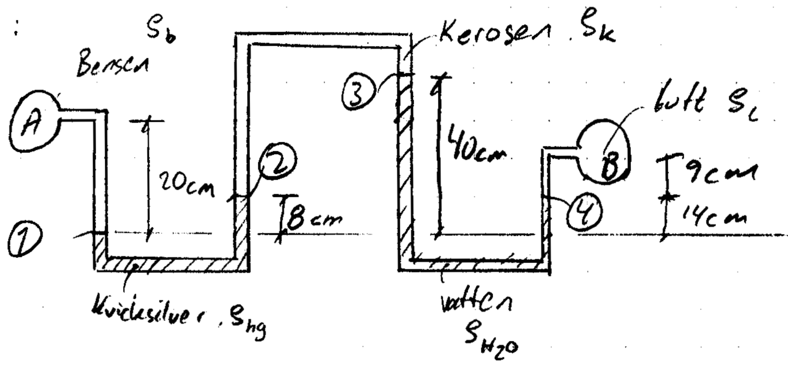
2.1

2.07

Givet:



Temperatur $T = 20°C$

Sökt: Tryckskillnaden mellan A och B

Lösning:

Hydrostatisk tryckfördelning $\nabla p = \rho g$ (2.15)

Densiteter Appendix A.3 $\Rightarrow$ (2.20) $p_2 - p_1 = -\rho g(z_2 - z_1)$

$\rho_b = 881 \frac{kg}{m^3}$

$\rho_{hg} = 13\,550 \frac{kg}{m^3}$

$\rho_k = 804 \frac{kg}{m^3}$

$\rho_{H_2O} = 998 \frac{kg}{m^3}$

(A.2) $\rho_l = 1{,}20 \frac{kg}{m^3}$

$p_A - p_1 = -\rho_b g \cdot 20 \cdot 10^{-2}$

$p_1 - p_2 = -\rho_{hg} g (-8 \cdot 10^{-2})$

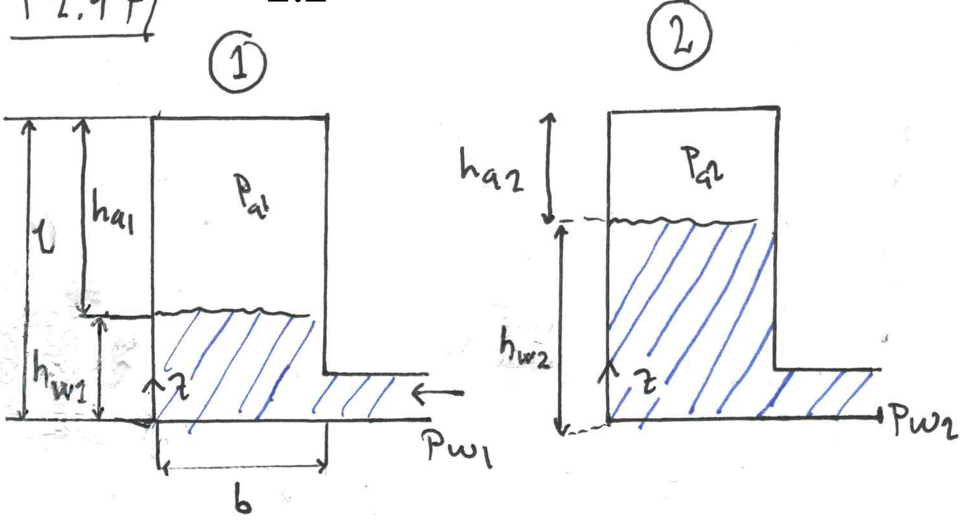
$p_2 - p_3 = -\rho_k g (-32 \cdot 10^{-2})$

$p_3 - p_4 = -\rho_{H_2O} g \cdot 26 \cdot 10^{-2}$

$p_4 - p_B = -\rho_l g (-9 \cdot 10^{-2})$

Summera: $p_A - p_B = -g(\rho_b \cdot 20 - \rho_{hg} \cdot 8 - \rho_k \cdot 32 + \rho_{H_2O} \cdot 26 - \rho_l \cdot 9) \cdot 10^{-2} = 8{,}9$ kPa

P2.47 2.2



Known:

$P_{a1} = 110 \text{ kPa}$

$P_{w2} = 175 \text{ kPa}$

$h_{a1} = 0.75 \text{ m}$

$h_{w1} = 0.35 \text{ m}$

$L = h_{a1} + h_{w1} = 1.1 \text{ m}$

$b = 0.5 \text{ m}$

$\rho_w = 998 \frac{kg}{m^3}$

Task: Calculate h_{w2} if P_{w2} is 175 kPa

Assume: Hydrostatic pressure in fluid (negligible in air)

Solution: Use hydrostatic pressure & the fact that the mass of air must remain!

