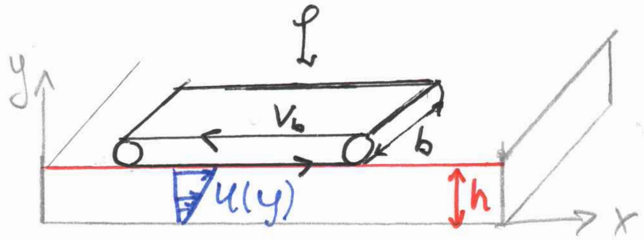
P1.1

1.1



Known:

$V_b = 2.5\,m/s$

$L = 2\,m$

$b = 60\,cm = 0.6\,m$

$h = 3\,cm = 0.03\,m$

$\mu_{SAE\,30} = \left\{ \begin{matrix} \text{Table A.3} \\ \text{F.S P. 31} \end{matrix} \right\} = 0.29 \frac{kg}{m\,s}$

Assume: Negligible effects from air, Linear flow profile, no slip

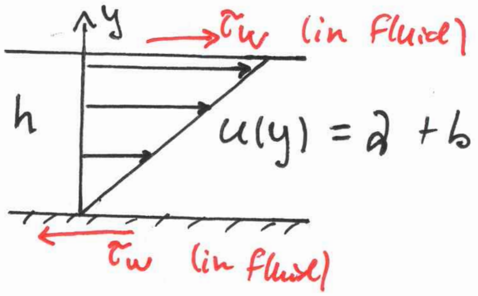
Find: $P(h, L, V, b, \mu)$, P in W

Solution: Use definition of shear stress & Force.

$$\left.\begin{aligned} F &= \text{pressure} \times \text{area} = \tau_w \cdot A \\ E &= \int F\,dL = F \cdot L \\ P &= \frac{dE}{dt} = F\frac{L}{t} = F \cdot V_b \end{aligned}\right\} \quad P = \tau_w \cdot A \cdot V_b \quad [A]$$

?

Eq. (1.23) $\quad \tau_w = \mu \dfrac{du}{dy} \quad [B]$



B.C: $u(y) = a + by$

$u(0) = 0 \Rightarrow a = 0 \quad$ (no slip)

$u(h) = V_b \Rightarrow V_b = bh \Rightarrow b = \dfrac{V_b}{h}$

$\therefore \; u(y) = V_b \dfrac{y}{h}$, in [B]

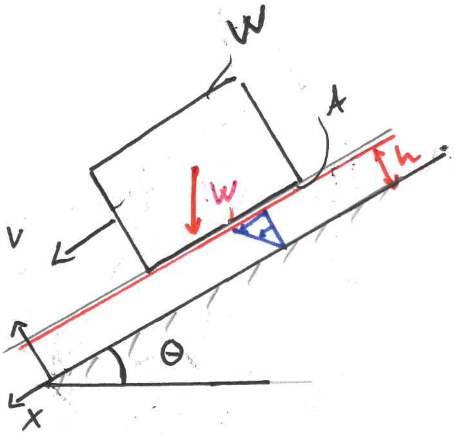
$\tau_w = \mu \dfrac{d}{dy}\left(V_b \dfrac{y}{h}\right) = \dfrac{\mu V_b}{h}$, in [A]

$P(h, L, b, \mu, V_b) = \dfrac{\mu V_b}{h} \cdot A \cdot V_b = \{A = b \cdot L\} = \mu \dfrac{V_b^2}{h} b \cdot L$

$P = 72.5 \; W$

P 1.2

1.2



known:

$m = 6\ kg \Rightarrow$

$\rightarrow F_g = 6 \cdot g = 58.86\ N = W$

$A = 35\ cm^2 = 0.0035\ m^2$

$\theta = 15°$

$h = 1mm = 0.001m$

$\mu = \mu_{SAE30} = \{\text{Table A.3}\} = 0.29 \frac{kg}{ms}$

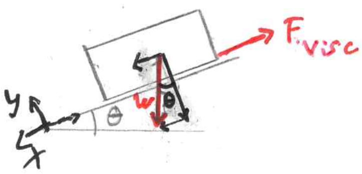
Linear flow profile

Assume: Negligible effects from air, no slip

Derive: The terminal velocity V and calculate V.

