Fluid Mechanics - MTF053

Lecture 20

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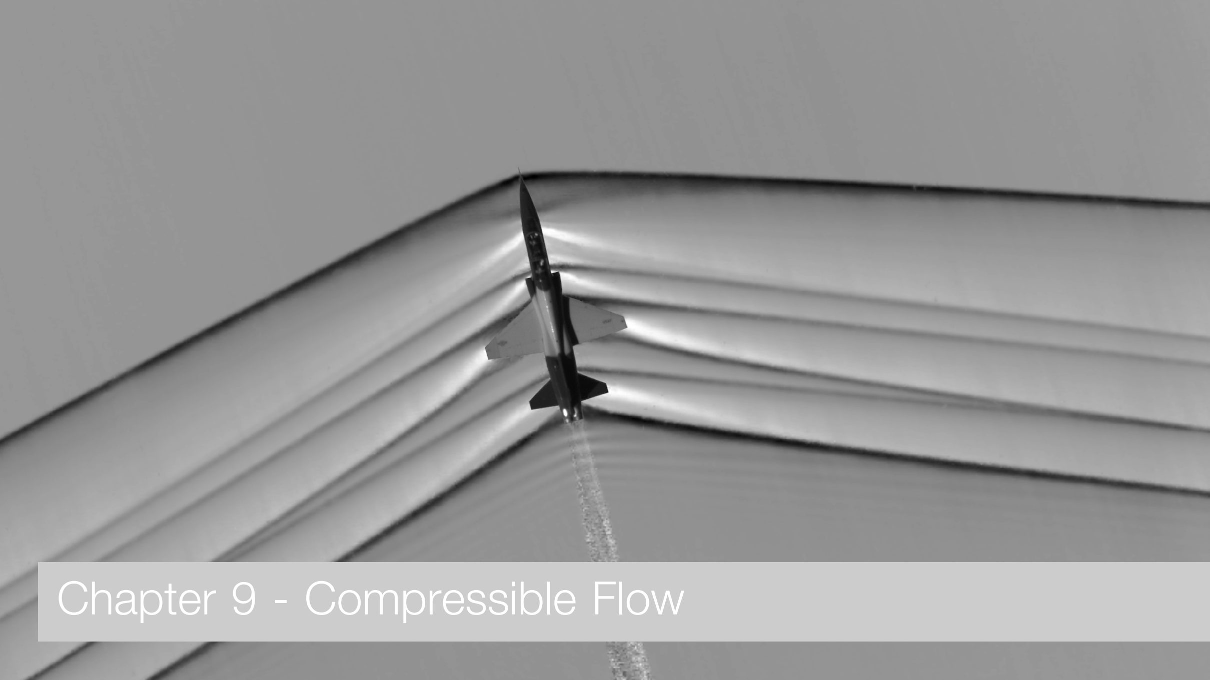
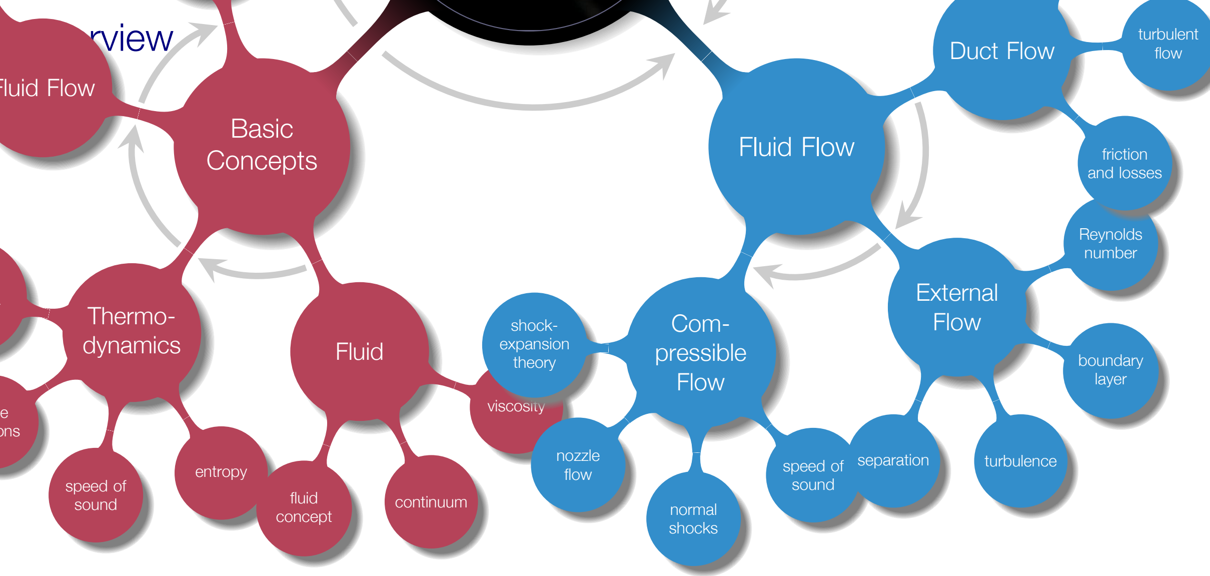