Hydrostatic pressure at 2:

$$P_{w2} = P_{a2} + \rho g h_{w2} \qquad [A]$$

Use the ideal gas law (1.10) & the fact that air mass constant.

$$P = \rho R T \Rightarrow \frac{P_{a2}}{P_{a1}} = \frac{\rho_{a2} R T}{\rho_{a1} R T} = \frac{\rho_{a2}}{\rho_{a1}} \qquad [B]$$

$$m_{a1} \equiv m_{a2} \Rightarrow \rho_{a1} V_{a1} \equiv \rho_{a2} V_{a2} \Rightarrow \frac{\rho_{a2}}{\rho_{a1}} = \frac{V_{a1}}{V_{a2}} = \frac{h_{a1} A}{h_{a2} A} = \frac{h_{a1}}{h_{a2}} \quad \text{in } [B]$$

$$P_{a2} = \frac{h_{a1}}{h_{a2}} P_{a1} \quad \text{in } [A]$$

$$P_{w2} = \frac{h_{a1}}{h_{a2}} P_{a1} + \rho_w g h_{w2} = [h_{a2} = L - h_{w2}] = P_{w2} = \frac{h_{a1}}{L - h_{w2}} P_{a1} + \rho_w g h_{w2}$$

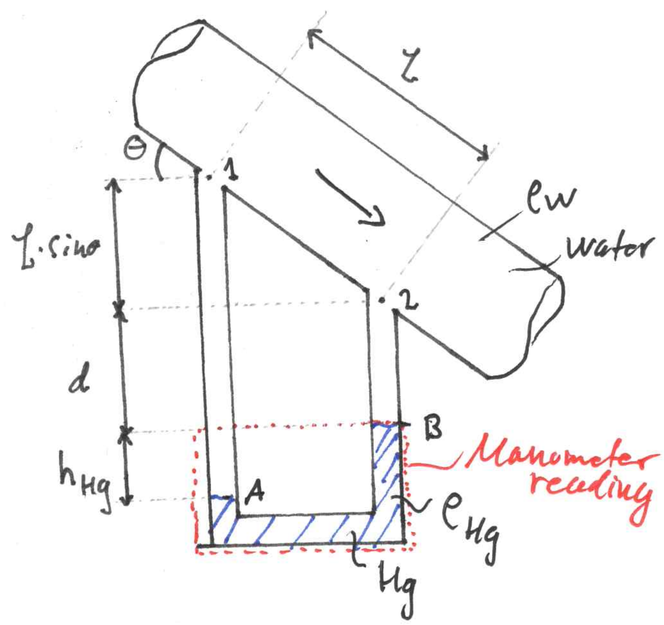
Multiply with $L - h_{w2}$: $P_{w2}(L - h_{w2}) = h_{a1} P_{a1} + \rho_w g h_{w2} (L - h_{w2}) \Rightarrow \ldots$

$$h_{w2}^2 - h_{w2} \underbrace{\left(L + \frac{P_{w2}}{\rho g}\right)}_{a} + \underbrace{\frac{P_{w2} L - P_{a1} h_{a1}}{\rho_w g}}_{b} = 0 \Rightarrow h_{w2} = \frac{a}{2} \pm \sqrt{\left(\frac{a}{2}\right)^2 - b}$$

$$h_{w2} = \begin{cases} 0.612 \text{ m} \\ 18.4 \text{ m} \leftarrow \text{Impossible!} \end{cases}$$

2.3

2.3



Known:

$L = 1.524\ m$

$\theta = 45°$

$h_{Hg} = 0,1524\ m$

$\rho_{Hg} = 13550\ \frac{kg}{m^3}$ | Table A.3

$\rho_w = 998\ \frac{kg}{m^3}$

Find: * Pressure difference between 1 & 2

* Pressure drop due to friction

* What pressure drop does the manometer reading correspond to?

Assume: Hydrostatic through bend & manometer, steady-state ($\tilde{a} = 0$)

Solution: Calculate the pressure drop 1 → 2 through manometer.