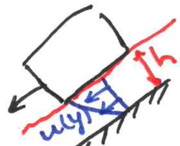
Terminal velocity: $a_x = 0 \Rightarrow \sum F_x = 0$

Force equlibrium of the block:



$\Rightarrow \sum F_x = W \sin\theta - F_{visc} = 0 \quad [1]$?

$F_{visc} = \tau_w \cdot A, \quad \tau_w = \mu \frac{du}{dy}$, Eq (1.23)



Linear flow profile $u(y) = a + by$

No slip: $u(0) = 0 = a$

$u(h) = V = b \cdot h \Rightarrow b = \frac{V}{h} \Big\} \Rightarrow u(y) = V\frac{y}{h}$

$\tau_w = \mu \frac{d}{dy}\left(V\frac{y}{h}\right) = \mu \frac{V}{h} \Rightarrow F_{visc} = \mu \frac{V}{h} \cdot A$ in [1]

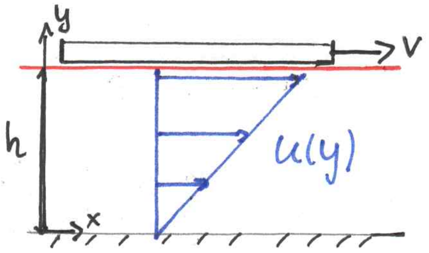
$W \sin\theta - \mu \frac{V}{h} \cdot A = 0 \Rightarrow$

$V = W \sin\theta \frac{h}{\mu A}$

$V = 15\ m/s$

P 1.3

1.3



Known:

$V = 5.5 \; m/s$

$h = 6 mm = 0.006 m$

$\mu = \left\{ \begin{matrix} \text{Table A.3} \\ \text{F.S. 31} \end{matrix} \right\} = 1.49 \; \frac{kg}{m\,s}$

$\rho = \{\text{Table A.3}\} = 1260 \; \frac{kg}{m^3}$

Assume: Linear flow profile, no-slip $\begin{cases} u(0) = 0 \\ u(h) = V \end{cases}$

Find: a) wall shear stress τ_w

b) Reynolds number (Re) if $L = h$

Solution: Use Eq. (1.23) to find τ_w

(1.23) $\quad \tau_w = \mu \frac{du}{dy}$ - ?

$u(y) = a + by$

B.C: $\left. \begin{matrix} u(0) = 0 = a \\ u(h) = V = bh \end{matrix} \right\} \; u(y) = V \frac{y}{h} \Rightarrow \frac{du}{dy} = \frac{V}{h}$

$\tau_w = \mu \frac{V}{h} = 1365.83... = 1366 \; Pa$

b) $Re = \frac{\text{Inertia}}{\text{Viscosity}} = \frac{\rho V L}{\mu} = \frac{\rho V h}{\mu} = 27.9... = 28$

Eq.(1.24)

P1.4

1.4

Is the Stokes-Oseen formula dimensionally homogeneous?

$$\underbrace{F}_{\mathrm{I}} = \underbrace{3\pi \mu D V}_{\mathrm{II}} + \underbrace{\frac{9\pi}{16} \rho V^2 D^2}_{\mathrm{III}}$$

Dimensions of units:

$F: \frac{kg \cdot m}{s^2}$

$\mu: \frac{kg}{m \cdot s}$ (e.g. see Table A.3)

$D: m$

$V: \frac{m}{s}$

$\rho: \frac{kg}{m^3}$ (e.g. see Table A.3)

I: $\begin{array}{c} F \\ \frac{kg \cdot m}{s^2} \end{array}$

II: $\begin{array}{ccccccc} 3\pi & \mu & D & V & & \\ - & \frac{kg}{\cancel{m} \cdot s} & \cancel{m} & \frac{m}{s} & = & \frac{kg \cdot m}{s^2} \end{array}$ ok!

III: $\begin{array}{ccccccc} \frac{9\pi}{16} & \rho & V^2 & D^2 & & \\ - & \frac{kg}{\cancel{m^3}} & \frac{\cancel{m^2}}{s^2} & \cancel{m^2} & = & \frac{kg \cdot m}{s^2} \end{array}$ ok!

The formula is dimensionally homogeneous!