Chapter 9 - Compressible Flow


Basic Concepts
Thermo-dynamics
Fluid
speed of sound
entropy
fluid concept
continuum
viscosity
Fluid Flow
Duct Flow
turbulent flow
friction and losses
Reynolds number
External Flow
boundary layer
separation
turbulence
Com-pressible Flow
shock-expansion theory
nozzle flow
normal shocks
speed of sound

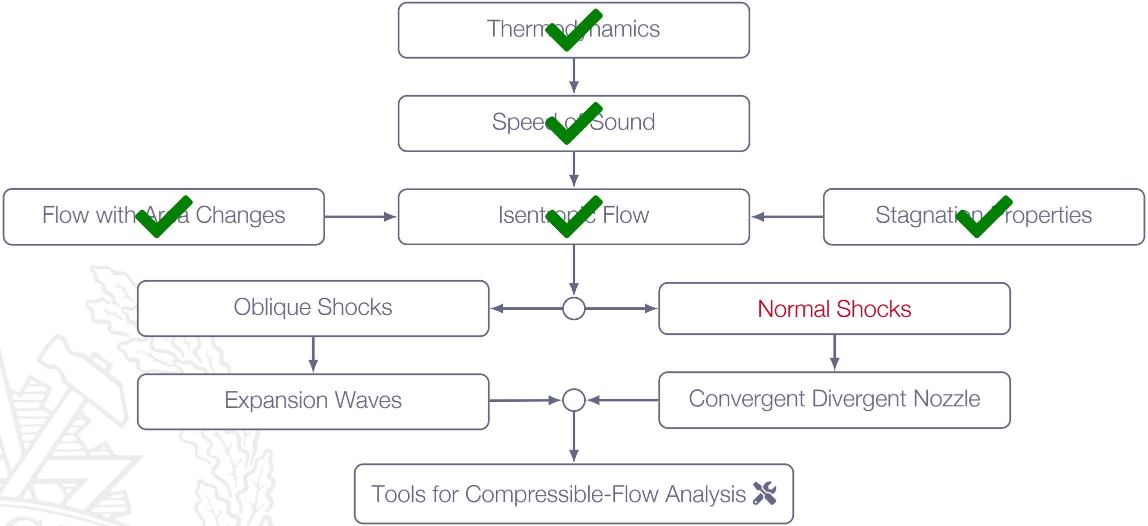
Learning Outcomes

4 Be **able to categorize** a flow and **have knowledge about** how to select applicable methods for the analysis of a specific flow based on category

37 **Understand and explain** basic concepts of compressible flows (the gas law, speed of sound, Mach number, isentropic flow with changing area, normal shocks, oblique shocks, Prandtl-Meyer expansion)

Let's go supersonic ...

Roadmap - Compressible Flow



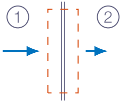
Shock Waves

"Shock waves are nearly discontinuous changes in a supersonic flow"

Reasons for the appearance of shocks in a flow can be for example:

- higher downstream pressure
- sudden changes in flow direction
- blockage by a downstream body
- explosion

Normal Shocks



Continuity:

$$\rho_1 u_1 = \rho_2 u_2$$

Momentum:

$$p_1 - p_2 = \rho_2 u_2^2 - \rho_1 u_1^2$$

Energy:

$$h_1 + \frac{1}{2} u_1^2 = h_2 + \frac{1}{2} u_2^2 = h_o$$

The Rankine-Hugoniot relation:

$$h_2 - h_1 = \frac{1}{2}(p_2 - p_1)\left(\frac{1}{\rho_2} + \frac{1}{\rho_1}\right)$$

Normal Shocks

$$h_2 - h_1 = \frac{1}{2}(p_2 - p_1)\left(\frac{1}{\rho_2} + \frac{1}{\rho_1}\right)$$

Note! The Rankine-Hugoniot relation only includes thermodynamic properties (no velocities) and gives a relation between the flow state upstream of the shock and the flow downstream of the shock

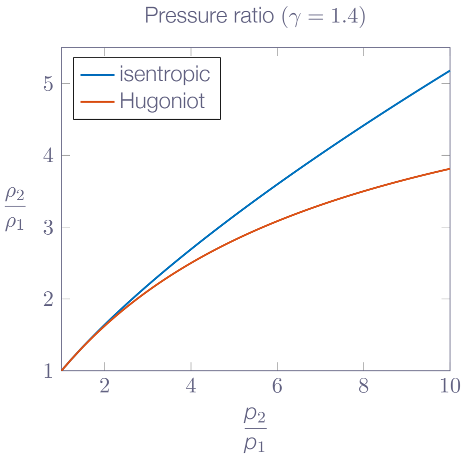
Normal Shocks

The Rankine-Hugoniot relation

$$\frac{\rho_2}{\rho_1} = \frac{1 + \left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{p_2}{p_1}\right)}{\left(\frac{\gamma+1}{\gamma-1}\right) + \left(\frac{p_2}{p_1}\right)}$$

