Fluid Mechanics - MTF053

Lecture 13

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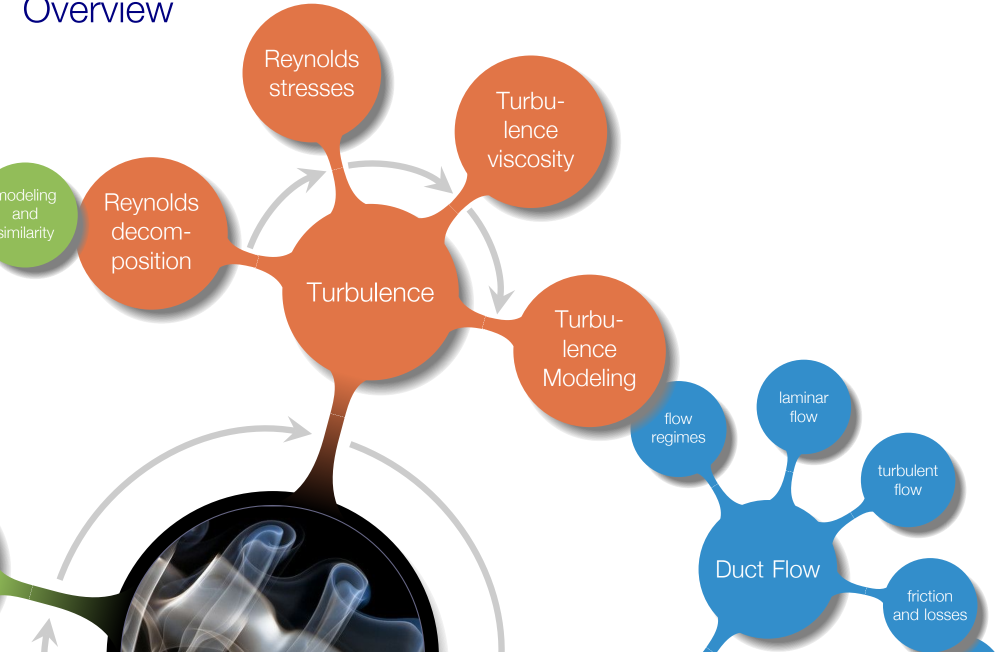
Chapter 6 - Viscous Flow in Ducts

Overview



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and
similarity
Reynolds
decom-
position
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stresses
Turbu-
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viscosity
Turbulence
Turbu-
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Modeling
flow
regimes
laminar
flow
turbulent
flow
Duct Flow
friction
and losses

Learning Outcomes

3 **Define** the Reynolds number

4 Be **able to categorize** a flow and **have knowledge about** how to select applicable methods for the analysis of a specific flow based on category

6 **Explain** what a boundary layer is and when/where/why it appears

8 **Understand** and be able to **explain** the concept shear stress

18 **Explain** losses appearing in pipe flows

19 **Explain** the difference between laminar and turbulent pipe flow

20 **Solve** pipe flow problems using Moody charts

24 **Explain** what is characteristic for a turbulent flow

25 **Explain** Reynolds decomposition and derive the RANS equations

26 **Understand** and **explain** the Boussinesq assumption and turbulent viscosity

27 **Explain** the difference between the regions in a boundary layer and what is characteristic for each of the regions (viscous sub layer, buffer region, log region)

if you think about it, pipe flows are everywhere (a pipe flow is not a flow of pipes)