Eq. (2.14) $P_1 - P_2 = (P_1 - P_A) + (P_A - P_B) + (P_B - P_2) =$

$$= -\rho_w g \underbrace{(z_1 - z_A)}_{L\sin\theta + \cancel{d} + h_{Hg}} - \rho_{Hg} g \underbrace{(z_A - z_B)}_{-h_{Hg}} - \rho_w g \underbrace{(P_B - P_2)}_{-\cancel{d}}$$

$$= -\rho_w g L\sin\theta - \rho_w g h_{Hg} + \rho_{Hg} g h_{Hg}$$

$$= \underbrace{-\rho_w g L\sin\theta}_{\text{gravity}} + \underbrace{g h_{Hg}(\rho_{Hg} - \rho_w)}_{\text{Manometer reading (Friction)}} = -10,5\ kPa + 18,8\ kPa$$

$P_1 - P_2 = 8,2\ kPa$

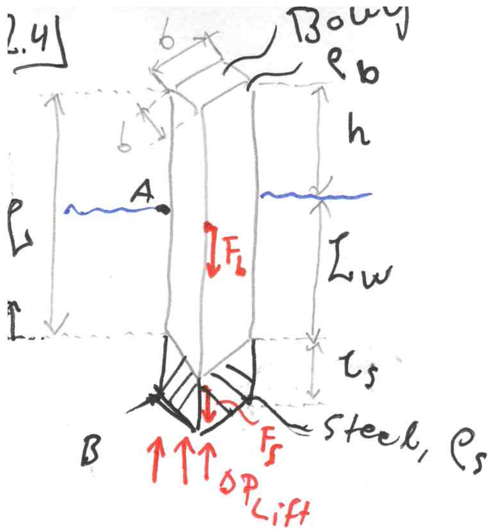
Proof: Use Newton 2nd per-unit volume, Eq. (2.8) through pipe

$$\Sigma \tilde{f} = \rho \cancel{\tilde{a}}^{\,0} = \tilde{f}_{press} + \tilde{f}_{grav} + \tilde{f}_{visc} = -\tilde{\nabla} p + \rho \tilde{g} + \tilde{f}_{visc} = 0$$ Integrate

$$\int_2^1 (-\tilde{\nabla} p + \rho \tilde{g} + \tilde{f}_{visc}) = -(P_1 - P_2) - \rho_w g (z_1 - z_2) + \Delta P_{fric} = 0$$

$$\Delta P_{fric} = (P_1 - P_2) + \rho_w g L\sin\theta = 18,8\ kPa$$

2.4



Known:

$b = 0.05\,m$

$\mathcal{L} = 3.5\,m$

$h = 0.5\,m$

Eq. (1.7) $SG = \frac{\rho_{fluid}}{\rho_{H2O}} = \frac{\rho_{fluid}}{1000}$

$SG_w = 1.024$; $SG_s = 7.85$

$SG_b = 0.6$

$\rho_w = 1000 \cdot SG_w = 1024\ kg/m^3$

$\rho_b = 600\ kg/m^3$

$\rho_s = 7850\ kg/m^3$

$\mathcal{L}_w = \mathcal{L} - h = 3\ m$

Find: Weight of the Steel

Assume: Hydrostatic & Steel assumed as square (instead of Sphere)

Solution: Set up force equilibrium for the bouy & calculate forces

$$\sum F_z = 0 = \Delta p_{Lift} \cdot b^2 - F_s - F_b \qquad [A]$$

$$F_s = m_s g = V_s \rho_s g = b^2 \mathcal{L}_s \rho_s g \qquad [B]$$

$$F_b = m_b g = V_b \rho_b g = b^2 \mathcal{L} \rho_b g$$

Δp_{Lift} ? → Use Newton 2nd per-unit volume, Eq (2.8)

$$\sum \tilde{f} = \rho \underbrace{\cancel{\tilde{a}}}_{0,\ Hydrostat} = -\tilde{\nabla} p + \rho \tilde{g} + \underbrace{\cancel{\tilde{f}_{visc}}}_{0,\ Hydrostat} \Rightarrow \tilde{\nabla} p = \rho \tilde{g} \Rightarrow \text{In } z\text{-direction,}$$

(2.12) $$\frac{dp}{dz} = -\rho g \Rightarrow \text{Integrate} \Rightarrow \int_A^B dp = \int_A^B -\rho g\, dz \Rightarrow \Delta p_{Lift} = -\rho_w g \underbrace{(z_A - z_B)}_{-(\mathcal{L}_w + \mathcal{L}_s)} \rightarrow$$