P1.13 1.5

Known: $\eta_{pump} = \frac{Q \Delta P}{P}$ | $\Delta p = 35\ lbf/in^2$ $P = 16\ hp$
$Q = 40\ l/s = 0{,}04\ \frac{m^3}{s}$

Task: Calculate the pump efficiency, η_{pump}

Solution: Convert all properties to SI units and calculate the efficiency.

Appendix C:

$$\Delta p = 35\ lbf/in^2 = 35 \cdot 6.8948e^3\ Pa = 241318\ Pa$$

$$P = 16\ hp = 16 \cdot 745{,}7\ W = 11931{,}2\ W$$

Insert everything in the efficiency formula:

$$\eta_{pump} = \frac{0{,}04 \cdot 241318}{11931{,}2} = 0{,}81 \quad \text{or} \quad 81\ \%$$

P1.25 1.6

Known: $T = 74°F$

$P = 14.5\ lbf/in^2$

Air

Task: Estimate the fluid density of the air

Solution: Convert the temperature & pressure values into SI units and use the equation of state (ideal gas law)

Ideal gas law, Eq. (1.10)

$$P = \rho R T \Rightarrow \rho = \frac{P}{RT} \quad \dots \quad [A]$$

Appendix C:

$$P = 14.5\ lbf/in^2 = 14.5 \cdot 6.8948e^3\ Pa = 99974.6\ Pa$$

$$T = 74°F = \frac{5}{9}(74°F - 32)\ °C = 23{,}3°C, \quad T = 23.3°C = 23{,}3 + 273\ K = 296{,}3\ K$$

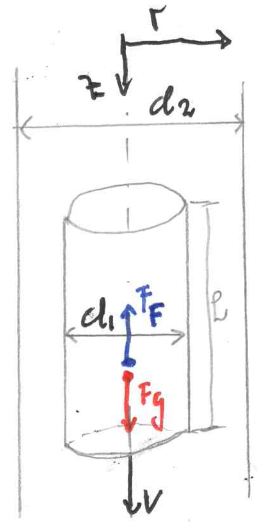
Appendix A:

Table A.2: $R_{air} = 287\ \frac{J}{kg\,K}$

P in Pa, T in K and R → Eq [A] gives:

$$\rho = 1{,}18\ \frac{kg}{m^3}$$

P 1.41   1.7



Known:

$F_g = 30\,N$ | SAE 50 oil:

$d_1 = 0.06\,m$ | Table A.3 →

$l = 0.4\,m$ | $\mu = 0.86\,\frac{kg}{ms}$

$d_2 = 0.0604\,m$

Task: Find velocity, V, for the cylinder at zero accelleration (terminal velocity)

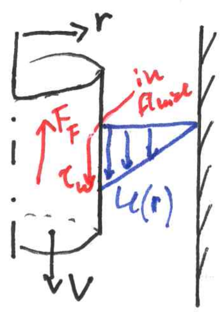
Assume: Linear flow profile

Solution: Use force equlibrium for the cylinder and wall shear stress

$$\sum F = 0 = F_g - F_F \Rightarrow F_g = F_F \quad [A]$$

Definition of pressure/force: $F = \int \tau \, dA = \tau \cdot A = \tau \, l \, \pi d_1$ [B]

Shear stress on cylinder:



$$\tau_w = \mu \frac{du}{dr} \qquad (1.23)$$

Linear flow profile gives: $u(r) = a r + b$

B.C: $u\left(\frac{b_2}{2}\right) = 0 = a\frac{b_2}{2} + b$

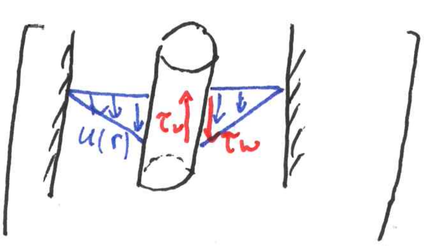
$u\left(\frac{b_1}{2}\right) = V = a\frac{b_1}{2} + b$

$$u(r) = \frac{2V}{d_1 - d_2} \cdot r - \frac{V d_2}{d_1 - d_2}$$

$\tau_w = \mu \frac{du}{dr} = \mu \frac{2V}{d_1 - d_2}$, The wall shear stress and F_F are in oposite direction: [A] & [B]

