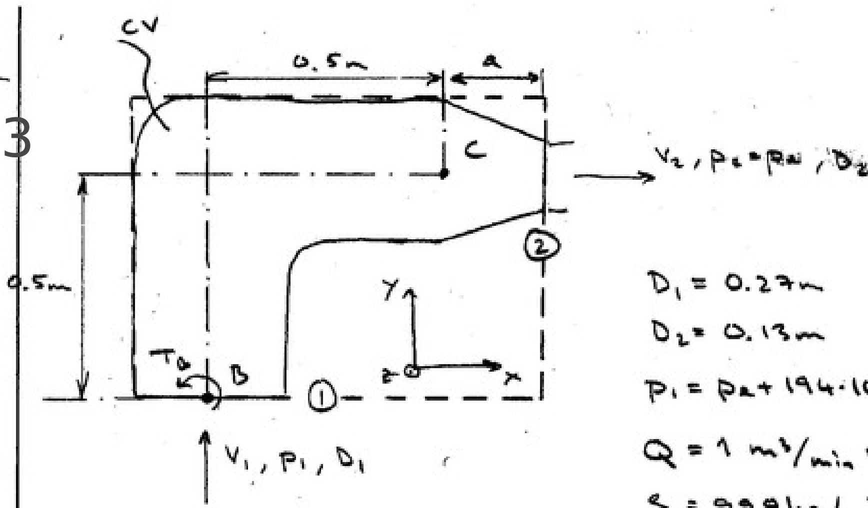
$$R\left(\frac{Q}{A}\cos\theta - \Omega R\right) = 0$$

$$\Omega = \frac{Q\cos\theta}{A\cdot R} = \frac{2.5\cdot 10^{-4}\cos\theta}{\pi\cdot 0.0035^2\cdot 0.15}$$

a) $\Omega \approx 43.3$ rad/s $\approx$ 414 varv/min

b) $\Omega \approx 33.2$ rad/s $\approx$ 317 varv/min

3.23



$D_1 = 0.27$ m

$D_2 = 0.13$ m

$p_1 = p_a + 194 \cdot 10^3$ Pa

$Q = 1$ m³/min $= \ldots$ m³/s

$\rho = 998$ kg/m³

Impulsmomentsatsen:

RTT med $B = \bar{H}_0 = \int_{sys} (\bar{r} \times \bar{V})\, dm$

$$b = \frac{d\bar{H}_0}{dm} = \bar{r} \times \bar{V}$$

$$\sum \bar{M}_B = \frac{\partial}{\partial t}\int_{CV} (\bar{r}\times\bar{V})\rho\, dV + \int_{CS} (\bar{r}\times\bar{V})\rho(\bar{V}\cdot\hat{n})\, dA$$

Fix C.V. och stationärt: $\frac{\partial}{\partial t} = 0$

$$\sum \bar{M} = \underbrace{\sum(\bar{r}\times\bar{F}_{tryck})}_{=0} + T_B\,\hat{z} = \int_{CS} (\bar{r}\times\bar{V})\rho(\bar{V}\cdot\hat{n})\, dA =$$

{räknar bara med övertryck och $\bar{r}_1 = \bar{0}$}

$$= \underbrace{\int_{1} (\bar{0}\times\bar{V}_1)\rho(\bar{V}_1\cdot(-\hat{y}))\, dA}_{=0} + \int_{2} (\bar{r}_2\times\bar{V}_2)\rho(\bar{V}_2\cdot\hat{x})\, dA$$

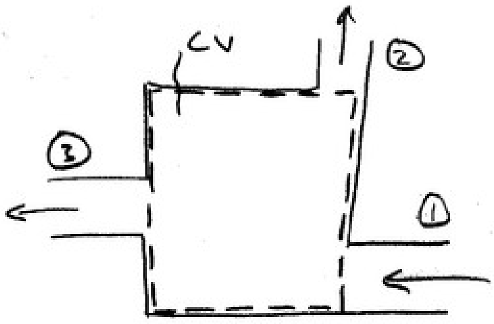
$$= \left\{ \begin{array}{l} \bar{r}_2 = (0.5+a,\ 0.5,\ 0),\quad \bar{V}_2 = (V_2, 0, 0) \\ \bar{r}_2\times\bar{V}_2 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0.5+a & 0.5 & 0 \\ V_2 & 0 & 0 \end{vmatrix} = -0.5 V_2\,\hat{z} \\ \bar{V}_2\cdot\hat{x} = V_2 \end{array} \right\} = \int -0.5 \ldots \hat{z}\,\rho V_2\, dA$$

$$-0.5\rho V_2^2 A_2\,\hat{z} = -\frac{2\rho Q^2}{\pi D_2^2}\,\hat{z} \approx 10.5 \text{ Nm medurs}$$

