Fluid Mechanics - MTF053

Lecture 22

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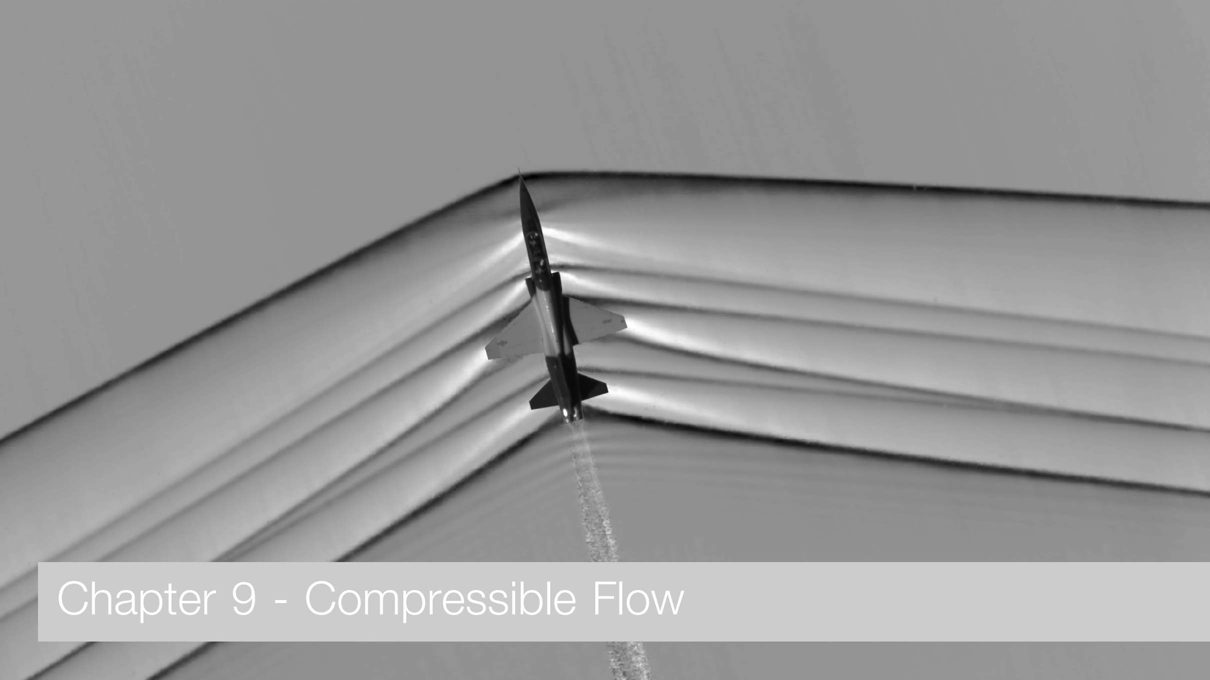
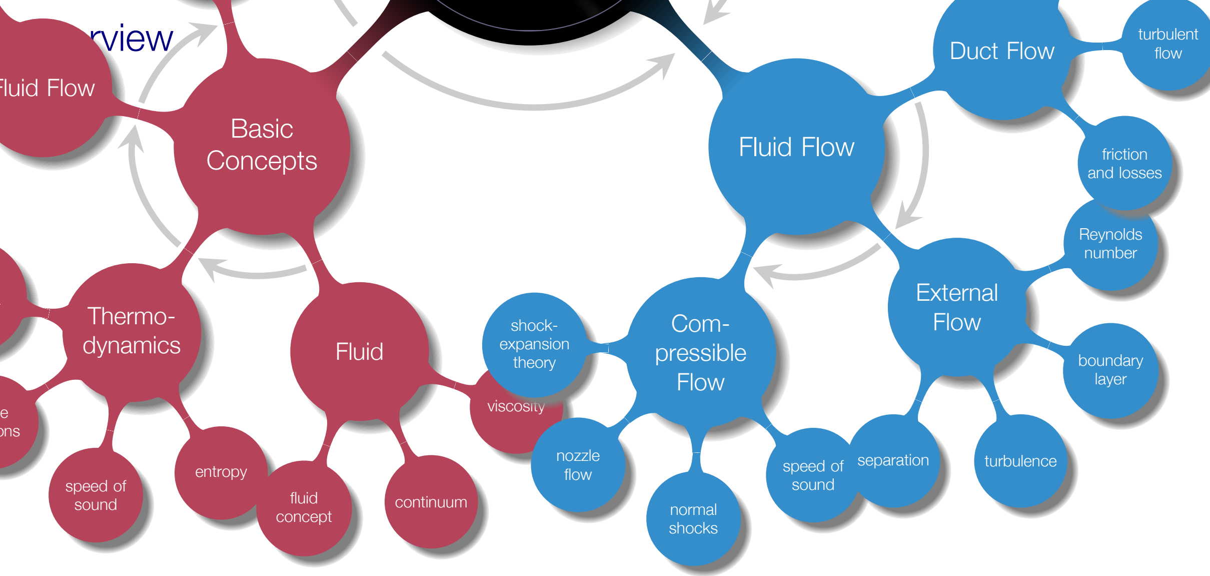