The isentropic relation

$$\frac{\rho_2}{\rho_1} = \left(\frac{p_2}{p_1}\right)^{1/\gamma}$$



Normal Shocks

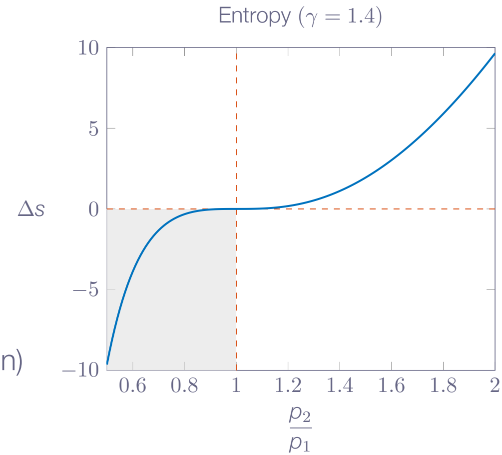
The second law of thermodynamics

$$s_2 - s_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

can be rewritten as

$$s_2 - s_1 = C_v \ln \left[\frac{p_2}{p_1} \left(\frac{\rho_1}{\rho_2} \right)^{\gamma} \right]$$

(ρ_2/ρ_1 from the Rankine-Hugoniot relation)



Note! a reduction of entropy is a violation of the second law of thermodynamics

Normal Shocks

For a perfect gas, it is possible to obtain relations for normal shocks that only include upstream variables

Momentum equation: $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$

$$p_2 - p_1 = \rho_1 u_1^2 - \rho_2 u_2^2 = \{\rho_1 u_1 = \rho_2 u_2\} = \rho_1 u_1 (u_1 - u_2) = \rho_1 u_1^2 \left(1 - \frac{u_2}{u_1}\right)$$

divide by p_1

$$\frac{p_2}{p_1} = 1 + \frac{\rho_1 u_1^2}{p_1}\left(1 - \frac{u_2}{u_1}\right)$$

$$u_1^2 = M_1^2 a_1^2 = M_1^2 \gamma R T_1 = \gamma M_1^2 \frac{p_1}{\rho_1} \Rightarrow \frac{p_2}{p_1} = 1 + \gamma M_1^2 \left(1 - \frac{u_2}{u_1}\right)$$

Normal Shocks

$$\frac{p_2}{p_1} = 1 + \gamma M_1^2 \left(1 - \frac{u_2}{u_1}\right)$$

Using the energy equation its possible obtain a relation for $\frac{u_2}{u_1}$ (the derivation is quite lengthy though)

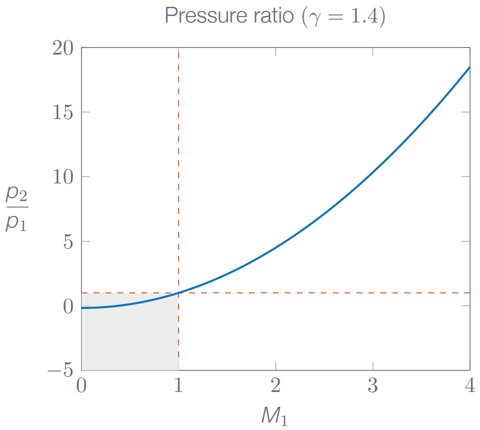
$$\frac{u_2}{u_1} = \frac{2 + (\gamma - 1)M_1^2}{(\gamma + 1)M_1^2}$$

and thus

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}\left(M_1^2 - 1\right)$$

Normal Shocks

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}\left(M_1^2 - 1\right)$$



Note! from before we know that p_2/p_1 must be greater than 1.0, which means that M_1 must be greater than 1.0

Normal Shocks

Momentum equation: $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$

$$M = \frac{u}{a} \Rightarrow p_1 + \rho_1 M_1^2 a_1^2 = p_2 + \rho_2 M_2^2 a_2^2$$

$$a = \sqrt{\gamma RT} = \sqrt{\frac{\gamma p}{\rho}} \Rightarrow p_1 + \rho_1 M_1^2 \frac{\gamma p_1}{\rho_1} = p_2 + \rho_2 M_2^2 \frac{\gamma p_2}{\rho_2}$$

$$p_1 \left(1 + \gamma M_1^2\right) = p_2 \left(1 + \gamma M_2^2\right)$$

$$\frac{p_2}{p_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}$$

Normal Shocks

Two ways to calculate the pressure ratio over the shock

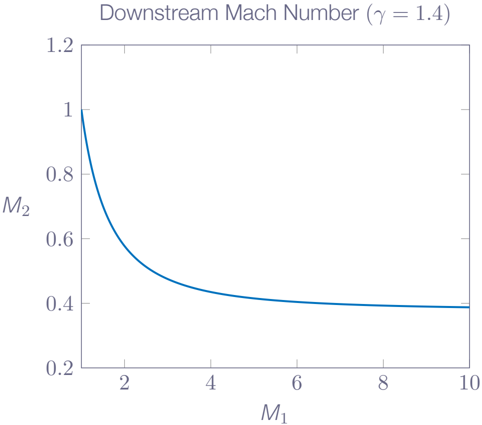
$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}\left(M_1^2 - 1\right) \qquad \frac{p_2}{p_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}$$

Setting the relations equal gives a relation for the downstream Mach number

$$\boxed{M_2^2 = \frac{(\gamma - 1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma - 1)}}$$

Normal Shocks

$$M_2^2 = \frac{(\gamma - 1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma - 1)}$$

