Fluid Mechanics - MTF053

Lecture 18

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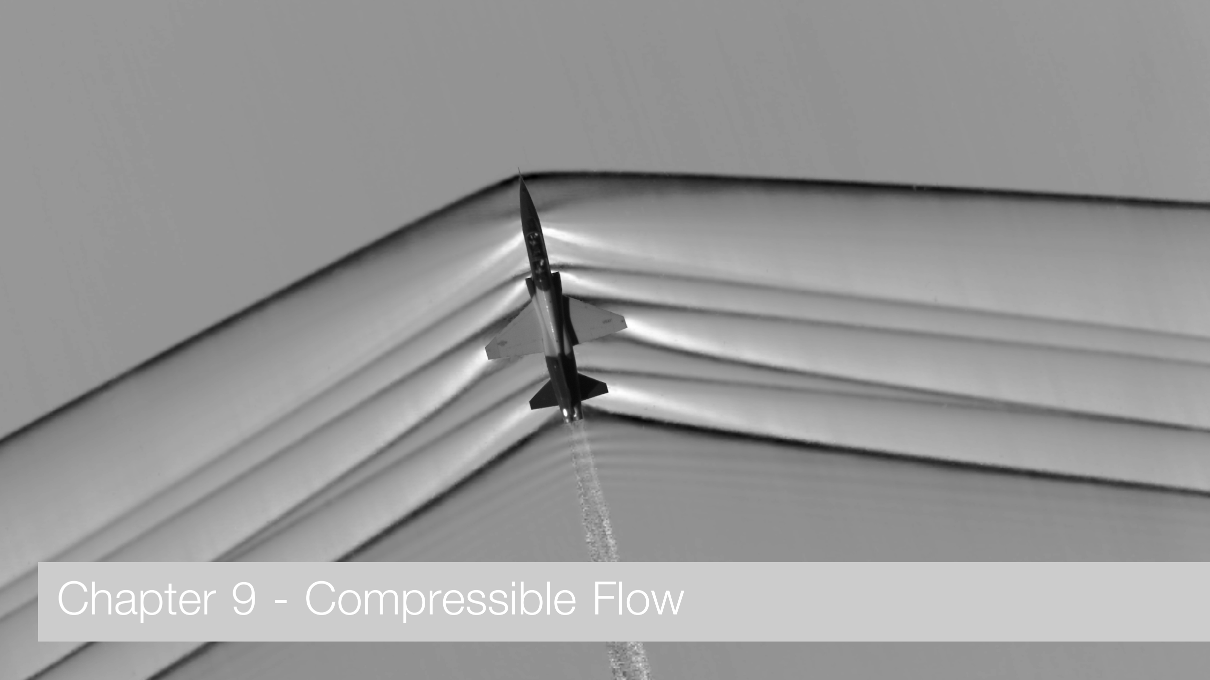
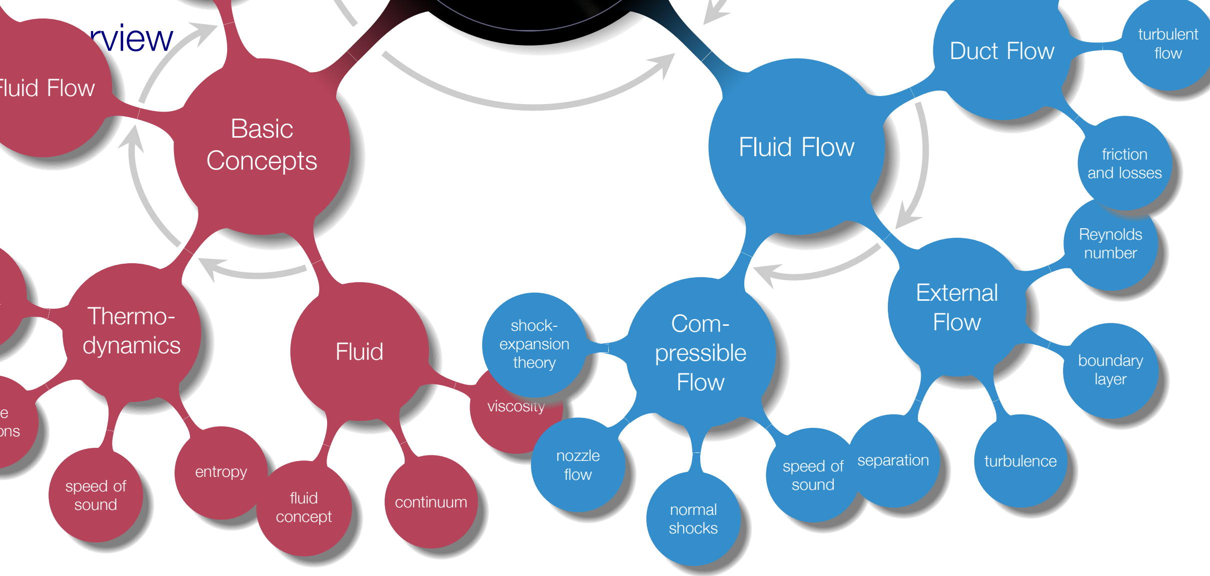