Chapter 9 - Compressible Flow



rview
Fluid Flow
Basic Concepts
Thermo-dynamics
Fluid
e
ons
speed of sound
entropy
fluid concept
continuum
viscosity
Fluid Flow
Duct Flow
turbulent flow
friction and losses
Reynolds number
External Flow
boundary layer
separation
turbulence
Com-pressible Flow
shock-expansion theory
nozzle flow
normal shocks
speed of sound

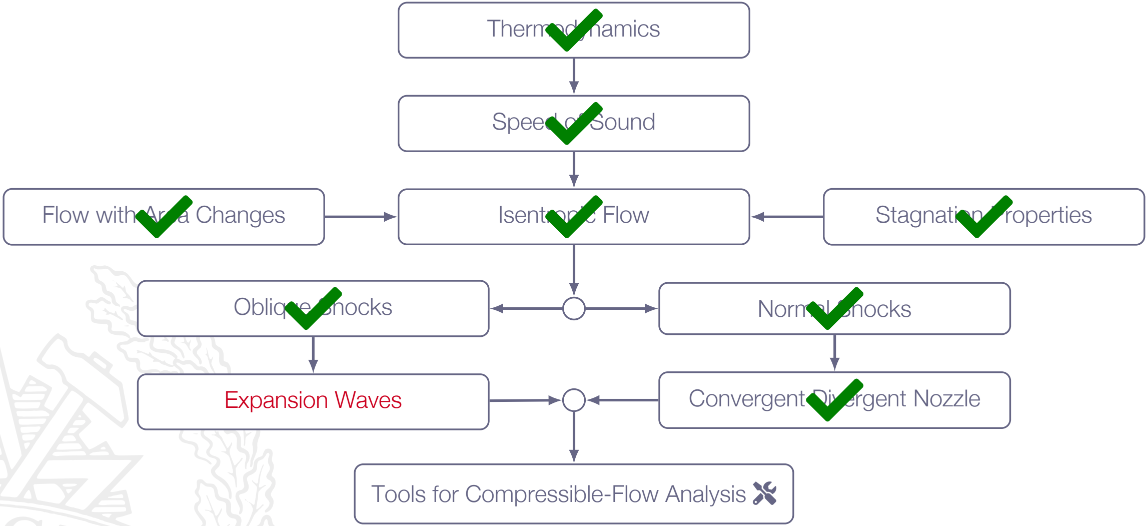
Learning Outcomes

4 Be **able to categorize** a flow and **have knowledge about** how to select applicable methods for the analysis of a specific flow based on category

37 **Understand and explain** basic concepts of compressible flows (the gas law, speed of sound, Mach number, isentropic flow with changing area, normal shocks, oblique shocks, Prandtl-Meyer expansion)

Let's go supersonic ...

