Fluid Mechanics - MTF053

Lecture 21

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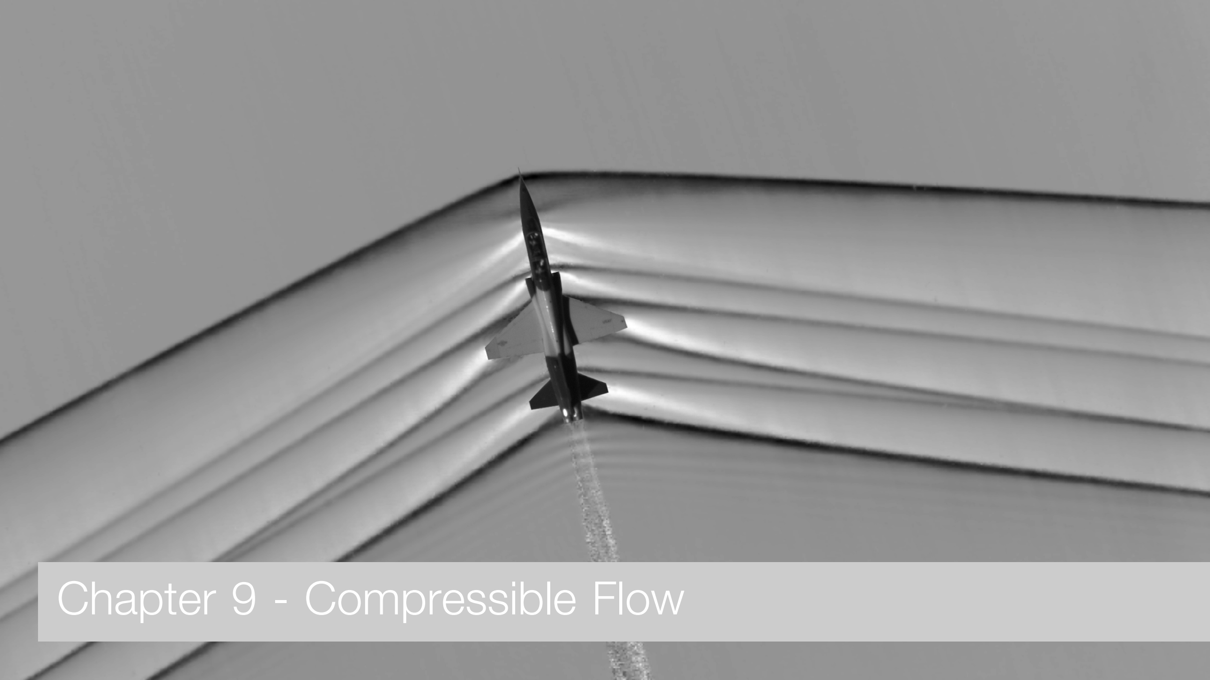
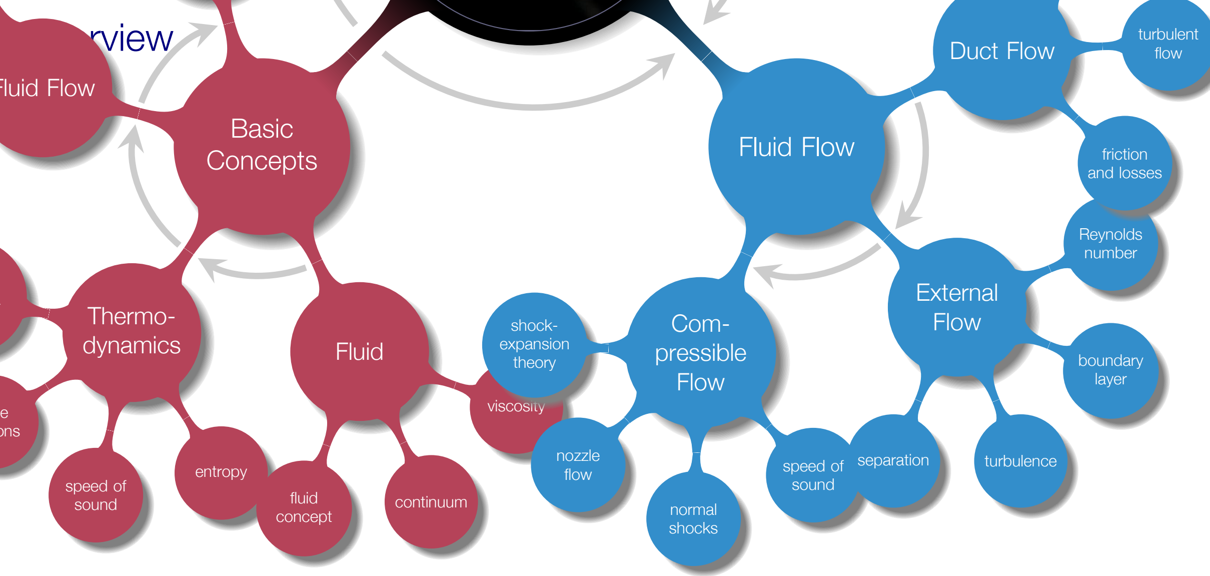
Chapter 9 - Compressible Flow




rview
Fluid Flow
Basic Concepts
Thermo-dynamics
Fluid
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ons
speed of sound
entropy
fluid concept
continuum
viscosity
Fluid Flow
Duct Flow
turbulent flow
friction and losses
Reynolds number
External Flow
boundary layer
separation
turbulence
Com-pressible Flow
shock-expansion theory
nozzle flow
normal shocks
speed of sound