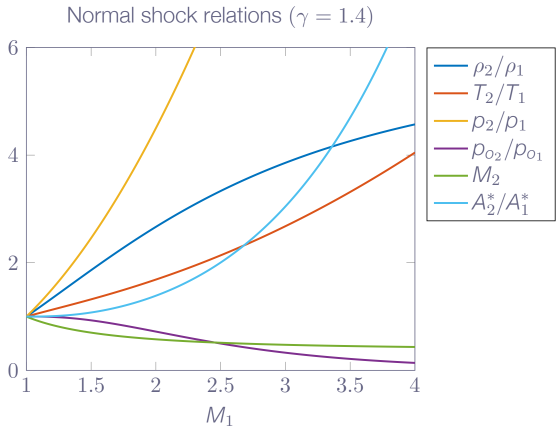


Note! for $\gamma > 1$ and $M_1 > 1$, the downstream Mach number must be less than 1.0, i.e we will always have subsonic flow behind a normal shock

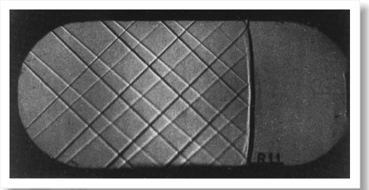
Normal Shocks - Summary

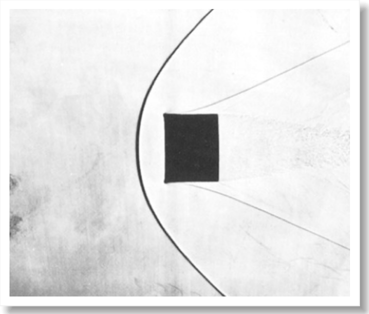
1. Supersonic flow upstream of normal shock
2. Subsonic flow downstream of normal shock
3. Entropy increases over the shock and consequently total pressure decreases
4. Sonic throat area increases
5. Very weak shock waves are nearly isentropic

Normal Shocks



Normal Shocks





Moving Normal Shocks

Change frame of reference

- coordinate system moving with the shock
- thermodynamic properties does not change

$$h_2 - h_1 = \frac{1}{2}(p_2 - p_1)\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$$

W →

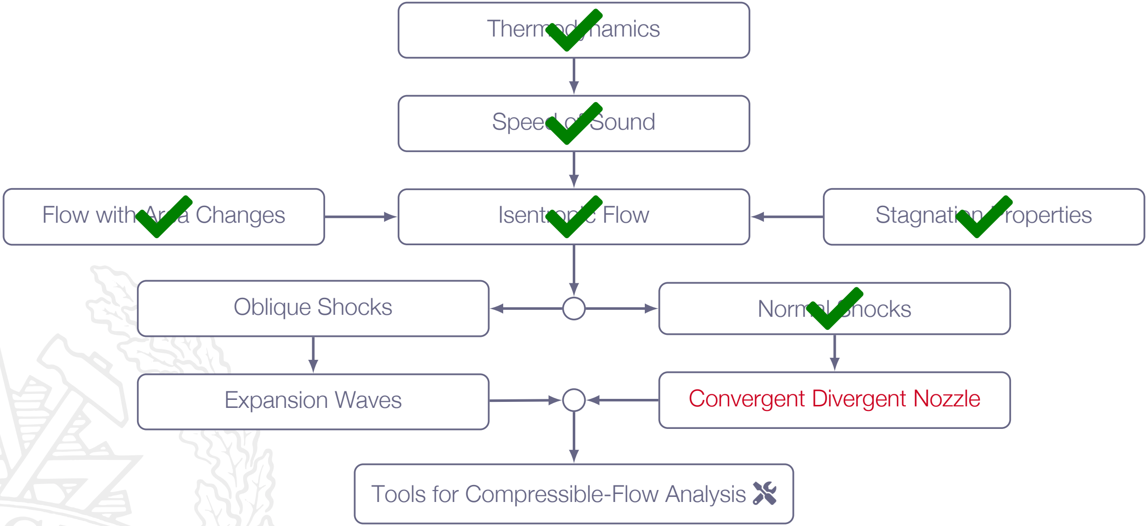
$p + \Delta p$	p
$\rho + \Delta \rho$	ρ
$T + \Delta T$	T
$u = u_p$	$u = 0$