Roadmap - Compressible Flow

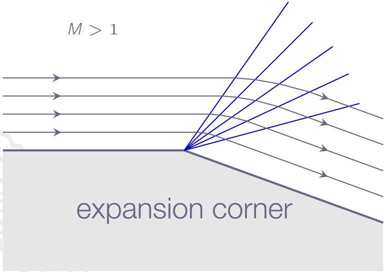


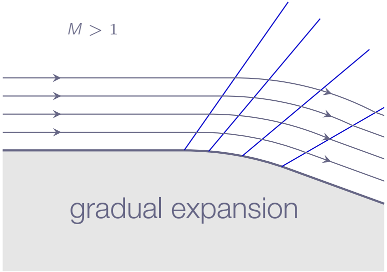
Expansion Waves

- Gradual change of flow angle
- Increasing flow area
- Increasing Mach number
- Accumulation of infinitesimal flow deflections - isentropic

Expansion Waves

What is an expansion wave or expansion region?





The Prandtl-Meyer Function

- The change of flow properties over an expansion region can be calculated using the Prandtl-Meyer function
- The Prandtl-Meyer function derivation is based on the fact that each expansion wave gives an infinitesimal change in flow angle and flow properties

Prandtl-Meyer Function Derivation (*for the interested*)

For small deflection angles, linearization of the θ-β-Mach relation gives

$$\frac{dp}{p} \approx \frac{\gamma M^2}{(M^2-1)^{1/2}} d\theta$$

The momentum equation for inviscid flows gives

$$dp = -d(\rho V^2) = -\rho V dV - V \underbrace{d(\rho V)}_{=0} = -\rho V dV = -\rho V^2 \frac{dV}{V} = -\rho a^2 M^2 \frac{dV}{V} \Rightarrow$$

$$\Rightarrow \{\rho a^2 = \rho\gamma RT = \gamma p\} \Rightarrow \frac{dp}{p} = -\gamma M^2 \frac{dV}{V}$$

Prandtl-Meyer Function Derivation (*for the interested*)

Now, setting the two expressions for dp/p equal

$$-\gamma M^2 \frac{dV}{V} = \frac{\gamma M^2}{(M^2-1)^{1/2}} d\theta \Rightarrow d\theta = -(M^2-1)^{1/2}\frac{dV}{V}$$

$$V = Ma \Rightarrow dV = adM + Mda \Rightarrow \frac{dV}{V} = \frac{dM}{M} + \frac{da}{a}$$

$$d\theta = -(M^2-1)^{1/2}\left(\frac{dM}{M} + \frac{da}{a}\right)$$

Prandtl-Meyer Function Derivation (*for the interested*)

$$d\theta = -(M^2-1)^{1/2}\left(\frac{dM}{M}+\frac{da}{a}\right)$$

$$\frac{a_o}{a} = \left(1+\frac{\gamma-1}{2}M^2\right)^{1/2}$$

$$da = \left(1+\frac{\gamma-1}{2}M^2\right)^{-1/2} da_o + a_o d\left[\left(1+\frac{\gamma-1}{2}M^2\right)^{-1/2}\right]$$

isentropic $\Rightarrow$ $da_o = 0$

$$\frac{da}{a} = \frac{d\left[\left(1+\frac{\gamma-1}{2}M^2\right)^{-1/2}\right]}{\left(1+\frac{\gamma-1}{2}M^2\right)^{-1/2}} = \frac{-\frac{1}{2}\left(1+\frac{\gamma-1}{2}M^2\right)^{-3/2}(\gamma-1)MdM}{\left(1+\frac{\gamma-1}{2}M^2\right)^{-1/2}}$$

Prandtl-Meyer Function Derivation (*for the interested*)

$$d\theta = -(M^2 - 1)^{1/2}\left(\frac{dM}{M} + \frac{da}{a}\right)$$

$$\frac{da}{a} = \frac{-\frac{1}{2}(\gamma - 1)MdM}{1 + \frac{\gamma-1}{2}M^2} \Rightarrow d\theta = -\frac{2(M^2-1)^{1/2}}{2 + (\gamma - 1)M^2}\frac{dM}{M}$$