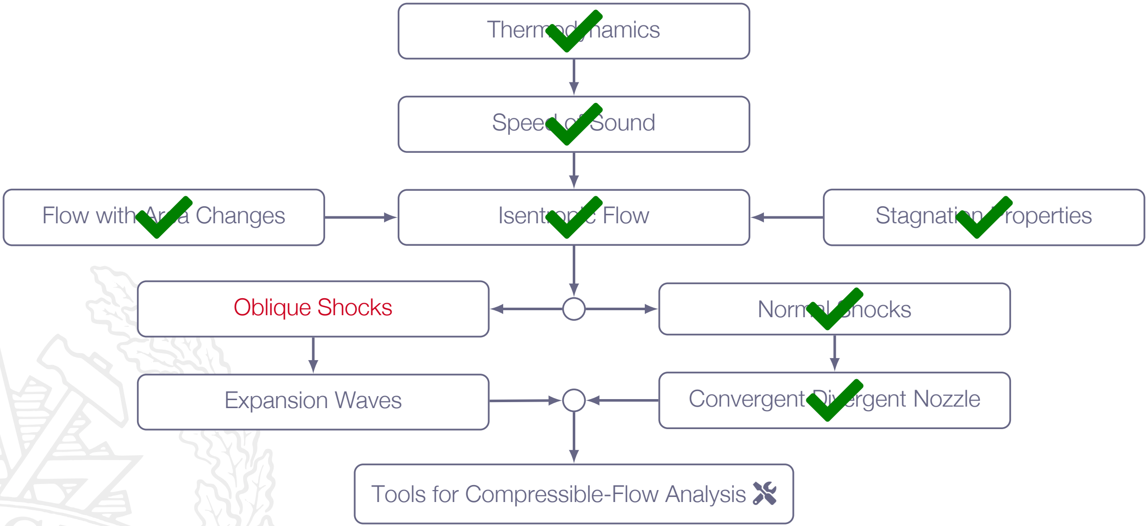
Learning Outcomes

4 Be **able to categorize** a flow and **have knowledge about** how to select applicable methods for the analysis of a specific flow based on category

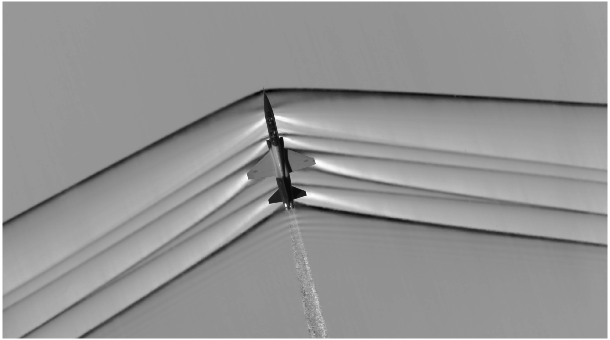
37 **Understand and explain** basic concepts of compressible flows (the gas law, speed of sound, Mach number, isentropic flow with changing area, normal shocks, oblique shocks, Prandtl-Meyer expansion)

Let's go supersonic ...

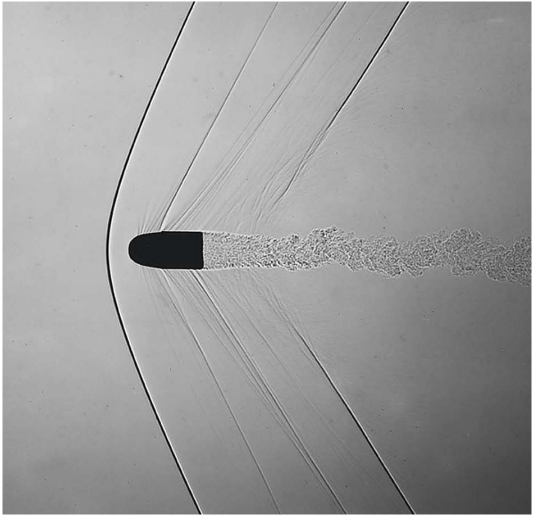
Roadmap - Compressible Flow



Oblique Shocks

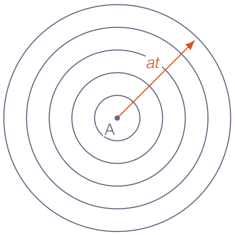


Oblique Shocks

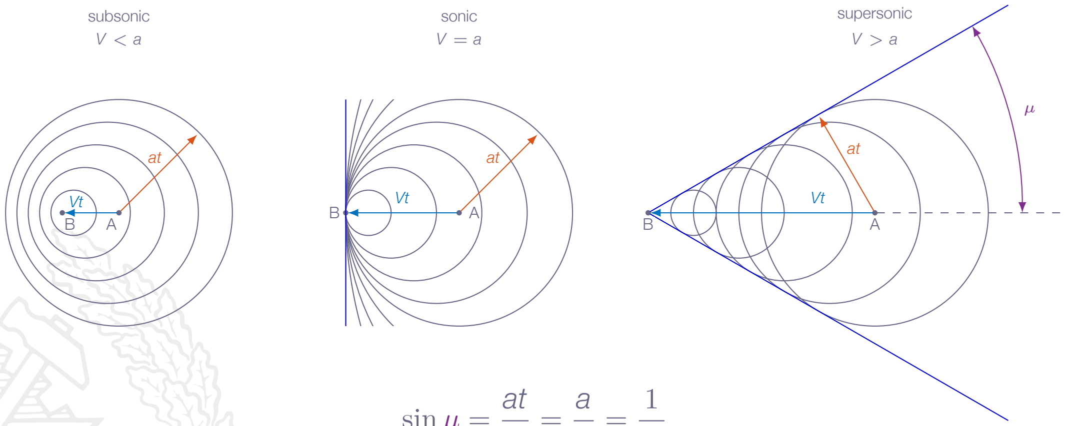


Mach Wave

Sound waves emitted from A (speed of sound a)