frame of reference: fixed to the fluid

$p + \Delta p$	p
$\rho + \Delta \rho$	ρ
$T + \Delta T$	T
$u = u_p - W$	$u = -W$

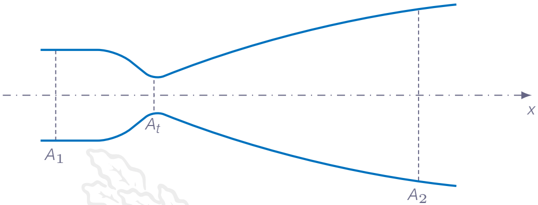
frame of reference: following the shock

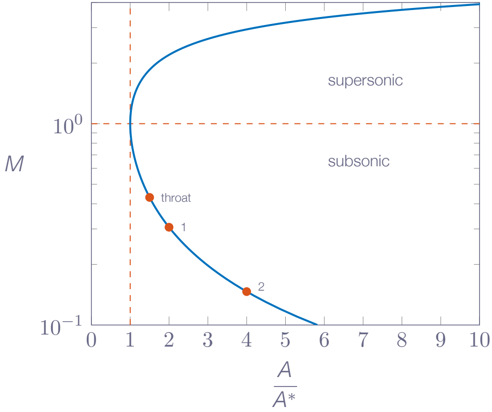
Roadmap - Compressible Flow



The Area-Mach-Number Relation

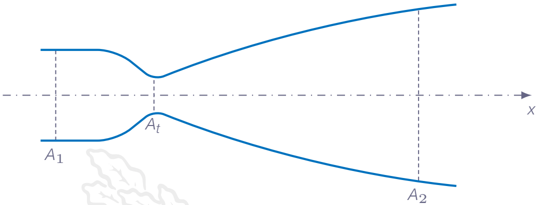
Sub-critical (non-choked) nozzle flow

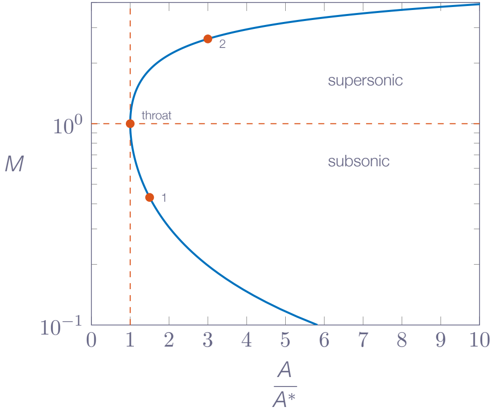




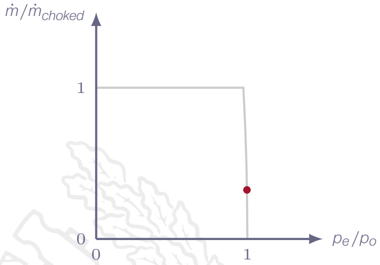
The Area-Mach-Number Relation

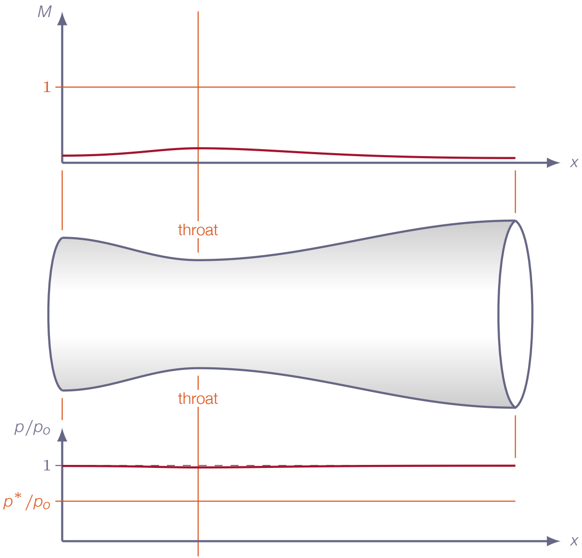
Critical (choked) nozzle flow





Convergent-Divergent Nozzle





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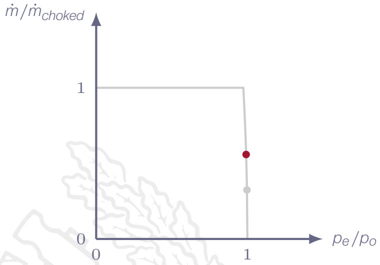

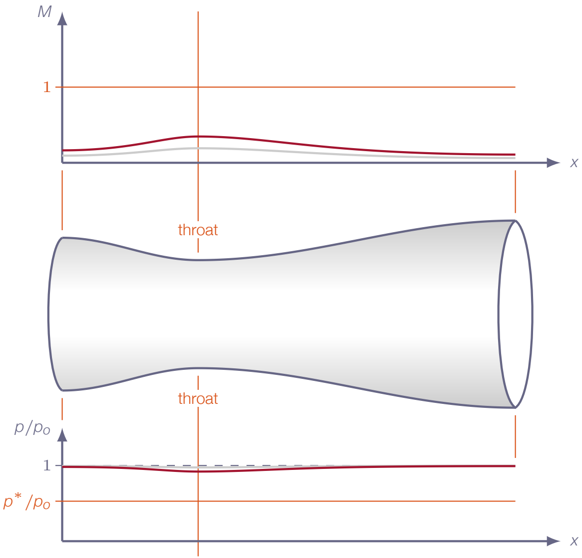

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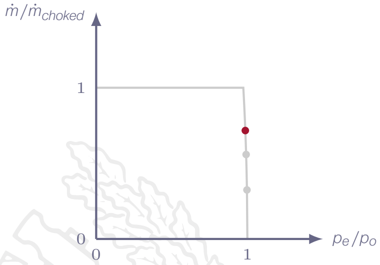

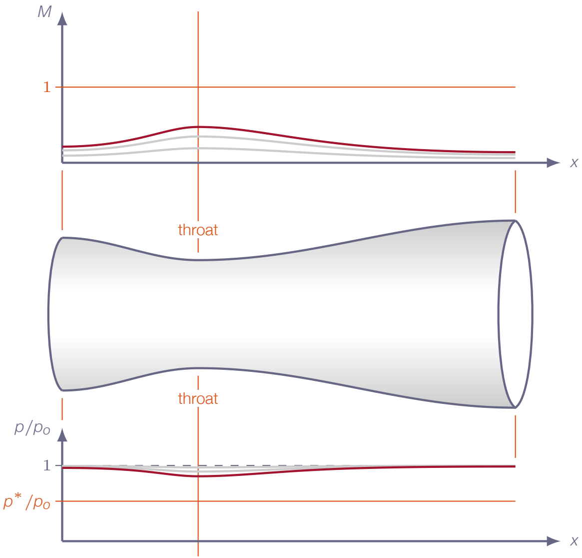

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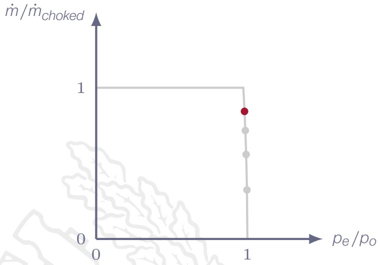