Introducing ω defined such that: $d\omega = -d\theta$, $\omega = 0$ when $M = 1$

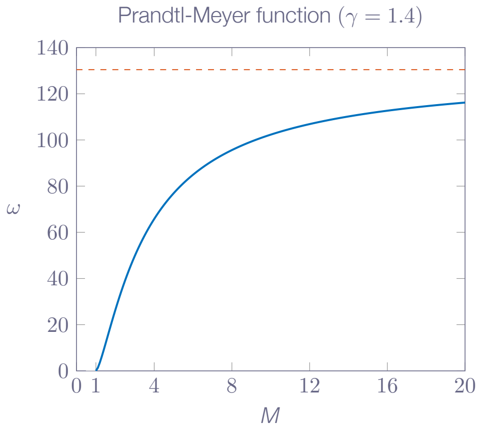
$$\int_0^{\omega} d\omega = \int_1^M \frac{2(M^2-1)^{1/2}}{2 + (\gamma - 1)M^2}\frac{dM}{M}$$

$$\omega(M) = \left(\frac{\gamma+1}{\gamma-1}\right)^{1/2} \tan^{-1}\left(\frac{M^2-1}{(\gamma+1)/(\gamma-1)}\right)^{1/2} - \tan^{-1}(M^2-1)^{1/2}$$

The Prandtl-Meyer Function

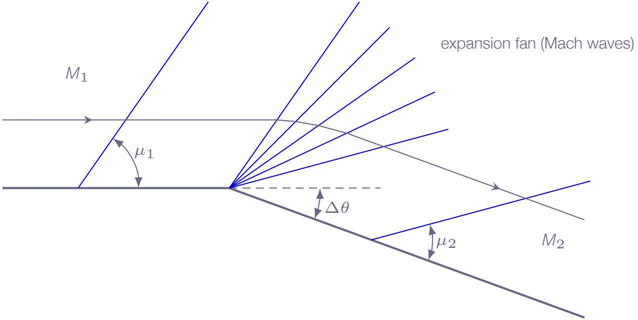
$$\omega(M) = \left(\frac{\gamma+1}{\gamma-1}\right)^{1/2} \tan^{-1}\left(\frac{M^2-1}{(\gamma+1)/(\gamma-1)}\right)^{1/2} - \tan^{-1}(M^2-1)^{1/2}$$

$$\omega(M)|_{M\to\infty} = 130.45^\circ$$



Prandtl-Meyer Expansion Waves

Example:



1. $\theta_1 = 0$, $M_1 > 1$ is given
2. θ_2 is given
3. find M_2 such that $\Delta\theta = \theta_2 - \theta_1 = \omega(M_2) - \omega(M_1)$

Prandtl-Meyer Expansion Waves

Since the flow is isentropic, the isentropic relations apply:

(T_o and p_o are constant)

Calorically perfect gas:

$$\frac{T_o}{T} = \left[1 + \frac{1}{2}(\gamma - 1)M^2\right]$$

$$\frac{p_o}{p} = \left[1 + \frac{1}{2}(\gamma - 1)M^2\right]^{\frac{\gamma}{\gamma-1}}$$