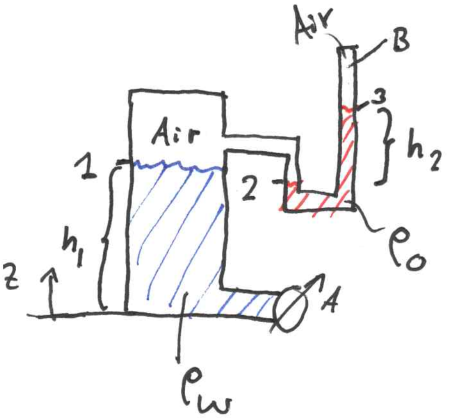
$\Delta p_{Lift} = \rho g (\mathcal{L}_w + \mathcal{L}_s)$, Every thing in [A]

$$\rho_w \cancel{g} (\mathcal{L}_w + \mathcal{L}_s) \cancel{b^2} - \cancel{b^2} \mathcal{L}_s \rho_s \cancel{g} - \cancel{b^2} \mathcal{L} \rho_b \cancel{g} = 0 \qquad \text{Solve for } \mathcal{L}_s$$

$$\mathcal{L}_s (\rho_s - \rho_w) = \mathcal{L}_w \rho_w - \mathcal{L} \rho_b \Rightarrow \mathcal{L}_s = \frac{\mathcal{L}_w \rho_w - \mathcal{L} \rho_b}{\rho_s - \rho_w} = 0.142\ldots\ m$$

$$\underset{[B]}{F_s =} b^2 \mathcal{L}_s \rho_s g = 27.4\ N$$

P2.40 2.5



Known:

$P_A = 140$ kPa
$h_1 = z_1 - z_A = 1.2$ m
$h_2 = z_3 - z_2 = 0.6$ m
$\rho_w = 998$ kg/m³
$\rho_o = 0.827 \cdot \rho_w$ kg/m³

Task: Find the pressure at B

Assume: Hydrostatic pressure, negligible pressure differance in air ($P_1 = P_2$; $P_3 = P_B$)

Solution: Set up hydrostatic pressure differance betwen A → B and calculate the pressure at B, withe eq. (2.14)

(2.14) $$P_A - P_B = (P_A - P_1) + \underbrace{(P_1 - P_2)}_{=0} + \underbrace{(P_2 - P_3)}_{P_2 - P_B} =$$

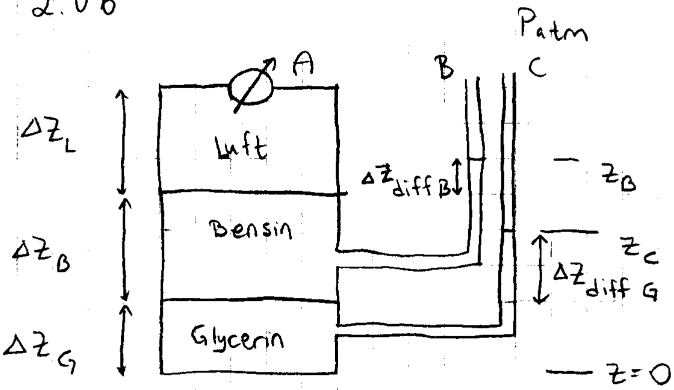
$$P_A - P_B = (P_A - P_1) + (P_2 - P_3) = -\rho_w g \underbrace{(z_A - z_1)}_{-h_1} - \rho_o g \underbrace{(z_2 - z_3)}_{-h_2} \Rightarrow$$

$$P_B = P_A - \rho_w g h_1 - \rho_o g h_2 = 140 \cdot 10^3 - 998 \cdot 9.81 \cdot 1.2 - 0.827 \cdot 998 \cdot 9.81 \cdot 0.6 \Rightarrow$$

$$P_B = 123.4 \text{ kPa}$$

2.6

2.06



$T = 20°C$

$P_A = 1{,}5$ kPa övertryck

$P_B = P_C = 0$ kPa övertryck

$\Delta Z_L = 2\,m$

$\Delta Z_B = 1{,}5\,m$

$\Delta Z_G = 1\,m$

Sökt: Höjderna Z_B och Z_C

Lösning: (A.3) $\rho_B = 680\ kg/m^3$ $\quad \rho_G = 1260\ kg/m^3$

* Vi försummar tryckskillnader i luften pga $\rho g h$ pga att luftens densitet är mkt låg i jämförelse med vätskor och h_r är i detta fallet mkt litet.