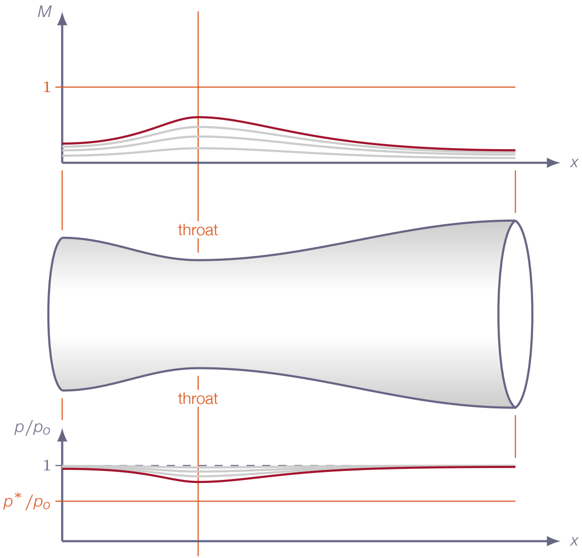

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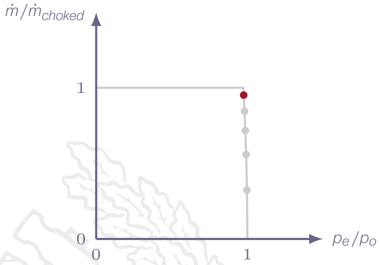

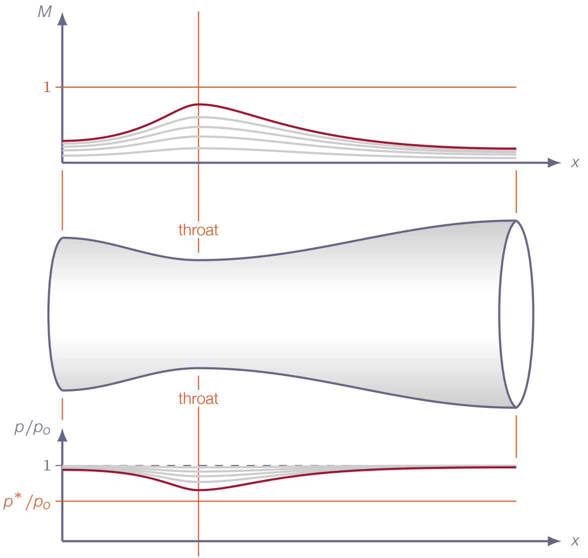

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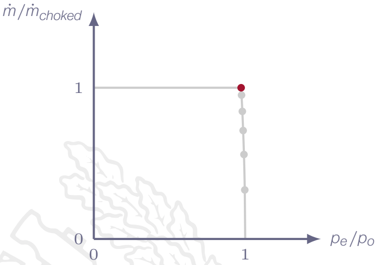

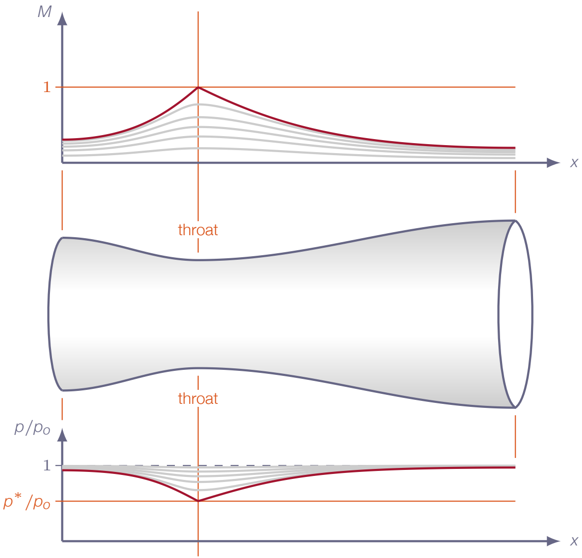

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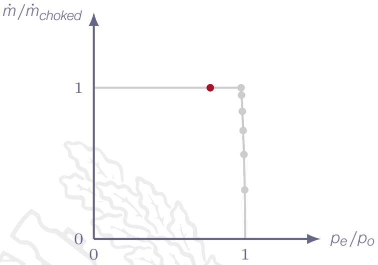