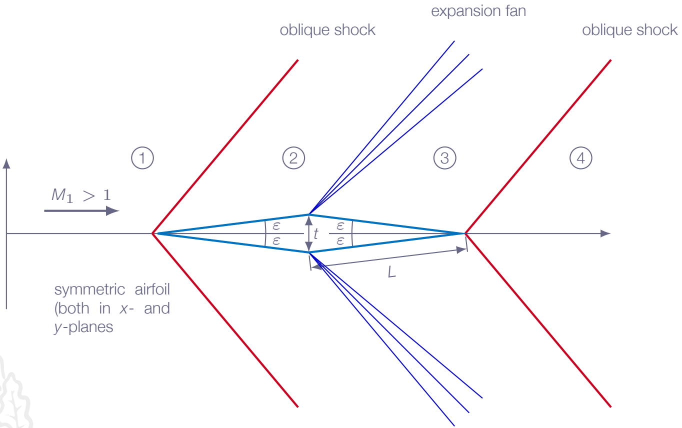
Prandtl-Meyer Expansion Waves

since $T_{o_1} = T_{o_2}$ and $p_{o_1} = p_{o_2}$

$$\frac{T_1}{T_2} = \frac{T_{o_2}}{T_{o_1}}\frac{T_1}{T_2} = \left(\frac{T_{o_2}}{T_2}\right) \Big/ \left(\frac{T_{o_1}}{T_1}\right) = \left[\frac{1+\frac{1}{2}(\gamma-1)M_2^2}{1+\frac{1}{2}(\gamma-1)M_1^2}\right]$$

$$\frac{p_1}{p_2} = \frac{p_{o_2}}{p_{o_1}}\frac{p_1}{p_2} = \left(\frac{p_{o_2}}{p_2}\right) \Big/ \left(\frac{p_{o_1}}{p_1}\right) = \left[\frac{1+\frac{1}{2}(\gamma-1)M_2^2}{1+\frac{1}{2}(\gamma-1)M_1^2}\right]^{\frac{\gamma}{\gamma-1}}$$

Diamond-Wedge Airfoil



Note! symmetric airfoil at zero incidence ⇒ zero lift but what about drag?

Diamond-Wedge Airfoil

1-2 standard oblique shock calculation for flow deflection angle ε and upstream Mach number M_1

2-3 Prandtl-Meyer expansion for flow deflection angle 2ε and upstream Mach number M_2

3-4 standard oblique shock calculation for flow deflection angle ε and upstream Mach number M_3

Diamond-Wedge Airfoil - Wave Drag

Since conditions 2 and 3 are constant in their respective regions, we obtain:

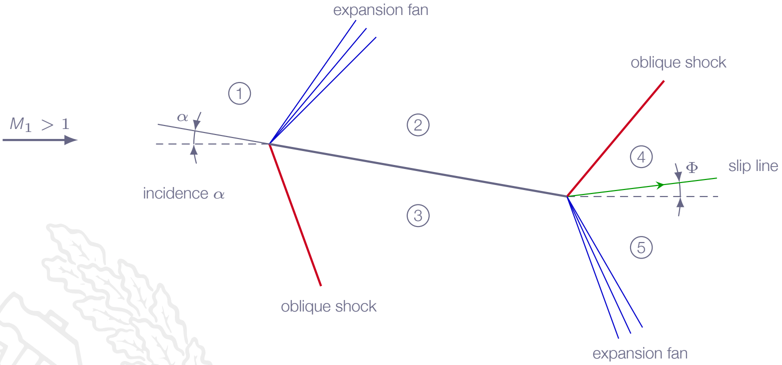
$$D = 2\left[p_2 L \sin(\varepsilon) - p_3 L \sin(\varepsilon)\right] = 2(p_2 - p_3)\frac{t}{2} = (p_2 - p_3)t$$

For supersonic free stream ($M_1 > 1$), with shocks and expansion fans according to figure we will always find that $p_2 > p_3$

which implies $D > 0$

Wave drag (drag due to flow loss at compression shocks)

Flat-Plate Airfoil



Flat-Plate Airfoil

It seems that the angle of the flow downstream of the flat plate would be different than the angle of the flow upstream of the plate. Can that really be correct?!

Flat-Plate Airfoil

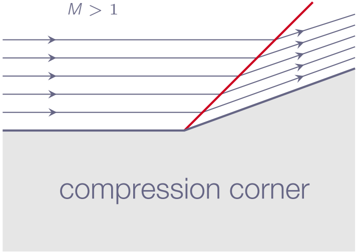
It seems that the angle of the flow downstream of the flat plate would be different than the angle of the flow upstream of the plate. Can that really be correct?!