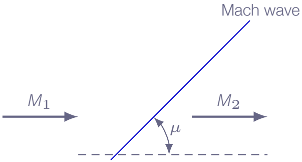
Mach Wave



$$\sin \mu = \frac{at}{Vt} = \frac{a}{V} = \frac{1}{M}$$

Mach Wave

A Mach wave is an infinitely weak oblique shock



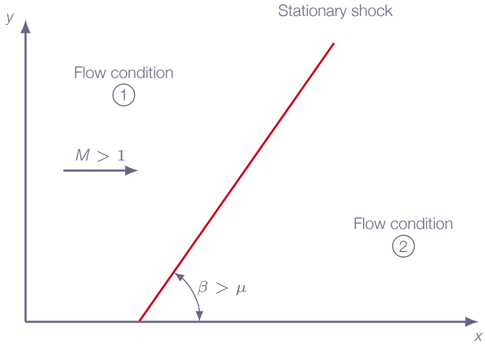
No substantial changes of flow properties over a single Mach wave

$M_1 > 1.0$ and $M_1 \approx M_2$

Isentropic

Oblique Shocks and Mach Waves

Two-dimensional steady-state flow



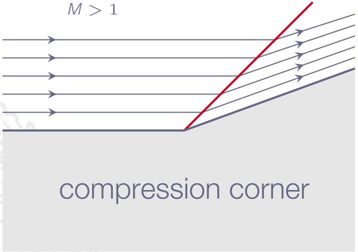
Significant changes of flow properties from 1 to 2

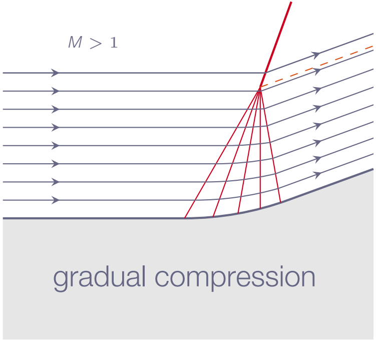
$M_1 > 1.0$, $\beta > \mu$, and $M_1 \neq M_2$

Not isentropic

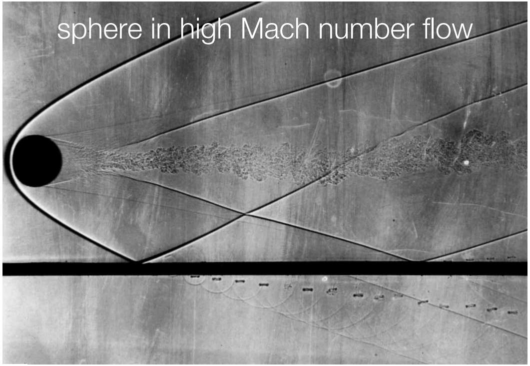
Oblique Shocks and Mach Waves

When does an oblique shock appear in a flow?

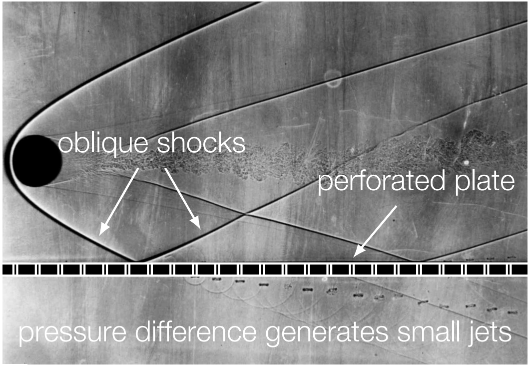




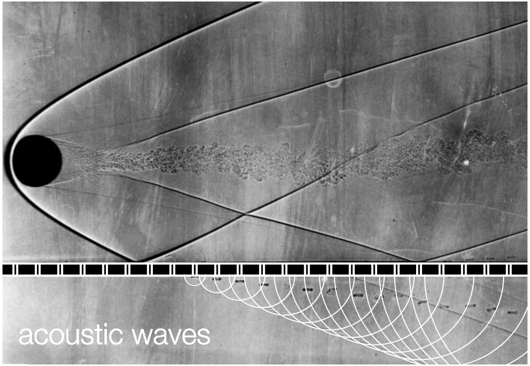
Oblique Shocks and Mach Waves



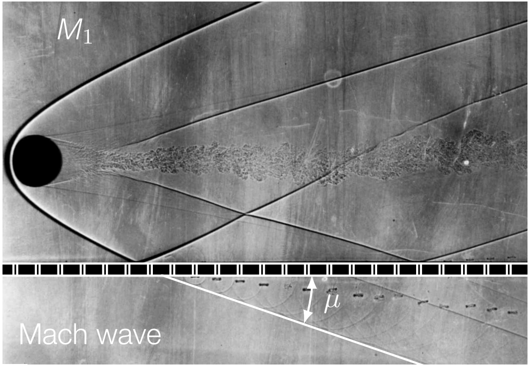
Oblique Shocks and Mach Waves



Oblique Shocks and Mach Waves

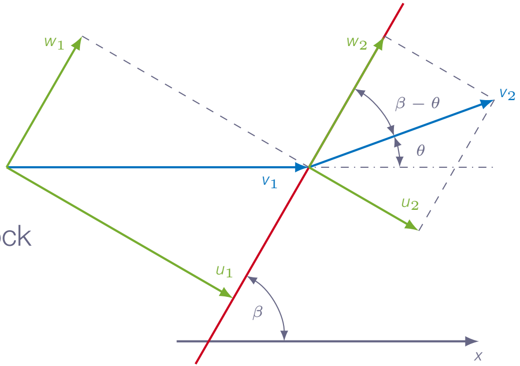


Oblique Shocks and Mach Waves



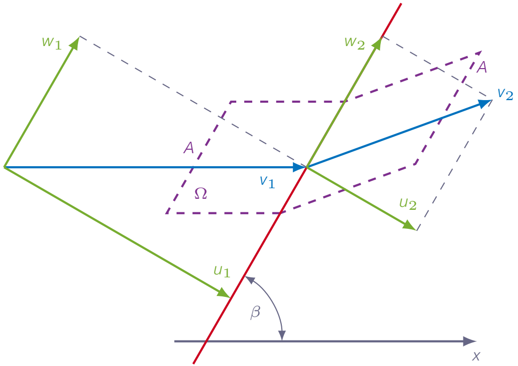
$$\mu = 19^\circ \Rightarrow M_1 = \frac{1}{\sin\mu} \approx 3.1$$

Oblique Shocks



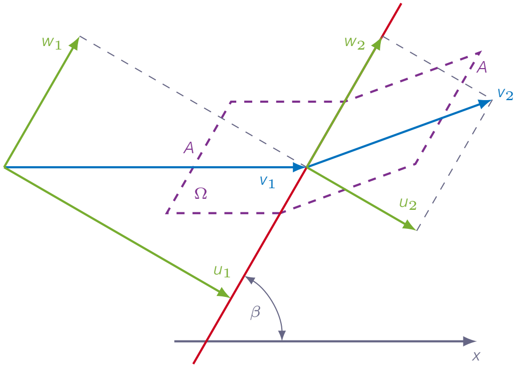
Stationary oblique shock

Oblique Shock Relations



- Two-dimensional steady-state flow
- Control volume aligned with flow stream lines

Oblique Shock Relations

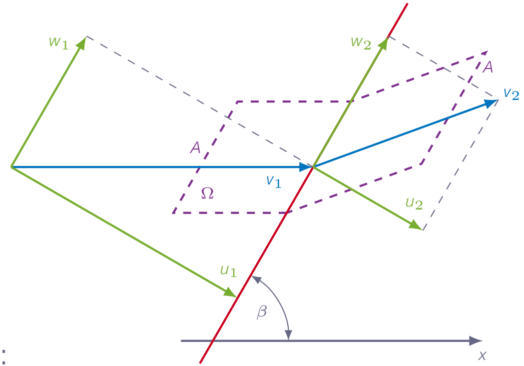


Velocity notations:

$$M_{n_1} = \frac{u_1}{a_1} = M_1 \sin(\beta) \qquad M_1 = \frac{v_1}{a_1}$$

$$M_{n_2} = \frac{u_2}{a_2} = M_2 \sin(\beta - \theta) \qquad M_2 = \frac{v_2}{a_2}$$

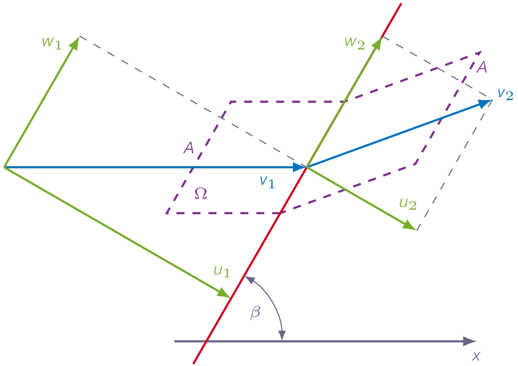
Oblique Shock Relations



Conservation of mass:

$$-\rho_1 u_1 A + \rho_2 u_2 A = 0 \Rightarrow \rho_1 u_1 = \rho_2 u_2$$

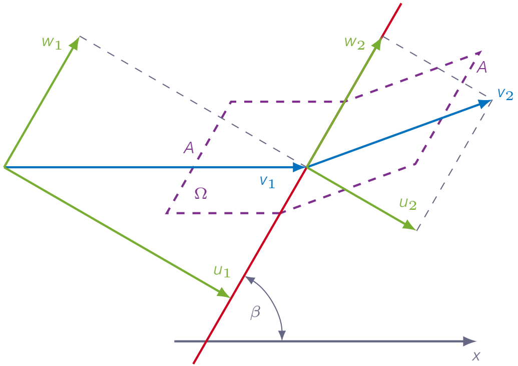
Oblique Shock Relations



Conservation of momentum (shock-normal direction):

$$-(\rho_1 u_1^2 + p_1)A + (\rho_2 u_2^2 + p_2)A = 0 \Rightarrow \rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2$$

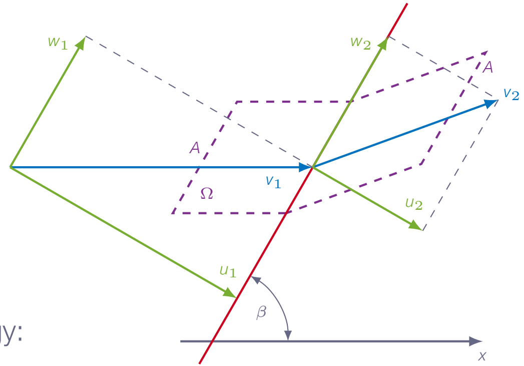
Oblique Shock Relations



Conservation of momentum (shock-tangential direction):

$$-\rho_1 u_1 w_1 A + \rho_2 u_2 w_2 A = 0 \Rightarrow w_1 = w_2$$

Oblique Shock Relations



Conservation of energy:

$$-\rho_1 u_1 [h_1 + \frac{1}{2}(u_1^2 + w_1^2)]A + \rho_2 u_2 [h_2 + \frac{1}{2}(u_2^2 + w_2^2)]A = 0 \Rightarrow h_1 + \frac{1}{2}u_1^2 = h_2 + \frac{1}{2}u_2^2$$

Oblique Shock Relations

We can use the same equations as for normal shocks if we replace M_1 with M_{n_1} and M_2 with M_{n_2}

$$M_{n_2}^2 = \frac{M_{n_1}^2 + [2/(\gamma - 1)]}{[2\gamma/(\gamma - 1)]M_{n_1}^2 - 1}$$

Ratios such as ρ_2/ρ_1, p_2/p_1, and T_2/T_1 can be calculated using the relations for normal shocks with M_1 replaced by M_{n_1}

Oblique Shock Relations

What about ratios involving stagnation flow properties, can we use the ones previously derived for normal shocks?

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P_{o_1}, ρ_{o_1}, etc are calculated using M_1 not M_{n_1} (the tangential velocity is included)

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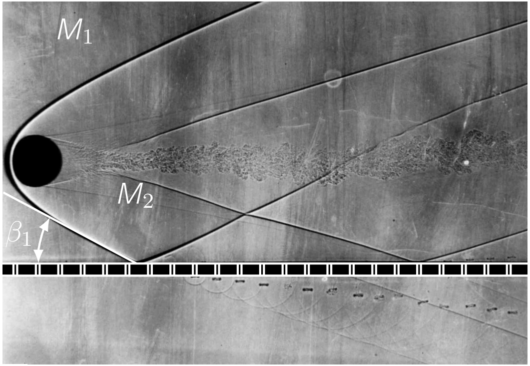
Note! Do not not use ratios involving total quantities, *e.g.* p_{o_2}/p_{o_1}, ρ_{o_2}/ρ_{o_1}, obtained from formulas or tables for normal shock

Oblique Shocks and Mach Waves

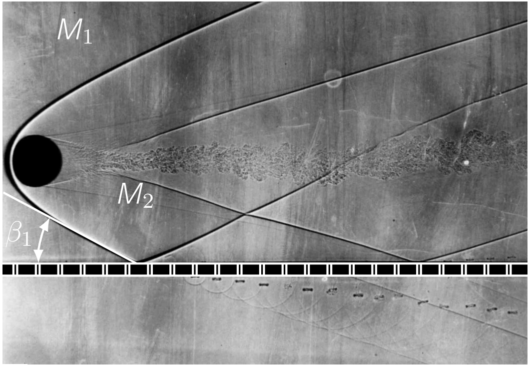


$M_1 > M_2$

$M_2 > 1.0$

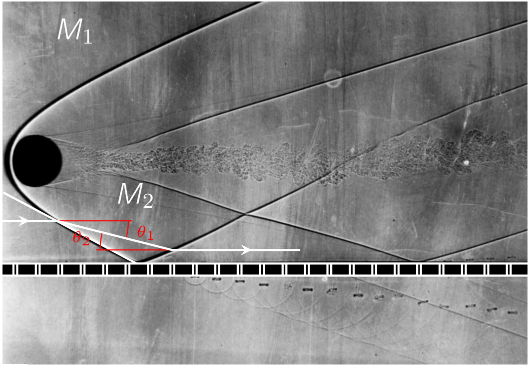
$$\theta_1 = f(M_1, \beta_1), \quad M_2 = f(M_1, \theta_1, \beta_1)$$

Oblique Shocks and Mach Waves



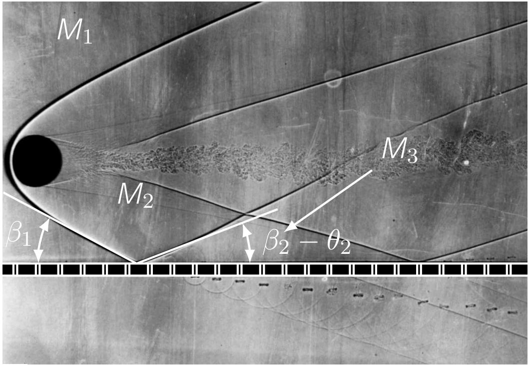
$$\left.\begin{aligned}\beta_1 &= 28^\circ \\ M_1 &= 3.1\end{aligned}\right\} \Rightarrow \theta_1 \approx 11.2^\circ, \quad M_2 \approx 2.5$$

Oblique Shocks and Mach Waves



$$\theta_1 = \theta_2$$

Oblique Shocks and Mach Waves



$$M_1 > M_2 > M_3$$

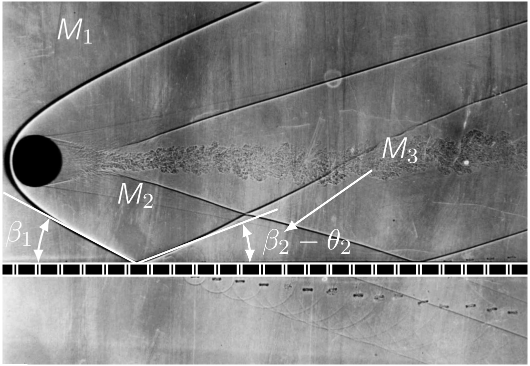
$$M_3 > 1.0$$

$$\beta_2 > \beta_1$$

$$\beta_2 = f(M_2, \theta_2), \quad M_3 = f(M_2, \theta_2, \beta_2)$$

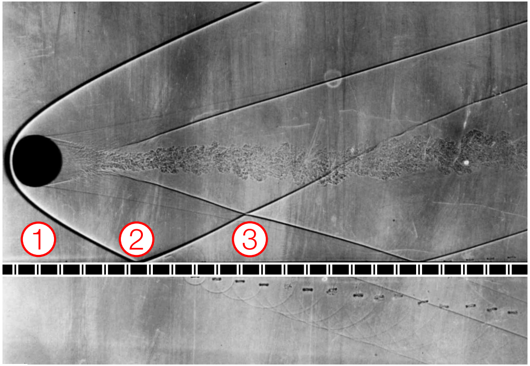
Note! Shock wave reflection at solid wall is **not** specular

Oblique Shocks and Mach Waves



$$\left.\begin{aligned} \theta_2 &= 11.2^\circ \\ M_2 &= 2.5 \end{aligned}\right\} \Rightarrow \beta_2 \approx 33^\circ, \quad M_3 \approx 2.0$$

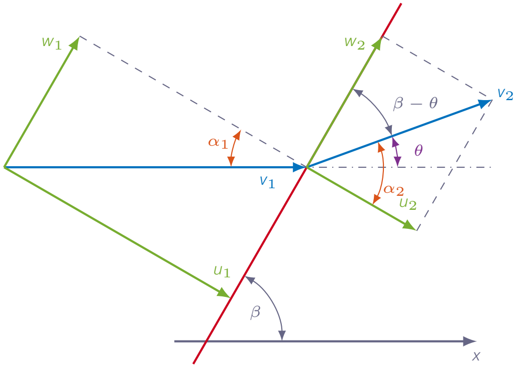
Oblique Shocks and Mach Waves



$$\frac{p_3}{p_1} = \frac{p_2}{p_1}\frac{p_3}{p_2} \approx 4.52$$

$$\frac{T_3}{T_1} = \frac{T_2}{T_1}\frac{T_3}{T_2} \approx 1.57$$

Deflection Angle (*for the interested*)



$$\theta = \alpha_2 - \alpha_1 = \tan^{-1}\left(\frac{w}{u_2}\right) - \tan^{-1}\left(\frac{w}{u_1}\right) \Rightarrow \frac{\partial \theta}{\partial w} = \frac{u_2}{w^2 + u_2^2} - \frac{u_1}{w^2 + u_1^2}$$

Deflection Angle (*for the interested*)

$$\frac{\partial\theta}{\partial w} = \frac{u_2}{w^2+u_2^2} - \frac{u_1}{w^2+u_1^2} = 0 \Rightarrow$$

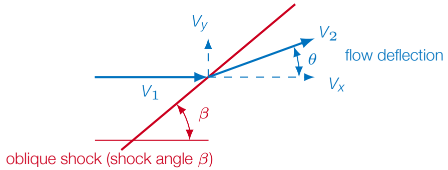
$$\frac{u_2(w^2+u_1^2) - u_1(w^2+u_2^2)}{(w^2+u_2^2)(w^2+u_1^2)} = 0 \Rightarrow \frac{(u_2-u_1)(w^2-u_1u_2)}{(w^2+u_2^2)(w^2+u_1^2)} = 0$$

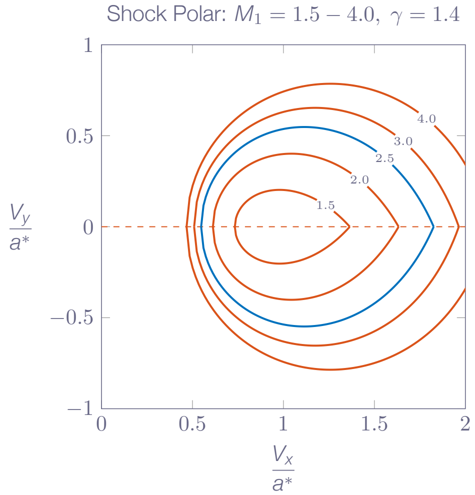
Two solutions:

1. $u_2 = u_1$ (no deflection)
2. $w^2 = u_1u_2$ (max deflection)

Shock Polar

Graphical representation of all possible deflection angles for a specific Mach number



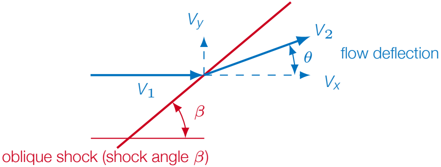


No deflection cases:

- normal shock (reduced shock-normal velocity)
- Mach wave (unchanged wave-normal velocity)

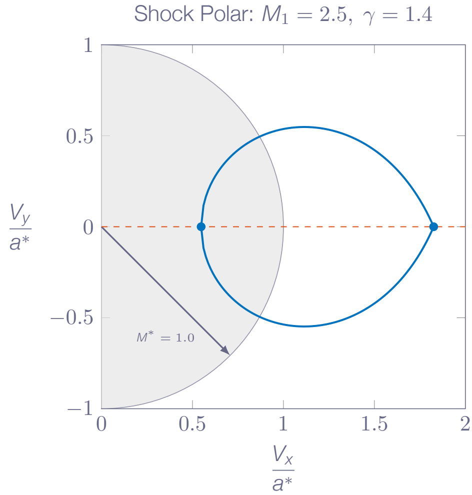
Shock Polar

Graphical representation of all possible deflection angles for a specific Mach number



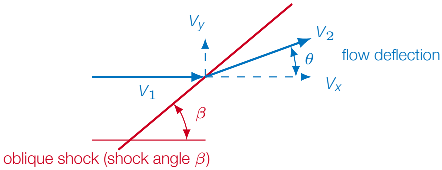
$$M^* = \frac{\sqrt{V_x^2 + V_y^2}}{a^*}$$

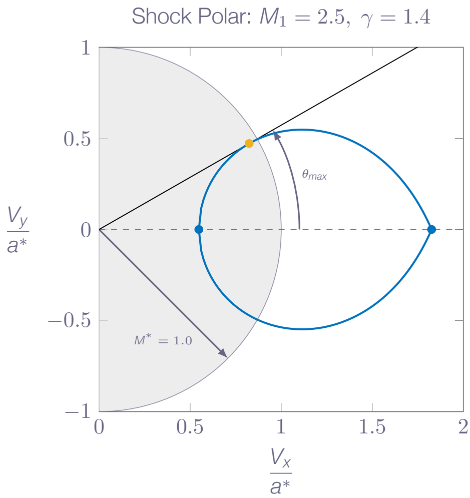
Solutions to the left of the sonic line are subsonic



Shock Polar - Flow Deflection - θ_{max}

Graphical representation of all possible deflection angles for a specific Mach number



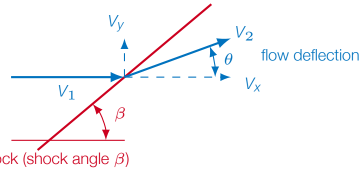


$$\tan\theta = \frac{V_y}{V_x}$$

It is not possible to deflect the flow more than θ_{max}

Shock Polar - Flow Deflection

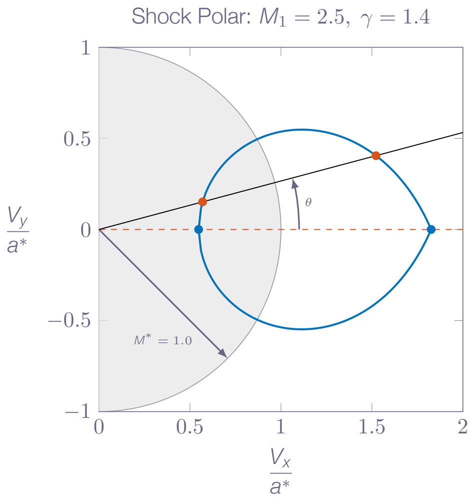
Graphical representation of all possible deflection angles for a specific Mach number



For each deflection angle $\theta < \theta_{max}$, there are two solutions

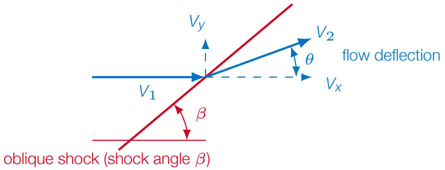
- strong shock solution
- weak shock solution

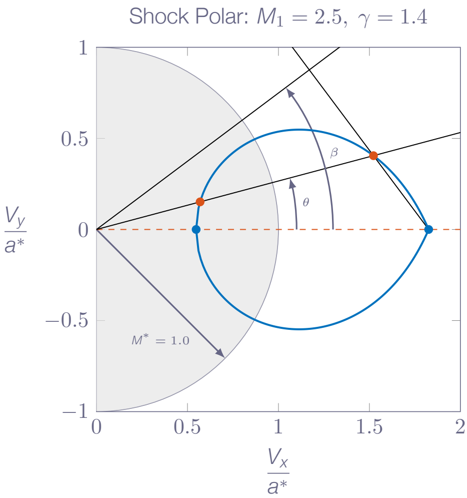
Weak shocks give lower losses and therefore the preferred solution



Shock Polar - Weak Solution

Graphical representation of all possible deflection angles for a specific Mach number

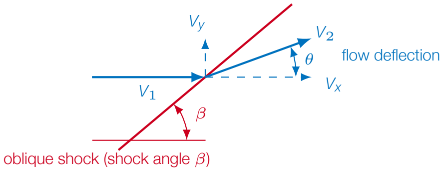


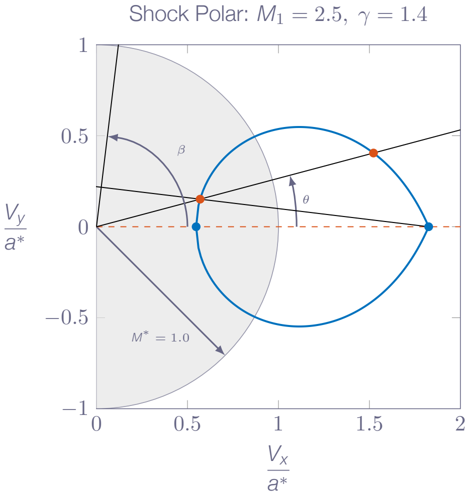


The shock polar can be used to calculate the shock angle β for a given deflection angle θ

Shock Polar - Strong Solution

Graphical representation of all possible deflection angles for a specific Mach number





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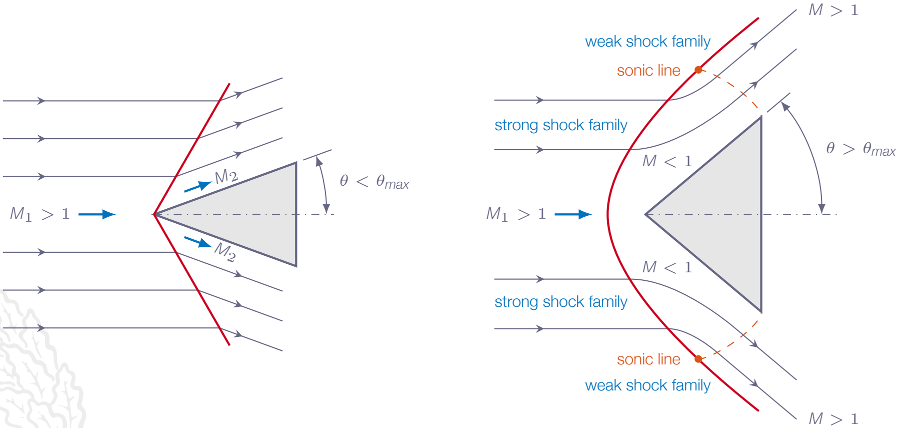
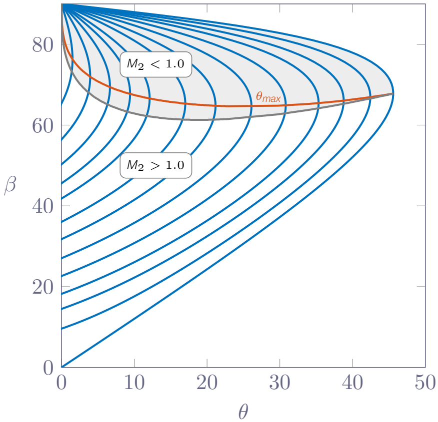
Flow Deflection



The θ-β-Mach Relation

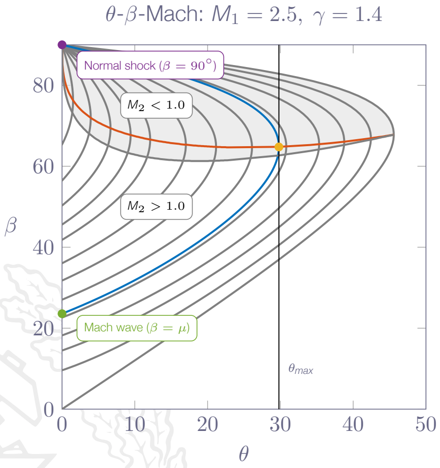
$$\tan(\theta) = \frac{2\cot(\beta)(M_1^2\sin^2(\beta) - 1)}{M_1^2(\gamma + \cos(2\beta)) + 2}$$

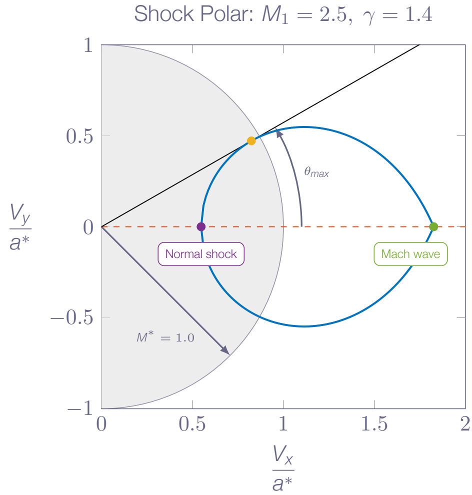


A relation between:

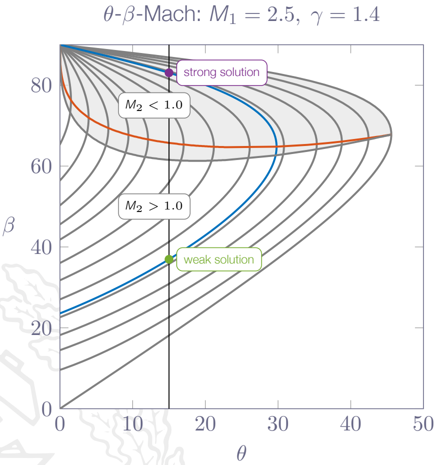
1. flow deflection angle θ
2. shock angle β
3. upstream flow Mach number M_1

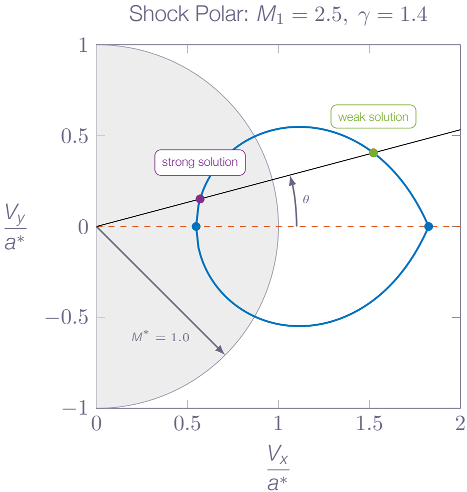
The θ-β-Mach Relation vs. Shock Polar





The θ-β-Mach Relation vs. Shock Polar





The θ-β-Mach Relation - Wedge Flow

Wedge flow oblique shock analysis:

1. θ-β-M relation $\Rightarrow$ β for given M_1 and θ
2. β gives M_{n_1} according to: $M_{n_1} = M_1 \sin(\beta)$
3. normal shock formula with M_{n_1} instead of M_1 $\Rightarrow$ M_{n_2} (instead of M_2)
4. M_2 given by $M_2 = M_{n_2} / \sin(\beta - \theta)$
5. normal shock formula with M_{n_1} instead of M_1 $\Rightarrow$ ρ_2/ρ_1, p_2/p_1, etc
6. upstream conditions + ρ_2/ρ_1, p_2/p_1, etc $\Rightarrow$ downstream conditions