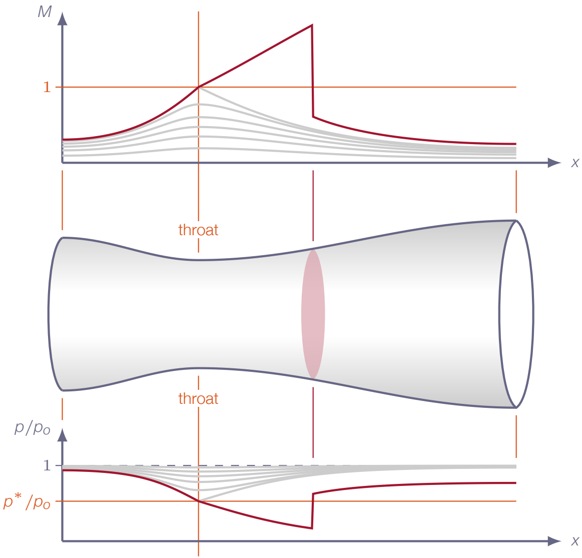

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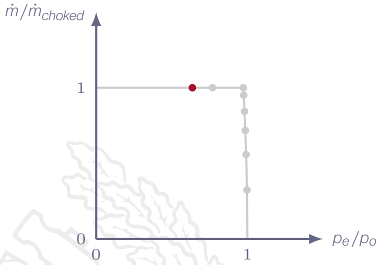

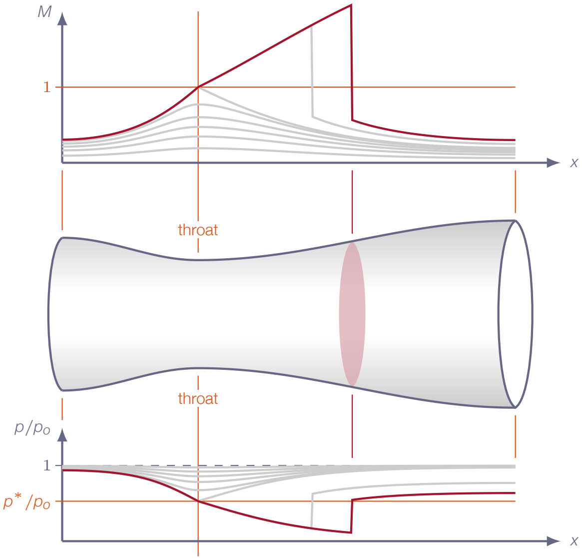

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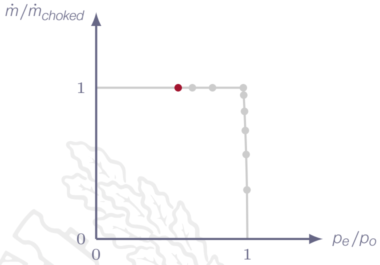

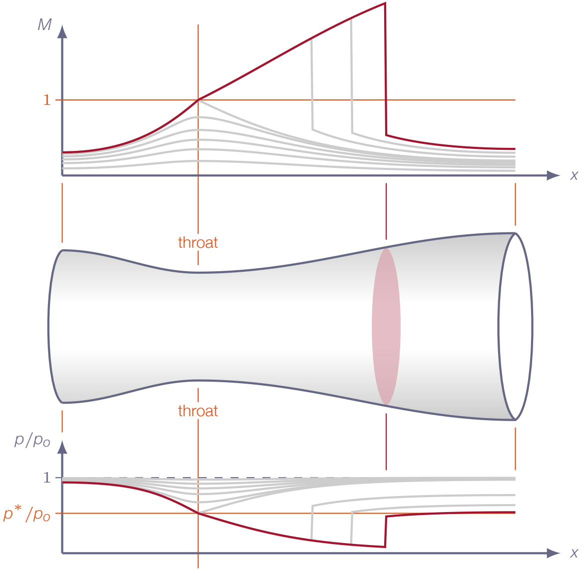

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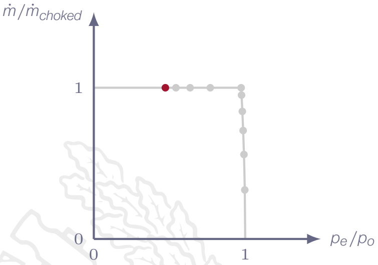

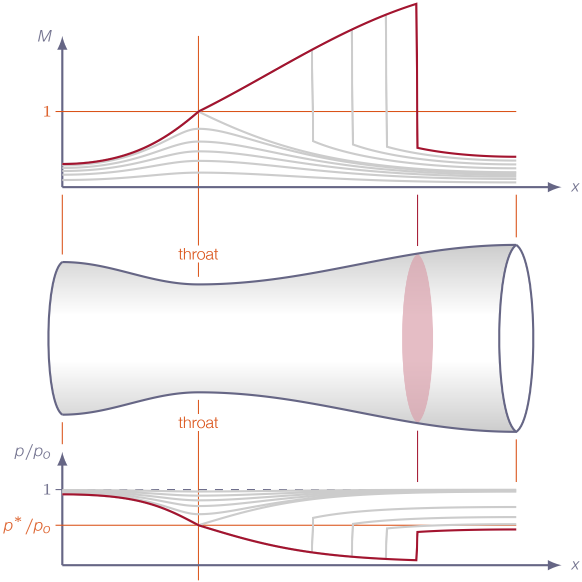

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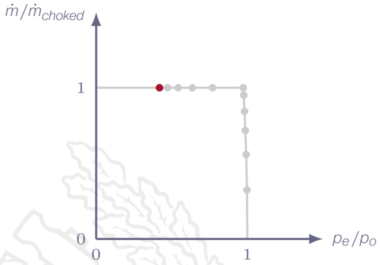