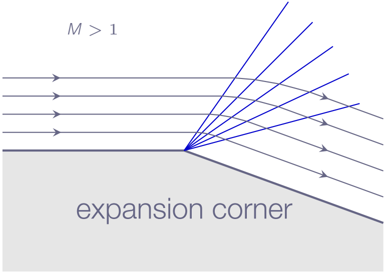
For the flow in the vicinity of the plate this is the correct picture. Further out from the plate, shock and expansion waves will interact and eventually sort the mismatch of flow angles out

Flat-Plate Airfoil

- Flow states 4 and 5 must satisfy:
  - $p_4 = p_5$
  - flow direction 4 equals flow direction 5 (Φ)
- Shock between 2 and 4 as well as expansion fan between 3 and 5 will adjust themselves to comply with the requirements
- For calculation of lift and drag only states 2 and 3 are needed
- States 2 and 3 can be obtained using standard oblique shock formulas and Prandtl-Meyer expansion

Oblique Shocks and Expansion Waves

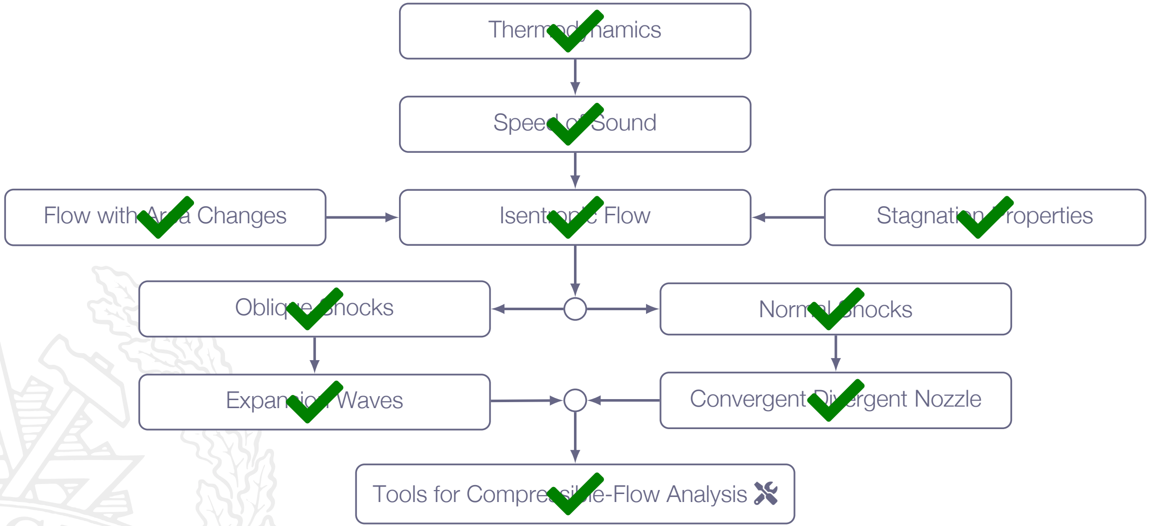




M	decrease
V	decrease
p	increase
ρ	increase
T	increase

M	increase
V	increase
p	decrease
ρ	decrease
T	decrease

Roadmap - Compressible Flow



Supersonic Stereo

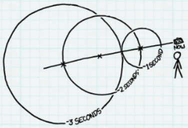
What if you somehow managed to make a stereo travel at twice the speed of sound, would it sound backwards to someone who was just casually sitting somewhere as it flies by?

—Tim Currie

Yes.

Technically, anyway. It would be pretty hard to hear.

The basic idea is pretty straightforward. The stereo is going faster than its own sound, so it will reach you first, followed by the sound it emitted one second ago, followed by the sound it emitted two seconds ago, and so forth.



The problem is that the stereo is moving at Mach 2, which means that two seconds ago, it was over a kilometer away. It's hard to hear music from that distance, particularly when your ears were just hit by (a) a sonic boom, and (b) pieces of a rapidly disintegrating stereo.

Wind speeds of Mach 2 would messily disassemble most consumer electronics. The force of the wind on the body of the stereo is roughly comparable to that of a dozen people standing on it: