Fluid Mechanics - MTF053

Lecture 19

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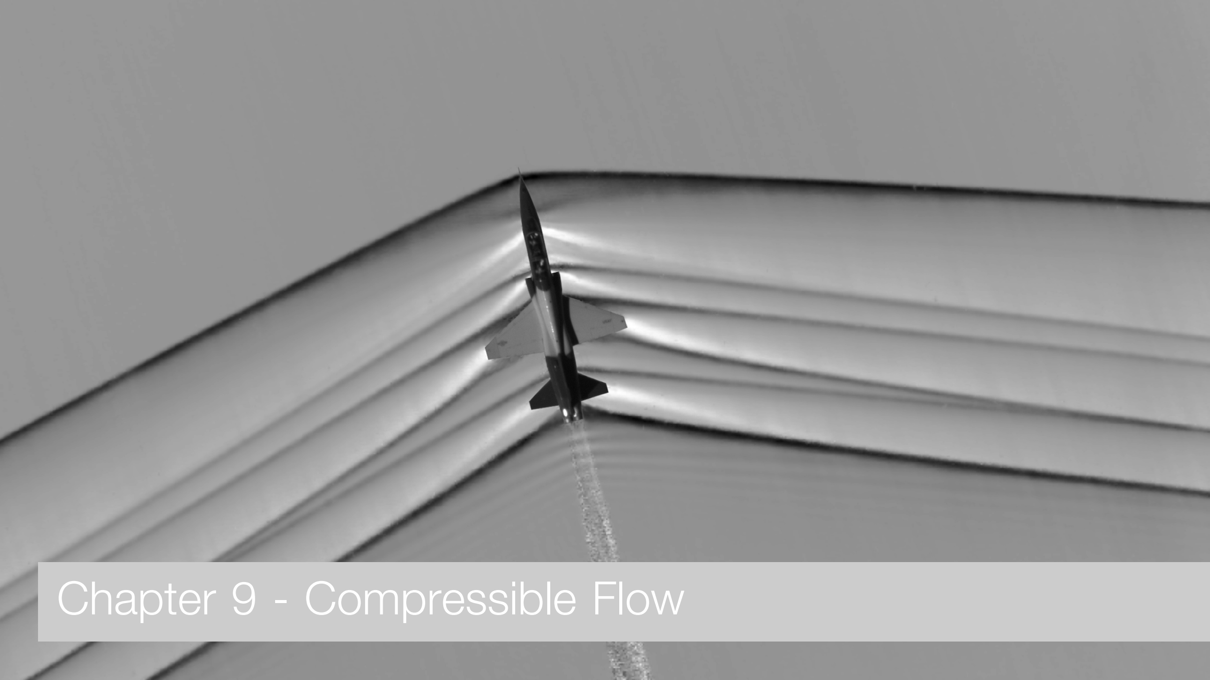
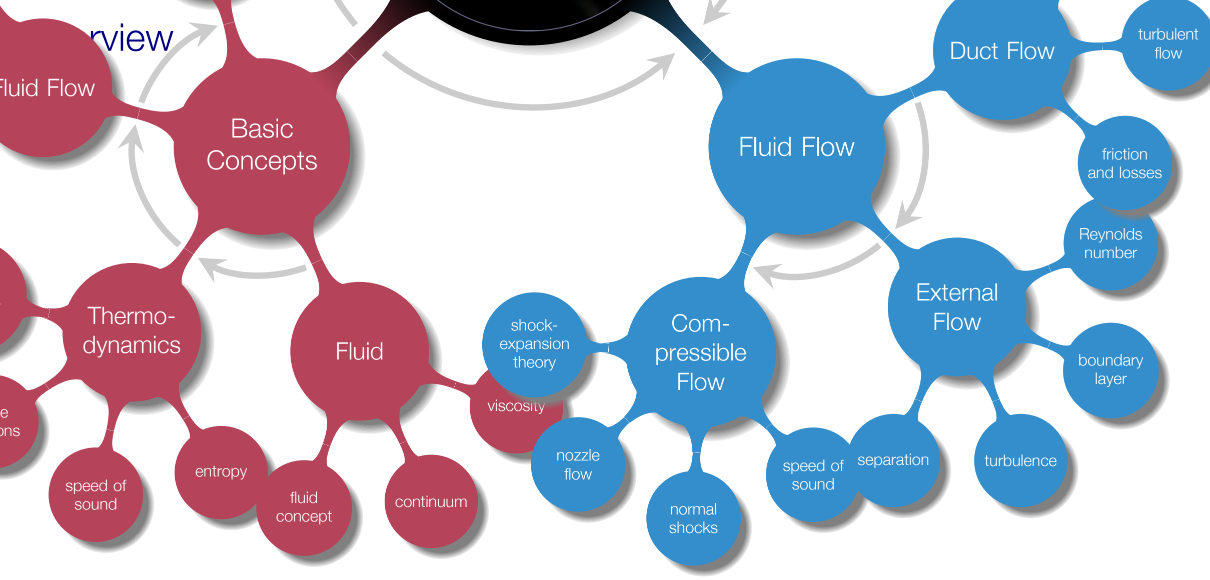