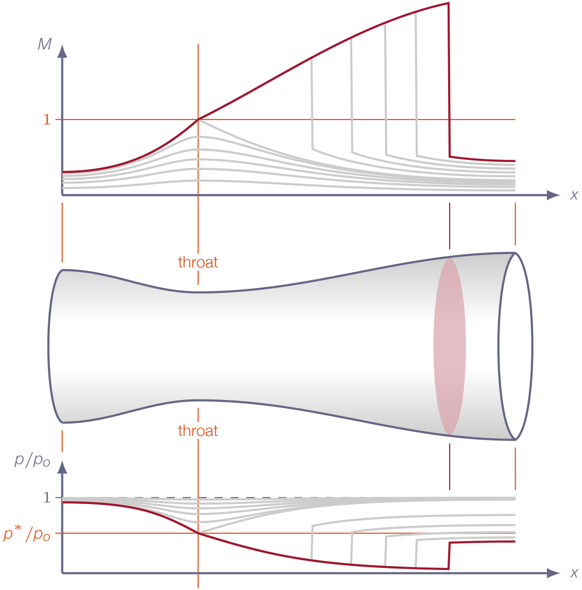

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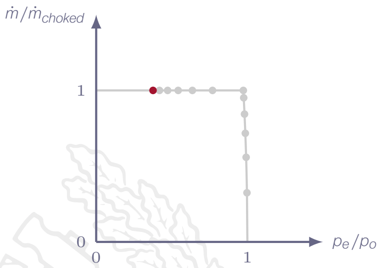

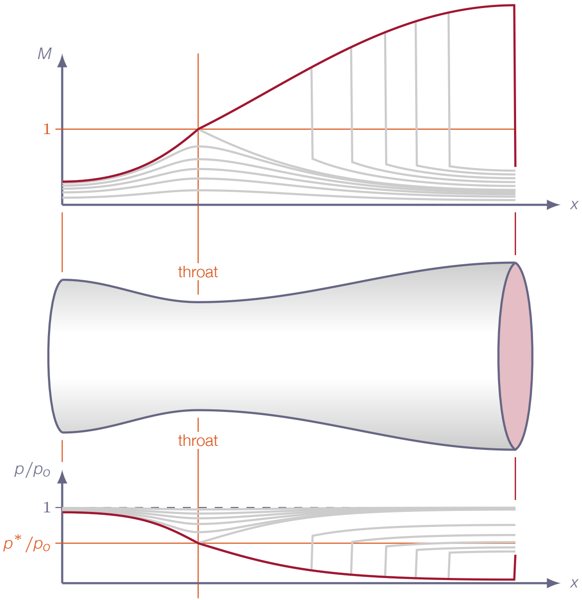

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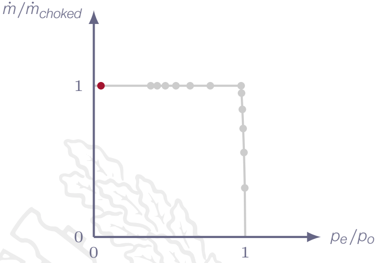
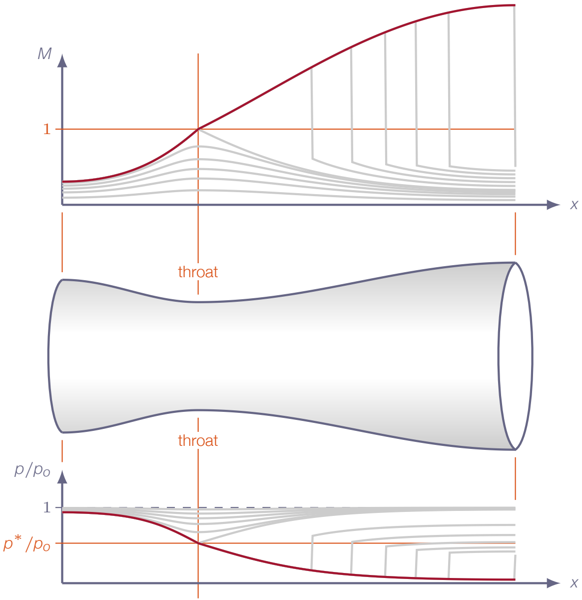
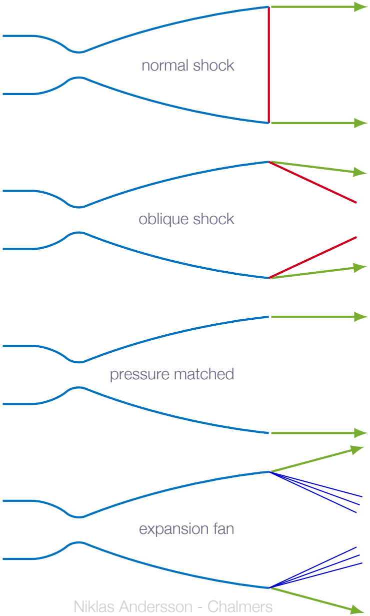




Convergent-Divergent Nozzle



$p_o/p_e = (p_o/p_e)_{ne}$
normal shock at nozzle exit

$(p_o/p_e)_{ne} < p_o/p_e < (p_o/p_e)_{sc}$
overexpanded nozzle flow

$p_o/p_e = (p_o/p_e)_{sc}$
pressure matched nozzle flow

$p_o/p_e > (p_o/p_e)_{sc}$
underexpanded nozzle flow

Convergent-Divergent Nozzle