$$P_A - P_B = -\rho_B g \big(\underbrace{\Delta Z_B + \Delta Z_G - Z_B}_{-\Delta Z_{diff\,B}}\big)$$

$$1{,}5 \cdot 10^3 = -680 \cdot 9{,}81\,(1{,}5 + 1 - Z_B) \Rightarrow Z_B = \frac{1{,}5 \cdot 10^3 + 16677}{6670{,}8} = \underline{\underline{2{,}72\ m}}$$

$$P_A - P_C = -\rho_B g \Delta Z_B - \rho_G g \big(\underbrace{\Delta Z_G - Z_C}_{-\Delta Z_{diff\,G}}\big)$$

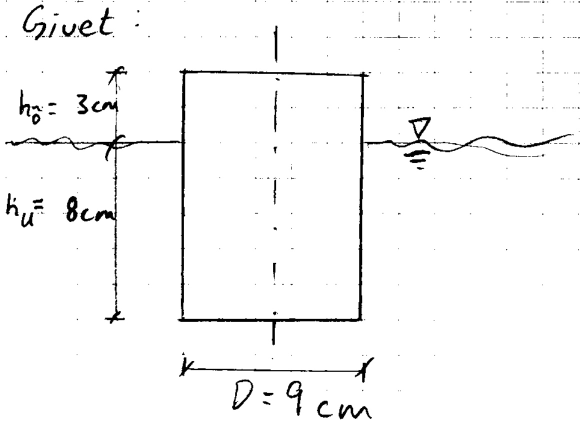
$$1{,}5 \cdot 10^3 = -680 \cdot 9{,}81 \cdot 1{,}5 - 1260 \cdot 9{,}81\,(1 - Z_C) \Rightarrow Z_C = \frac{1{,}5 \cdot 10^3 + 10\,006{,}2 + 12360{,}6}{12360{,}6}$$

$$= \underline{\underline{1{,}93\ m}}$$

2.7

2.07

Givet:



Sökt: Vad väger burken?

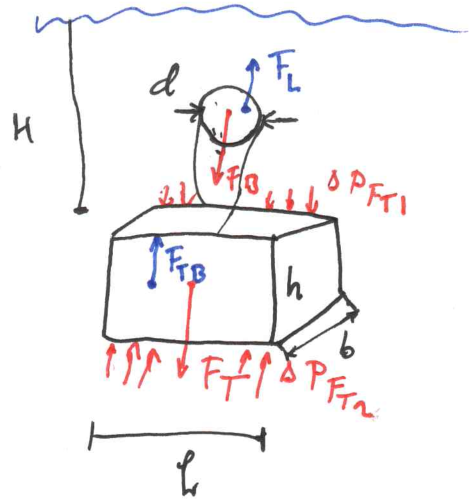
Lösning:

Kraftbalans ↓: $m_B \cdot g - m_u g = 0$

$$\Rightarrow \quad m_B = m_u = \rho_{H_2O} \cdot h_u \cdot \frac{\pi D^2}{4} = 998 \cdot 8 \cdot 10^{-2} \cdot \frac{\pi \cdot (9 \cdot 10^{-2})^2}{4} = 0{,}508\ kg$$

$$W_B = 4{,}98\ N$$

P2.118 | 2.8



Known:

$F_B = 1\,kN$

$F_T = 30\,kN$

$\rho_w = 1025\ kg/m^3$ (Table A.3)

$L = 1{,}5\,m$, $b = 0{,}6\,m$, $h = 0{,}5\,m$

* Balloon must lift 20% extra of treasure weight

Task: Find the balloon diameter d

Assume: Hydrostatic, negligible air weight.

Solution: Use force equilibrium & the definition of Buoyancy

$$\sum F = 0 = F_L + \underbrace{(F_{TB} - F_T)1{,}2}_{\text{Weight of box}} - F_B \Rightarrow F_L = 1{,}2(F_T - F_{TB}) + F_B \quad [A]$$

$$F_L = \rho_w g V_{Balloon} = \rho_w g \frac{3}{2} r^3 \pi = \rho_w g \frac{1}{6} d^3 \pi$$

, Lift force on balloon if negligible air weight.

$$F_B = \rho_w g V_{Box} = \rho_w g L \cdot b \cdot h$$

, Lift force on treasure box

In [A] gives:

$$\rho_w g \frac{1}{6} d^3 \pi = 1{,}2(F_T - \rho_w g L b h) + F_B \Rightarrow \text{Solve for } d \Rightarrow$$

$$d = \left(\frac{[1{,}2(F_T - \rho_w g L b h) + F_B]\,6}{\rho_w g \pi}\right)^{1/3} = 1{,}82\,m$$