Chapter 9 - Compressible Flow



rview
Fluid Flow
Basic Concepts
Thermo-dynamics
Fluid
e
ons
speed of sound
entropy
fluid concept
continuum
viscosity
Fluid Flow
Duct Flow
turbulent flow
friction and losses
Reynolds number
External Flow
boundary layer
separation
turbulence
Com-pressible Flow
shock-expansion theory
nozzle flow
normal shocks
speed of sound

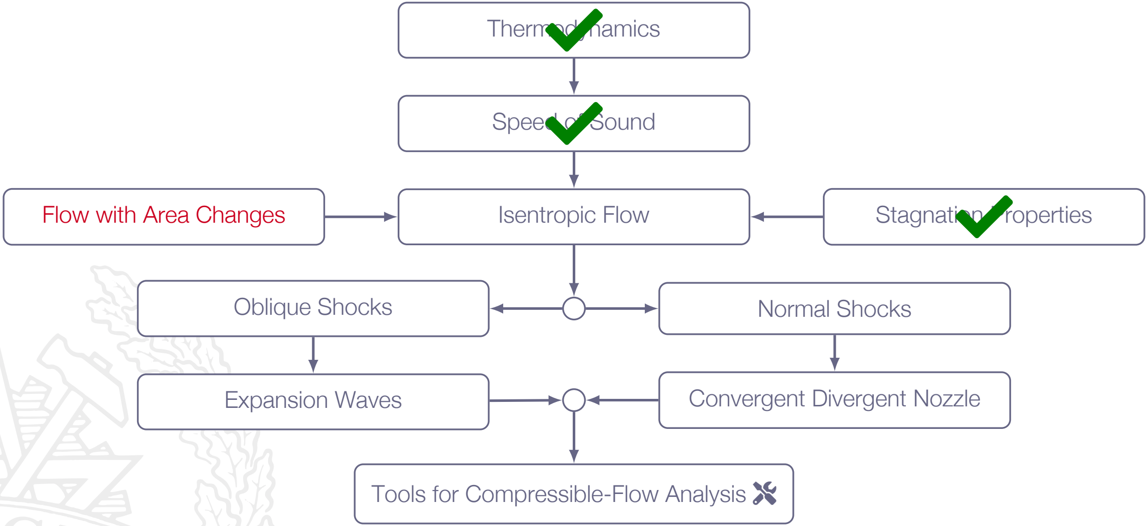
Learning Outcomes

4 Be **able to categorize** a flow and **have knowledge about** how to select applicable methods for the analysis of a specific flow based on category

37 **Understand and explain** basic concepts of compressible flows (the gas law, speed of sound, Mach number, isentropic flow with changing area, normal shocks, oblique shocks, Prandtl-Meyer expansion)

Let's go supersonic ...

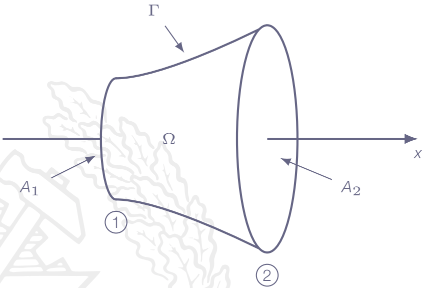
Roadmap - Compressible Flow



Isentropic Quasi-1D Flow

Quasi-1D:

- Flow properties varies in one direction only (x)
- The flow area is a smooth function $A = A(x)$
- Steady-state, inviscid and isentropic flow



The Area-Velocity Relation

Continuity:

$$\rho(x)V(x)A(x) = const \Rightarrow d(\rho VA) = 0 \Rightarrow AVd\rho + \rho AdV + \rho VdA = 0$$

divide by ρVA gives

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0$$

The Area-Velocity Relation

In the following, isentropic flow is assumed

Stagnation enthalpy:

$$h_o = h + \frac{1}{2}V^2 = const \Rightarrow dh + VdV = 0$$

The first and second law of thermodynamics:

$$Tds = 0 = dh - \frac{dp}{\rho} \Rightarrow dh = \frac{dp}{\rho}$$

and thus

$$\frac{dp}{\rho} + VdV = 0$$

The Area-Velocity Relation

$$\frac{dp}{\rho} + VdV = 0$$

From the definition of the **speed of sound**

$$dp = a^2 d\rho \Rightarrow a^2 \frac{d\rho}{\rho} + VdV = 0 \Rightarrow \frac{d\rho}{\rho} = -\frac{1}{a^2} VdV$$

The Area-Velocity Relation

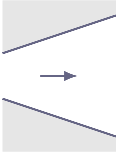
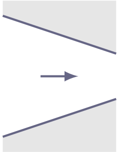
$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = \frac{dV}{V} - \frac{1}{a^2}VdV + \frac{dA}{A} = 0$$

$$\frac{dV}{V}\left(\frac{V^2}{a^2} - 1\right) = \frac{dA}{A}$$

$$\boxed{\frac{dV}{V} = \frac{1}{M^2 - 1}\frac{dA}{A} = -\frac{dp}{\rho V^2}}$$

The Area-Velocity Relation

$$\frac{dV}{V}(M^2 - 1) = \frac{dA}{A}$$

		Subsonic $M < 1$	**Supersonic** $M > 1$
	$dA > 0$	subsonic diffuser $dV < 0$ $dp > 0$	supersonic nozzle $dV > 0$ $dp < 0$
	$dA < 0$	subsonic nozzle $dV > 0$ $dp < 0$	supersonic diffuser $dV < 0$ $dp > 0$

The Area-Velocity Relation

$$\frac{dV}{V}(M^2 - 1) = \frac{dA}{A}$$

What happens when $M = 1$?

The Area-Velocity Relation

$$\frac{dV}{V}(M^2-1)=\frac{dA}{A}$$

What happens when $M=1$?

$M=1$ when $dA=0$

The Area-Velocity Relation

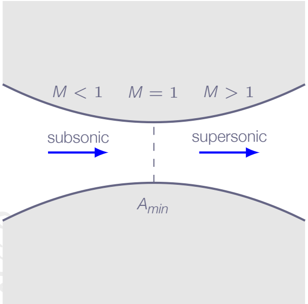
$$\frac{dV}{V}(M^2 - 1) = \frac{dA}{A}$$

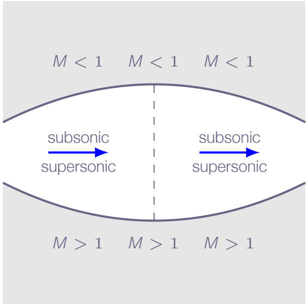
What happens when $M = 1$?

$M = 1$ when $dA = 0$

maximum or minimum area

The Area-Velocity Relation





The Area-Mach-Number Relation

$$\rho AV = \rho^* A^* V^* \Rightarrow \frac{A}{A^*} = \frac{\rho^*}{\rho}\frac{V^*}{V}$$

$$\frac{\rho^*}{\rho} = \frac{\rho^*}{\rho_o}\frac{\rho_o}{\rho} = \left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{1/(\gamma-1)}$$

$$\frac{V^*}{V} = \frac{(\gamma R T^*)^{1/2}}{V} = \frac{(\gamma R T)^{1/2}}{V}\left(\frac{T^*}{T_o}\right)^{1/2}\left(\frac{T_o}{T}\right)^{1/2} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{1/2}$$

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2+(\gamma-1)M^2}{\gamma+1}\right]^{(\gamma+1)/(\gamma-1)}$$

The Area-Mach-Number Relation

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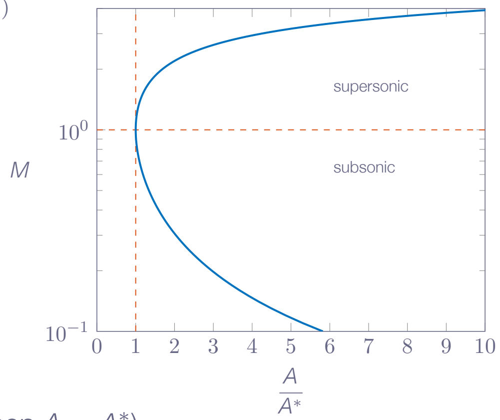


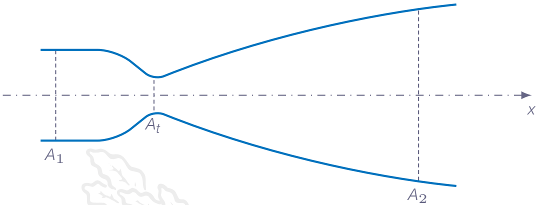
Note!

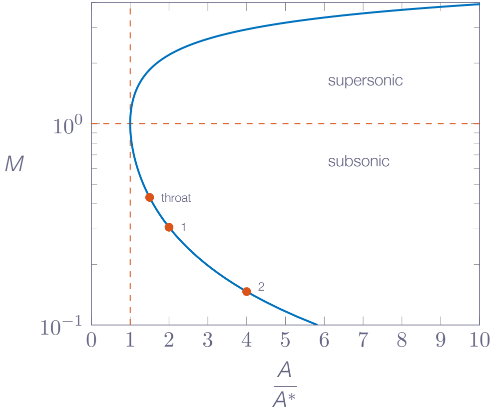
Two possible solutions for each value of $\dfrac{A}{A^*}$:

one subsonic and one supersonic (except when $A = A^*$)

The Area-Mach-Number Relation

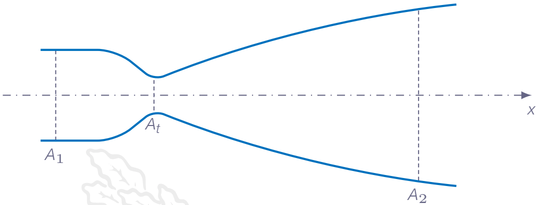
Sub-critical (non-choked) nozzle flow

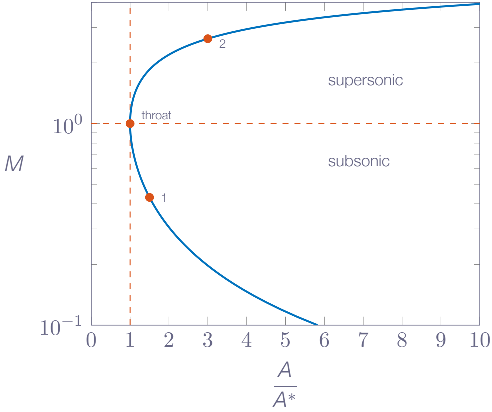




The Area-Mach-Number Relation

Critical (choked) nozzle flow



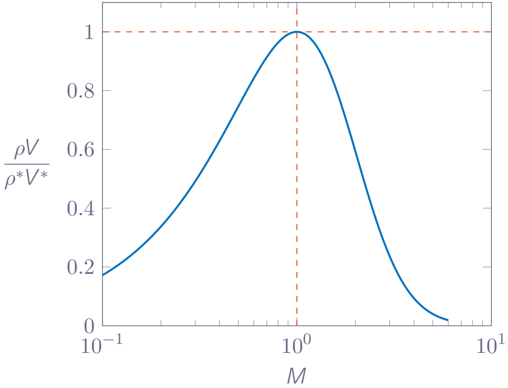


Choking

$$\rho V A = \rho^* A^* V^* \Rightarrow \frac{A^*}{A} = \frac{\rho V}{\rho^* V^*}$$

From the area-Mach-number relation

$$\frac{A^*}{A} = \begin{cases} < 1 & \text{if} \quad M < 1 \\ 1 & \text{if} \quad M = 1 \\ < 1 & \text{if} \quad M > 1 \end{cases}$$



The maximum possible mass flow through a duct is achieved when its throat reaches sonic conditions

Choking

$$\dot{m}_{max} = \rho^* A^* V^*$$

$$\frac{\rho^*}{\rho_o} = \left(\frac{2}{\gamma + 1}\right)^{1/(\gamma-1)}$$

$$V^* = \sqrt{\gamma R T^*}$$

$$\frac{T^*}{T_o} = \frac{2}{\gamma + 1}$$

$$\boxed{\dot{m}_{max} = \frac{A^* p_o}{\sqrt{T_o}} \sqrt{\frac{\gamma}{R} \left(\frac{2}{\gamma + 1}\right)^{\frac{(\gamma+1)}{(\gamma-1)}}}}$$