Chapter 9 - Compressible Flow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Fluid Flow
Basic Concepts
Thermo-dynamics
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ons
speed of sound
entropy
fluid concept
Fluid
continuum
viscosity
shock-expansion theory
Com-pressible Flow
nozzle flow
normal shocks
speed of sound
Fluid Flow
Duct Flow
turbulent flow
friction and losses
Reynolds number
External Flow
boundary layer
separation
turbulence

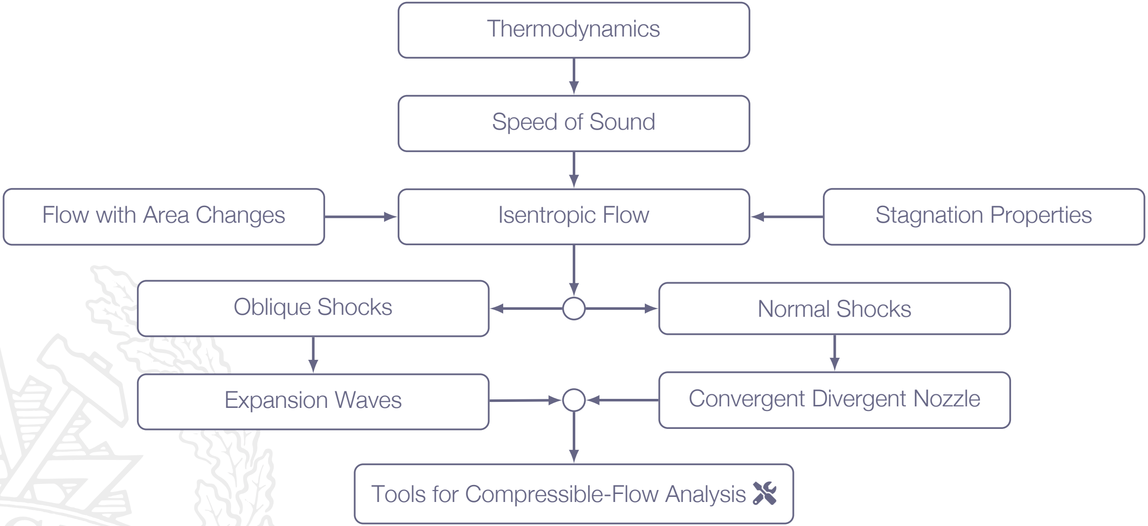
Learning Outcomes

4 Be **able to categorize** a flow and **have knowledge about** how to select applicable methods for the analysis of a specific flow based on category

37 **Understand and explain** basic concepts of compressible flows (the gas law, speed of sound, Mach number, isentropic flow with changing area, normal shocks, oblique shocks, Prandtl-Meyer expansion)

Let's go supersonic ...

Roadmap - Compressible Flow



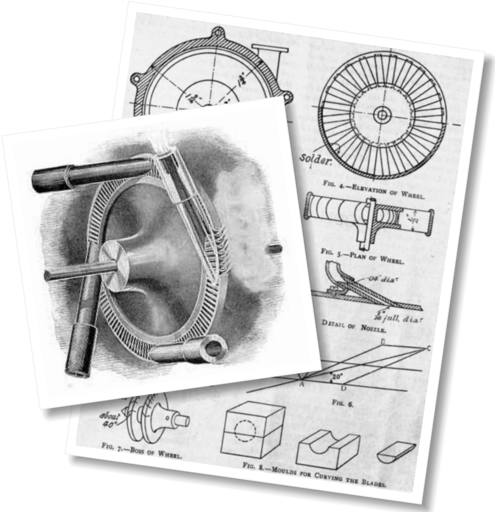
Motivation

Compressible flow:

- flows where variations in density are significant
- most often high-speed gas flows (gas dynamics)
- fluids moving at speeds comparable to the speed of sound
- not common in liquids (would require very high pressures)

Historical Milestones

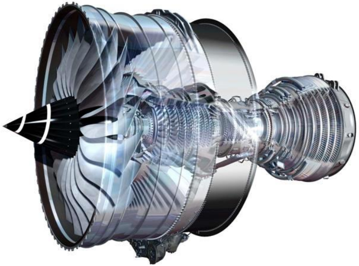
First supersonic flight - Charles Yeager 1947



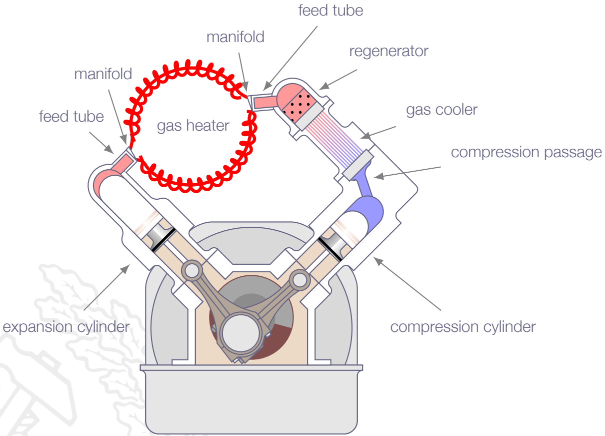
Steam turbine with convergent-divergent nozzles - Carl Gustav de Laval 1893

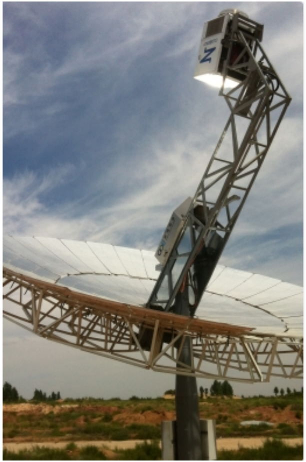
Compressible Flow Applications

Compressible Flow Applications



Compressible Flow Applications





Governing Equations

- With significant density changes follows substantial changes in pressure and temperature
- The energy equation must be included
- Four equations:
  1. Continuity
  2. Momentum
  3. Energy
  4. Equation of state
- Unknowns: ρ, p, T, and $\mathbf{V}$
- The four equations must be solved simultaneously

Mach Number Regimes

Incompressible flow

- insignificant density changes

Subsonic flow

- local and global Mach number less than unity

Transonic flow

- subsonic flow with regions of supersonic flow (local Mach number can be higher than one)
- supersonic flow with regions of subsonic flow (local Mach number can be less than one)

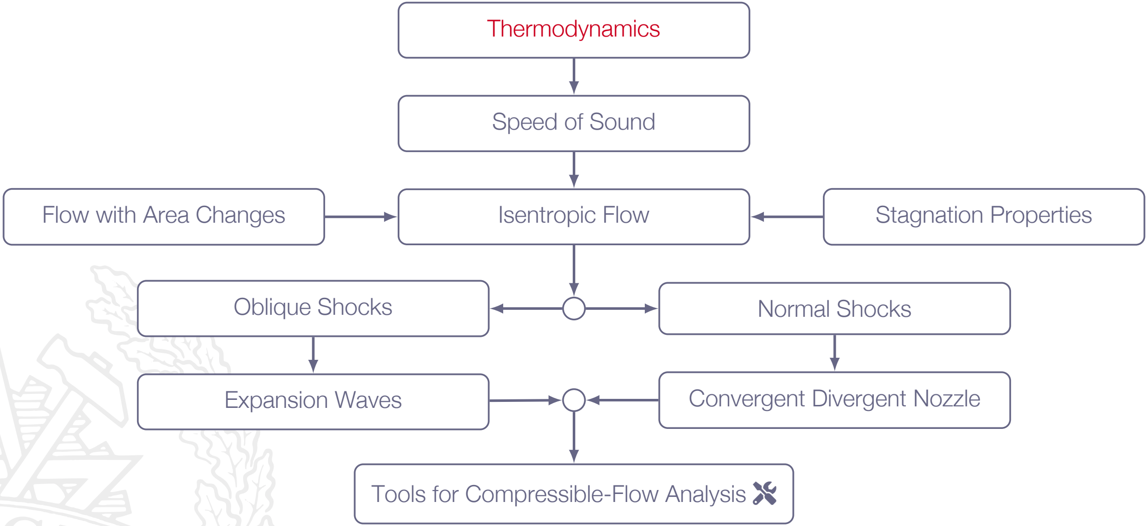
Supersonic flow

- local and global Mach number higher than one

Hypersonic flow

- Mach number higher than 5.0

Roadmap - Compressible Flow



Ratio of Specific Heats

- The ratio of specific heats is important in compressible flow

$$\gamma = \frac{C_p}{C_v}$$

- γ is a fluid property
- For moderate temperatures γ is a constant
- For higher temperatures γ varies with temperature
- For air, $\gamma = 1.4$

Equation of State

In the following, we will assume that the ideal gas law is applicable and that the specific heats are constants:

$$p = \rho RT$$

$$R = C_p - C_v = const$$

$$\gamma = \frac{C_p}{C_v} = const$$

Auxiliary relations:

$$C_v = \frac{R}{\gamma - 1}, C_p = \frac{\gamma R}{\gamma - 1}$$

Internal Energy and Enthalpy

Constant specific heats:

$$d\hat{u} = C_v dT$$

$$dh = C_p dT$$

Variable specific heats:

$$\hat{u} = \int C_v dT$$

$$h = \int C_p dT$$

Isentropic Relations

First law of thermodynamics

$$\delta q + \delta w = de$$

For reversible processes: $\delta w = -pd\upsilon$ (where $\upsilon = p/\rho$)

$$h = e + \frac{p}{\rho} = e + p\upsilon \Rightarrow dh = de + pd\upsilon + \upsilon dp$$

$$\delta q = dh - \upsilon dp$$

Second law of thermodynamics

$$ds = \frac{\delta q_{rev}}{T} = \frac{\delta q}{T} + ds_{irev} \Rightarrow ds \geq \frac{\delta q}{T}$$

Isentropic Relations

compute entropy change from the first and second law of thermodynamics (assuming reversible heat addition)

$$Tds = dh - \frac{dp}{\rho}$$

for perfect gases, $dh = C_p dT$

$$\int_1^2 ds = \int_1^2 C_p \frac{dT}{T} - R \int_1^2 \frac{dp}{p}$$

for constant specific heats (calorically perfect)

$$s_2 - s_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = C_v \ln \frac{T_2}{T_1} - R \ln \frac{\rho_2}{\rho_1}$$

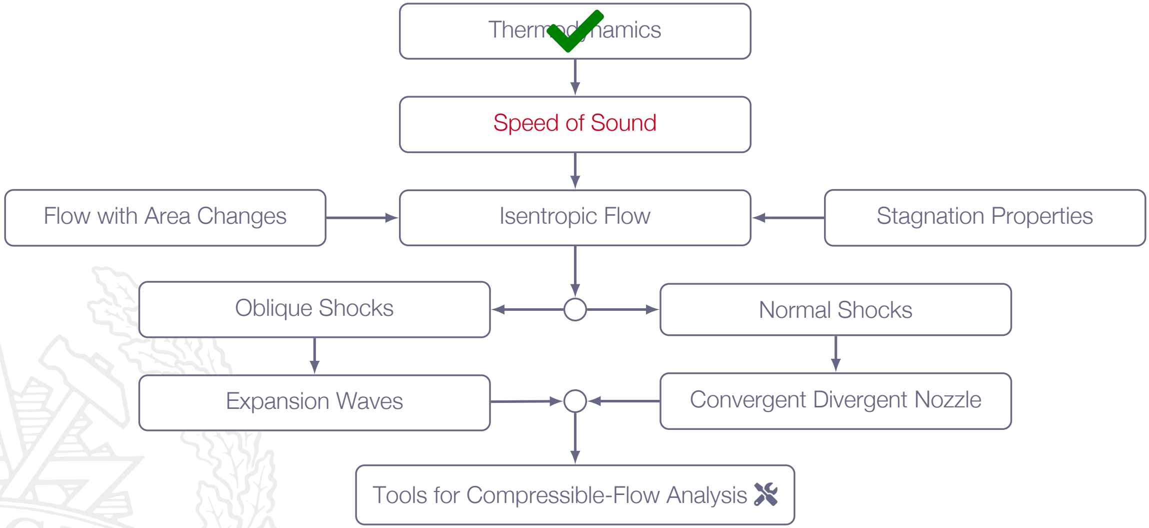
Isentropic Relations

$$s_2 - s_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = C_v \ln \frac{T_2}{T_1} - R \ln \frac{\rho_2}{\rho_1}$$

for isentropic flow ($s_2 = s_1$) we get

$$\boxed{\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}}$$

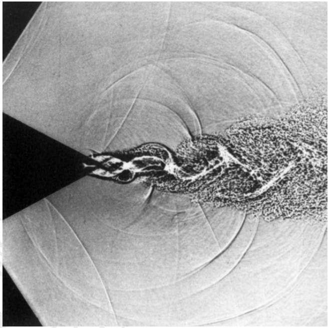
Roadmap - Compressible Flow

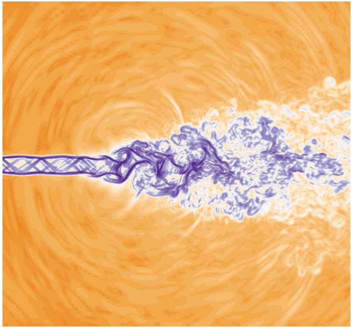


Speed of Sound

- The rate of propagation of a pressure pulse of infinitesimal strength through a fluid at rest
- Related to the molecular activity of the fluid
- A thermodynamic property

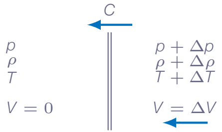
Speed of Sound



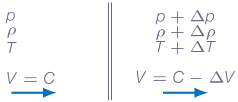


Speed of Sound

frame of reference fixed to fluid



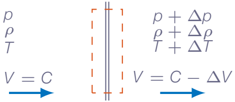
frame of reference following the wave



Speed of Sound

frame of reference following the wave

continuity:



$$\rho AC = (\rho + \Delta\rho)A(C - \Delta V)$$

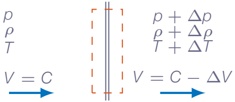
$$\Delta V = C\frac{\Delta\rho}{\rho + \Delta\rho}$$

Note! there are no gradients in the flow so viscous effects are confined to the interior of the wave

Speed of Sound

frame of reference following the wave

momentum:



$$pA - (p + \Delta p)A = (\rho AC)(C - \Delta V - C) \Rightarrow \Delta p = \rho C \Delta V$$

with ΔV from the continuity equation we get

$$C^2 = \frac{\Delta p}{\Delta \rho}\left(1 + \frac{\Delta \rho}{\rho}\right)$$

Note! the larger $\Delta\rho/\rho$, the higher the propagation velocity

Speed of Sound

In the limit of infinitesimal strength $\Delta\rho \rightarrow 0$ and thus

$$C^2 = a^2 = \frac{\partial p}{\partial \rho}$$

- There is no added heat and thus the process adiabatic
- For weak waves the process can also be assumed to be reversible

$$a^2 = \left.\frac{\partial p}{\partial \rho}\right|_s$$

Speed of Sound

$$a^2 = \left.\frac{\partial p}{\partial \rho}\right|_s$$

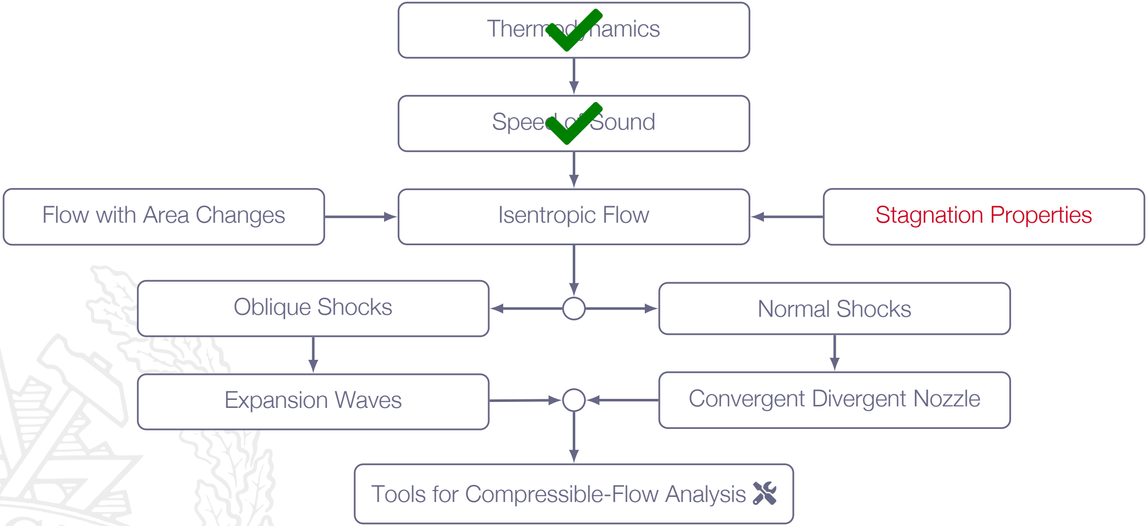
The isentropic relation gives

$$p = \rho^\gamma \Rightarrow \frac{\partial p}{\partial \rho} = \gamma \rho^{\gamma - 1} = \gamma \frac{p}{\rho} = \gamma RT$$

and thus

$$\boxed{a = \sqrt{\gamma RT}}$$

Roadmap - Compressible Flow



Stagnation Enthalpy

Consider high-speed gas flow past an insulated wall

$$h_1 + \frac{1}{2}V_1^2 + gz_1 = h_2 + \frac{1}{2}V_2^2 + gz_2 - q + w_\nu$$

- differences in potential energy extremely small
- outside of the boundary layer, heat transfer and viscous work are zero

$$h_1 + \frac{1}{2}V_1^2 = h_2 + \frac{1}{2}V_2^2 = const$$

Stagnation Enthalpy

$$h + \frac{1}{2}V^2 = h_o$$

"The maximum enthalpy that the fluid would achieve if brought to rest adiabatically"

Stagnation Temperature

For a calorically perfect gas $h = C_p T$

$$h + \frac{1}{2}V^2 = h_o$$

$$C_p T + \frac{1}{2}V^2 = C_p T_o$$

Where T_o is the stagnation temperature

Mach Number Relations

$$C_pT + \frac{1}{2}V^2 = C_pT_o \Rightarrow 1 + \frac{V^2}{2C_pT} = \frac{T_o}{T}$$

$$C_pT = \frac{\gamma R}{\gamma - 1}T = \frac{\gamma RT}{\gamma - 1} = \frac{a^2}{\gamma - 1}$$

$$\boxed{\frac{T_o}{T} = 1 + \left(\frac{\gamma - 1}{2}\right)M^2}$$

Mach Number Relations

Since $a \propto T^{1/2}$ we get

$$\frac{a_o}{a} = \left(\frac{T_o}{T}\right)^{1/2} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]^{1/2}$$

Mach Number Relations

If the flow is adiabatic and reversible (isentropic), we may use the isentropic relations

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\gamma/(\gamma-1)} = \left[1 + \left(\frac{\gamma-1}{2}\right)M^2\right]^{\gamma/(\gamma-1)}$$

$$\frac{\rho_o}{\rho} = \left(\frac{T_o}{T}\right)^{1/(\gamma-1)} = \left[1 + \left(\frac{\gamma-1}{2}\right)M^2\right]^{1/(\gamma-1)}$$

Stagnation Properties

- p_o and ρ_o - the pressure and density that the flow would achieve if brought to rest isentropically
- All stagnation properties are constants in an isentropic flow
- h_o, T_o, and a_o are constants in an adiabatic flow but not necessarily p_o and ρ_o
- p_o and ρ_o will vary throughout an adiabatic flow as the entropy changes due to friction or shocks

Critical Properties

Another useful set of reference variables is the critical properties (sonic conditions)

$$\frac{T_o}{T} = 1 + \left(\frac{\gamma - 1}{2}\right) M^2 = \{M = 1.0\} = 1 + \left(\frac{\gamma - 1}{2}\right) = \left(\frac{2 + \gamma - 1}{2}\right) = \left(\frac{\gamma + 1}{2}\right)$$

Critical Properties

$$\frac{T^*}{T_o} = \left(\frac{2}{\gamma + 1}\right)$$

$$\frac{a^*}{a_o} = \left(\frac{2}{\gamma + 1}\right)^{1/2}$$

$$\frac{p^*}{p_o} = \left(\frac{2}{\gamma + 1}\right)^{\gamma/(\gamma-1)}$$

$$\frac{\rho^*}{\rho_o} = \left(\frac{2}{\gamma + 1}\right)^{1/(\gamma-1)}$$

Critical Properties

Air $\gamma = 1.4$

$$\frac{T^*}{T_o} = \left(\frac{2}{\gamma+1}\right) = 0.8333$$

$$\frac{a^*}{a_o} = \left(\frac{2}{\gamma+1}\right)^{1/2} = 0.9129$$

$$\frac{p^*}{p_o} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)} = 0.5283$$

$$\frac{\rho^*}{\rho_o} = \left(\frac{2}{\gamma+1}\right)^{1/(\gamma-1)} = 0.6339$$