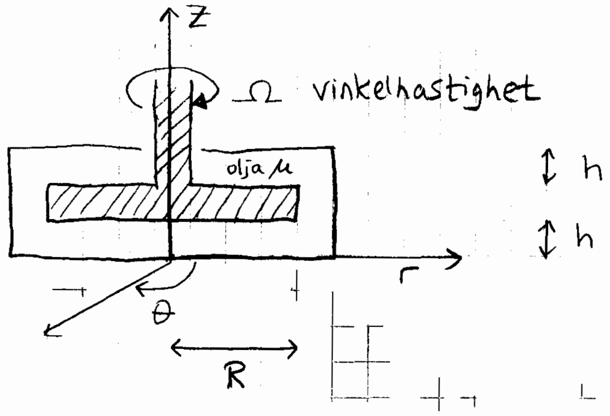
$$F_F = -\tau_w \, l \pi d_1 = F_g$$

$$\Rightarrow F_g = -\mu \frac{2V}{d_1 - d_2} \cdot l \pi d_1 \Rightarrow V = \frac{-F_g (d_1 - d_2)}{l \pi d_1 \, 2\mu} = 0.0925\ m/s$$



1,08

<u>Givet:</u>



Antag linjär hastighetsprofil och försumma skjuvspänningen vid skivändarna.

<u>Sökt:</u> Uttryck för vridmomentet på skivan M_{TOT}

<u>Lösning:</u> Vridmoment $M = F \cdot r$

I detta fall är kraften F, som krävs för att rotera skivan med en viss hastighet, samma kraft som krävs för att övervinna oljans motstånd till rörelse $F = \tau A$

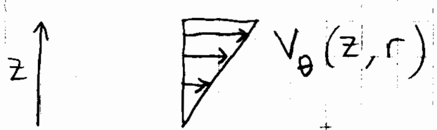
$$M(r) = \tau(r, z=h) \cdot A \cdot r$$

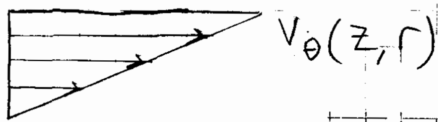
(1) $$M_{TOT} = \left(\int_0^{2\pi} \int_0^R \tau(r, z=h) \cdot r \cdot dA \right) \cdot 2$$ ← för plattans under och översida

* Observera att skjuvspänningen τ är en funktion av avståndet från plattans centrum, detta pga att den tangenta hastigheten $V_\theta = \Omega r$ $\left(\tau = \mu \frac{dV_\theta}{dz} \right)$

för varje punkt längs plattans radie gäller en annorlunda linjär hastighetsprofil

Nära plattans centrum | Långt ifrån plattans c





För linjär hastighetsprofil $\tau = \tau(r)$

$$V_\theta(r,z) = az + b$$

Randvillkor $\begin{cases} V_\theta(r, z=0) = 0 \\ V_\theta(r, z=h) = \Omega r \end{cases}$

$$V_\theta(r, z=0) = a \cdot 0 + b = 0 \Rightarrow b = 0$$

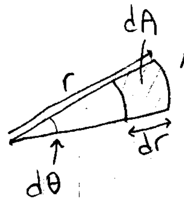
$$V_\theta(r, z=h) = a \cdot h = \Omega r \Rightarrow a = \frac{\Omega r}{h}$$

$$V_\theta(r,z) = \frac{\Omega r}{h}\cdot z$$

$$\tau = \mu\,\frac{d V_\theta(r,z)}{dz} = \mu\cdot\frac{\Omega r}{h}$$

(1) ger:

$$M_{TOT} = 2\cdot\int_0^{2\pi}\int_0^{R} \underbrace{\mu\cdot\frac{\Omega r}{h}}_{\tau}\cdot r\cdot \underbrace{r\cdot d\theta\, dr}_{dA}$$



$\updownarrow d\theta\cdot r$ = bågens längd $\quad dA = d\theta\cdot r\cdot dr$

$$M_{TOT} = 2\cdot\int_0^{2\pi}\int_0^{R} \frac{\mu\Omega}{h}\cdot r^3\cdot d\theta\, dr = 2\cdot\frac{\mu\Omega 2\pi}{h}\frac{R^4}{4} = \underline{\underline{\frac{\mu\Omega\pi R^4}{h}}}$$