3.20

3.24

$$\dot{Q}-\dot{W}_s = \frac{\partial}{\partial t}\int_{cv}(\hat{u}+\tfrac{1}{2}V^2+gz)\rho\, d\mathcal{V} + \int_{cs}(\hat{u}+\tfrac{1}{2}V^2+gz)\rho(\bar{V}\cdot\hat{n})dA$$



$\rho = 998\ kg/m^3$

$D_1 = 0.09\ m$, $Q_1 = 0.0611\ m^3/s$, $p_1 = 150\cdot 10^3\ Pa$

$D_2 = 0.07\ m$, $Q_2 = 0.0278\ m^3/s$, $p_2 = 225\cdot 10^3\ Pa$

$D_3 = 0.04\ m$, $Q_3 = 0.0333\ m^3/s$, $p_3 = 265\cdot 10^3\ Pa$

$$V_1 = \frac{4Q_1}{\pi D_1^2} = 9.60\ m/s$$

$$V_2 = \frac{4Q_2}{\pi D_2^2} = 7.22\ m/s$$

$$V_3 = \frac{4Q_3}{\pi D_3^2} = 26.50\ m/s$$

Stationärt : $\frac{\partial}{\partial t} = 0$

Försumma värmeöverföring : $\dot{Q} = 0$

Försumma gravitation : $gz = 0$

Försumma temp. effekter : $\hat{u} = const.$

$$-\dot{W}_s = \int_{cs}(\hat{u}+\frac{p}{\rho}+\tfrac{1}{2}V^2)\rho(\bar{V}\cdot\hat{n})dA$$

1-dim in- och utlopp:

$$-\dot{W}_s = \rho Q_2(\hat{u}+\frac{p_2}{\rho}+\tfrac{1}{2}V_2^2)+\rho Q_3(\hat{u}+\frac{p_3}{\rho}+\tfrac{1}{2}V_3^2) - \rho Q_1(\hat{u}+\frac{p_1}{\rho}+\tfrac{1}{2}V_1^2) =$$

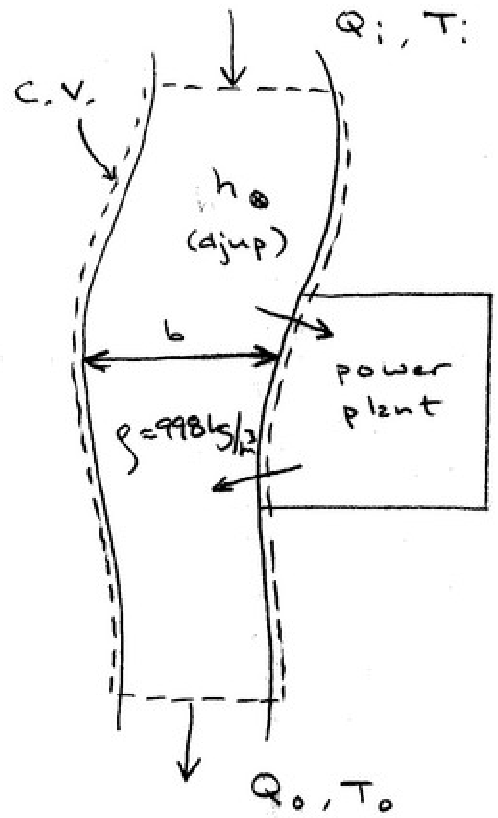
$$= Q_2(p_2+\tfrac{1}{2}\rho V_2^2)+Q_3(p_3+\tfrac{1}{2}\rho V_3^2)-Q_1(p_1+\tfrac{1}{2}\rho V_1^2)$$

$$= 15497\ W$$

$$\underline{\dot{W}_s = -15.5\ kW}$$

3.26

3.25



KANT

$Q_i = 2.5 m^3/s$

$T_i = 18°C$

$b = 45 m$

$h = 2.7 m$

$d\hat{h} = c_p dT$

$\dot{Q} = 55 MW$

$c_p = 4182 J/kgK$

Kontinuitet ger: $Q_i = Q_o \Rightarrow \dot{m} = \dot{m}_i = \dot{m}_o$

Använd nu energiekvationen:

RTT för $\begin{cases} B = E \\ \beta = \frac{dE}{dm} = e \end{cases}$

(3.63) $\underbrace{\dot{Q} - \dot{W}_s - \dot{W}_v}_{=0} = \underbrace{\frac{\partial}{\partial t} \int_{C.V.} (\hat{u} + \frac{1}{2}V^2 + gz)\rho dV}_{=0 \text{ ty stationärt}}$

$+ \int_{C.S} (\hat{h} + \frac{1}{2}V^2 + gz)\rho(\bar{V} \cdot \hat{n}) dA$

(3.64) 1-dim. in- och utlopp:

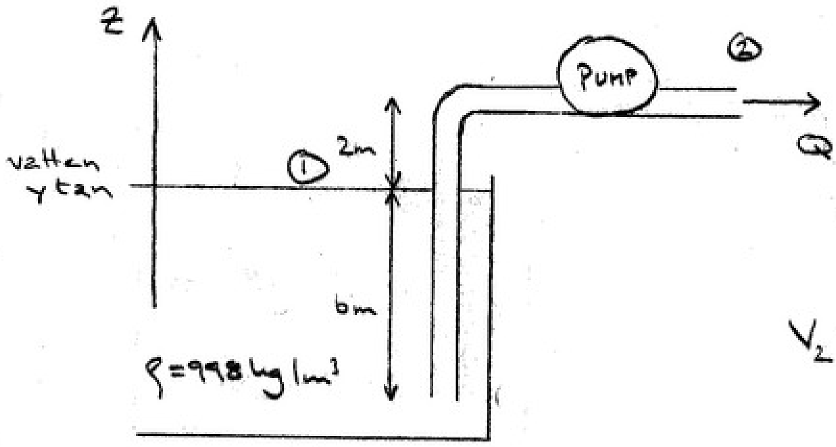
$\dot{Q} = (\hat{h} + \frac{1}{2}V^2 + gz)_o \dot{m}_o - (\hat{h} + \frac{1}{2}V^2 + gz)_i \dot{m}_i$

$\dot{Q} = \dot{m}(\hat{h}_o - \hat{h}_i) = \dot{m} c_p (T_o - T_i)$

$T_o = \frac{\dot{Q}}{\dot{m} c_p} + T_i = \frac{55 \cdot 10^6}{998 \cdot 2.5 \cdot 4182} + 18 \approx \underline{23.3°C}$

3.25

3.26



$Q = 220\, m^3/h$

$\approx 0.0611\, m^3/s$

$V_2 \approx 31.12\, m/s$

$h_f = 5\,m \Rightarrow \Delta p_f = \varrho g h_f = 48951.9\, Pa$

ANVÄND BERNOULLIS UTVIDGADE EKV.

$$p_1 + \underbrace{\frac{\varrho V_1^2}{2}}_{\approx 0} + \varrho g z_1 = p_2 + \frac{\varrho V_2^2}{2} + \varrho g z_2 + \Delta p_f - \varrho w_{pump}$$

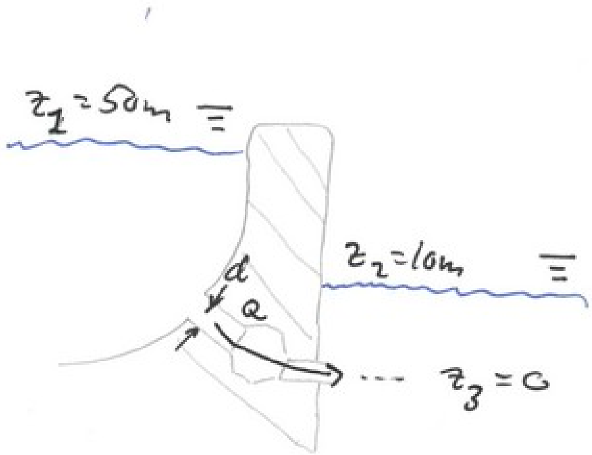
(−) since a pump adds energy

$$\varrho w_{pump} = \frac{\varrho V_2^2}{2} + \varrho g (z_2 - z_1) + \Delta p_f$$

$$P_{pump} = Q \varrho w_{pump} = 0.0611 \left(\frac{998 \cdot 31.12^2}{2} + 998 \cdot 9.81 \cdot 2 + 48951.9 \right)$$

$$\approx 33.7\, kW$$

P 3.184 | 3.27 (1)



Known:

$P = 25\,MW$ | $d = 4m$
$z_1 = 50m$ | $\rho = 998\,kg/m^3$ (water)
$z_2 = 10m$
$z_3 = 0m$
$h_f = \frac{3,5\,V^2}{2g}$

Task: * Find the flow rate through the turbine (two possibilities).
* Which flow rate has a better "conversion efficency"?

Assume: Steady-state, incompressible flow

Solution: Use the definition of pump/turbine power & combine this with the energy equation (7.75)

The turbine power: $P = \rho g h_{turb} Q \Rightarrow h_{turb} = \frac{P}{\rho g Q}$ [A]

Use the energy equation between 1 and 2. (7.75)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_{turb} - h_{pump} + h_f \Rightarrow$$

$P_1 = P_2$ $V_1 = V_2$ $P_1 = P_2$ $V_1 = V_2$ no pump

$$z_1 = z_2 + h_{turb} + h_f \Rightarrow 0 \overset{[A]}{=} \underbrace{\frac{P}{\rho g Q}}_{h_{turb}} + \underbrace{\frac{3,5\,V^2}{2g}}_{h_f} - \underbrace{\Delta z}_{z_1 - z_2}$$

Use the relation between velocity & flow rate, i.e. $Q = V \cdot A \Leftrightarrow V = \frac{Q}{A} = \frac{4Q}{\pi d^2} \Rightarrow$

$$0 = \frac{P}{\rho g Q} + \frac{3,5\left(\frac{4Q}{\pi d^2}\right)^2}{2g} - \Delta z \Rightarrow \frac{P}{\rho g Q} + \frac{28\,Q^2}{g \pi^2 d^4} - \Delta z = 0$$

Multiply by Q: $\frac{P}{\rho g} + \frac{28\,Q^3}{g \pi^2 d^4} - \Delta z\,Q = 0$

Multiply by $\frac{g \pi^2 d^4}{28}$: $\underbrace{\frac{P}{\rho g} \frac{g \pi^2 d^4}{28}}_{A} + Q^3 - \underbrace{\Delta z \frac{g \pi^2 d^4}{28}}_{B} Q \Rightarrow$

3.27 (2)

$A = 2,260\ldots \cdot 10^6$, $B = 3.540\ldots \cdot 10^3$

We have: $Q^3 - BQ + A = 0$ [B]

This equation needs to be solved iteratively (or numerically)

Start by guessing Q to satisfy the [B] equation

Initial guess $Q = 50\ m^3/s$ in [B] ≈ 6,15e6

New guess $Q = 60\ m^3/s$ in [B] ≈ 3,52e5

— " — $Q = 70$ — " — [B] = 1,25e5

— " — $Q = 80$ — " — [B] = −6,03e4 } Root must be between 70 & 80 m^3/s

— " — $Q = 75$ — " — [B] = 2,66e4

— " — $Q = 76$ — " — [B] ≈ 8,34e3

By continue guessing/iterate the three roots are found

$Q_1 = 76,46\ m^3/s$

$Q_2 = 137.90\ m^3/s$

$Q_3 = -214,37\ m^3/s$ ← Impossible!!!

The Q_1 (76.46 m^3/s) must have a better "conversion efficiency" since the head loss term, h_f, will be smaller with a smaller flow rate

$h_f(Q_1) = 6.6\ m$

$h_f(Q_2) = 21.5\ m$ ← larger energy loss in for the turbine.