1.9 $\quad$ 1.09 $\quad$ Givet: $\quad u = B\,\frac{\Delta P}{\mu}\,(r_0^2 - r^2)$

tryck (ΔP), hastighet (u), viskositet (μ), radie (r)

Sökt: Dimensionen på konstanten B

Lösning: $\left[\frac{m}{s}\right] = B\cdot\left[\frac{kg}{m\,s^2}\right]\left[\frac{ms}{kg}\right]\left[m^2\right] = B\left[\frac{m^2}{s}\right]$

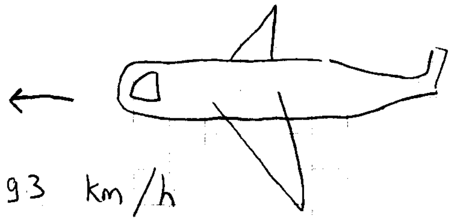
$\Delta P = \frac{F}{A} = \frac{kg\cdot m}{m^2\, s^2} = \left[\frac{kg}{ms^2}\right]$ $\quad \Rightarrow \underline{\underline{B = \left[\frac{1}{m}\right]}}$

$\mu = \left[\frac{kg}{ms}\right]$

eller i de primära dimensionerna $\underline{\underline{B = \left[\frac{1}{L}\right]}}$

1.10

1.10



$v = 893$ km/h

$= \frac{893 \cdot 000}{60 \cdot 60} = 248{,}1$ m/s

Sökt: Vid vilken höjd är planets mach-tal $M = 0{,}8$ då planets hastighet är $\vec{V} = 248{,}1$ m/s ?

Lösning: Machtalet ges av $M = \frac{V}{a}$ där a = ljudets hastighet

Det finns 2 olika sätt att lösa problemet:

1. Ljudets hastighet $a = \frac{V}{M} = \frac{248{,}1}{0{,}8} = 310{,}1$ m/s

   Vid vilken höjd är ljudets hastighet $a = 310$ m/s ?

   $\Rightarrow$ Tabell [A.6] s. 773 $\underline{\underline{z = 7500 \text{ m}}}$

2. Ekv. [1.39] s. 37 $a = \sqrt{kRT}$ för ideal gas, där

   tabell [A.4] torr luft $\Rightarrow \begin{cases} k = \frac{c_p}{c_v} = 1{,}4 & \text{specific-heat ratio} \\ R = 287 \ \frac{m^2}{s^2 K} \\ T \quad \text{okänd} \end{cases}$

Lös ut T

$$M = \frac{V}{a} = \frac{V}{\sqrt{kRT}} \Rightarrow M^2 = \frac{V^2}{kRT} \Rightarrow kRTM^2 = V^2 \Rightarrow T = \frac{V^2}{kRM^2}$$

$$T = \frac{248{,}1^2}{1{,}4 \cdot 287 \cdot 0{,}8^2} = 239 \text{ K}$$ Tabell [A.6] $\Rightarrow$ $\underline{\underline{z = 7500 \text{ m}}}$

1.11

1.11

Givet: Vatten som kokar vid 84 °C

Sökt: Höjd över havet

Lösning: Tabell [A.5] $x_0 = 80^\circ C$, $y_0 = 47{,}35$ kPa; $x_1 = 90^\circ C$, $y_1 = 70{,}11$ kPa

interpolation $P_v(84°C) = y = 47{,}35 + \frac{(70{,}11 - 47{,}35)(84-80)}{(90-80)}$

$= 47{,}35 + 9{,}104 = \underline{\underline{56{,}454 \text{ kPa}}}$

(formel för interpolation

$$y = y_0 + \frac{y_1 - y_0}{x_1 - x_0}(x - x_0)$$
)

Vid detta tryck kokar vatten vid 84°C. Vid vilken höjd över havet återfinns detta tryck? ⇒ Tabell A.6

Tabell [A.6] $x_0 = 57\,718$ Pa, $y_0 = 4500$ m; $x_1 = 54\,008$ Pa, $y_1 = 5000$ m

Interpolation för $Z(56{,}454 \text{ kPa}) = y = 4500 + \frac{(5000-4500)(56{,}454 - 57{,}7)}{(54{,}008 - 57{,}718)}$

$= 4500 + 170{,}35 = \underline{\underline{4670{,}35 \text{ m}}}$