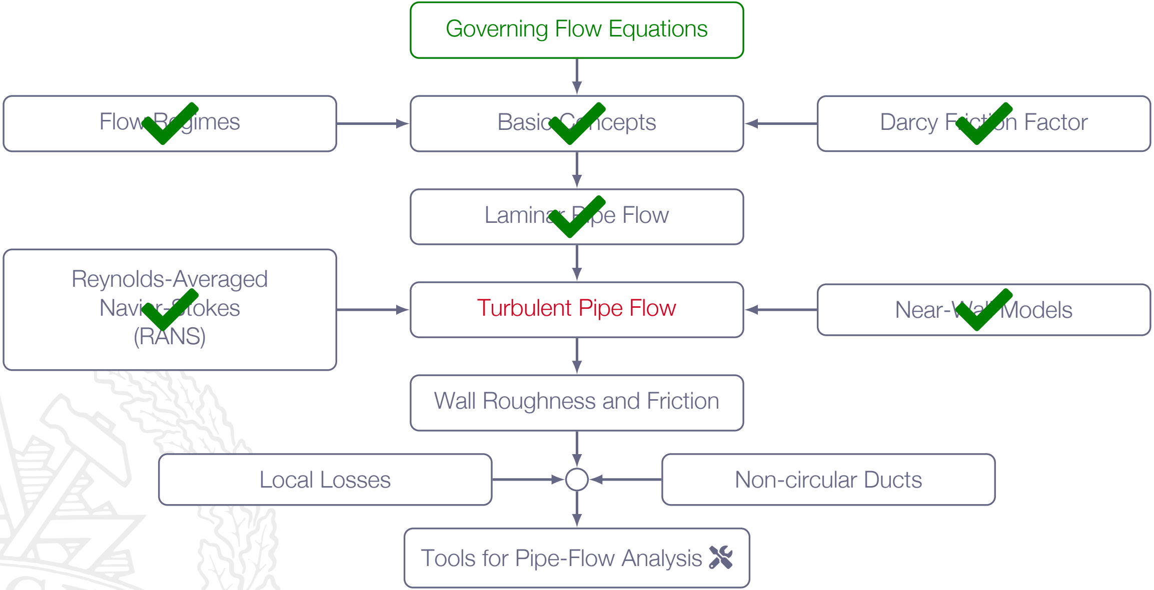
Complementary Course Material

These lecture notes covers chapter 6 in the course book and additional course material that you can find in the following documents

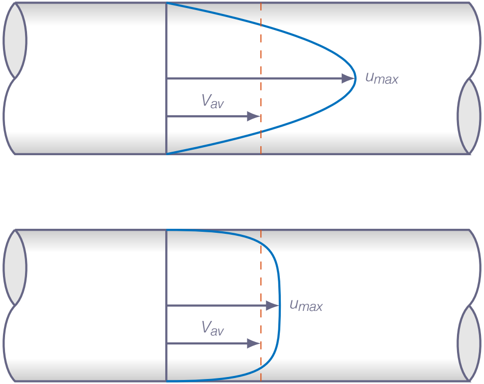
MTF053_Equation-for-Boundary-Layer-Flows.pdf

MTF053_Turbulence.pdf

Roadmap - Viscous Flow in Ducts



Turbulent Pipe Flow



Laminar flow

Turbulent flow

Turbulent Pipe Flow

As we did for laminar pipe flow, we will now obtain the friction factor for turbulent pipe flow

$$\tau_w = f_D \frac{\rho V^2}{8} = \rho u^{*2} \Rightarrow f_D = 8 \left(\frac{V}{u^*} \right)^{-2}$$

So, what we need now is an estimate of the average flow velocity in the pipe (V) ...

There are different ways to do this and here is one example:

1. Assume that we can use the log-law all the way across the pipe
2. Integrate to get the average velocity
3. Insert the calculated average velocity into the relation above

Turbulent Pipe Flow

$$f_D = 8\left(\frac{V}{u^*}\right)^{-2}$$

$$\left.\begin{aligned} \frac{\overline{u}(r)}{u^*} &\approx \frac{1}{\kappa}\ln\frac{(R-r)u^*}{\nu} + B \\ V = \frac{Q}{A} &= \frac{1}{\pi R^2}\int_0^R \overline{u}(r)2\pi r dr \end{aligned}\right\} \Rightarrow \frac{V}{u^*} \approx \frac{1}{\pi R^2}\int_0^R \left[\frac{1}{\kappa}\ln\frac{(R-r)u^*}{\nu} + B\right] r dr$$

with $\kappa = 0.41$ and $B = 5.0$ we get

$$\frac{V}{u^*} \approx 2.44\ln\frac{Ru^*}{\nu} + 1.34$$

details on next slide

Turbulent Pipe Flow

$$\frac{V}{u^*} = \frac{2}{R^2}\int_0^R \left[\frac{r}{\kappa}\ln\left(\frac{(R-r)u^*}{\nu}\right) + Br\right] dr = \frac{2}{\kappa R^2}\int_0^R \left[\ln(R-r) + \ln\left(\frac{u^*}{\nu}\right) + B\kappa\right] r dr =$$

$$= \frac{1}{\kappa}\left(\ln\left(\frac{u^*}{\nu}\right) + B\kappa\right) + \frac{2}{\kappa R^2}\int_0^R r\ln(R-r)dr =$$

$$= \frac{1}{\kappa}\ln\left(\frac{u^*}{\nu}\right) + B + \frac{2}{\kappa R^2}\left[\frac{1}{4}\left(-2(R^2-r^2)\ln(R-r) - r(2R+r)\right)\right]_0^R =$$

$$= \frac{1}{\kappa}\ln\left(\frac{Ru^*}{\nu}\right) + B - \frac{3}{2\kappa} = \{\kappa = 0.41,\ B = 5.0\} = 2.44\ln\frac{Ru^*}{\nu} + 1.34$$

Turbulent Pipe Flow

$$\frac{V}{u^*} \approx 2.44 \ln \frac{Ru^*}{\nu} + 1.34$$

The argument of the logarithm can be rewritten as

$$\frac{Ru^*}{\nu} = \frac{VD}{2\nu}\frac{u^*}{V} = \left\{Re_D = \frac{VD}{\nu},\ f_D = 8\left(\frac{u^*}{V}\right)^2\right\} = \frac{1}{2}Re_D\left(\frac{f_D}{8}\right)^{1/2}$$

and thus:

$$\boxed{\frac{1}{\sqrt{f_D}} \approx 2.0 \log_{10}(Re_D\sqrt{f_D}) - 0.8}$$

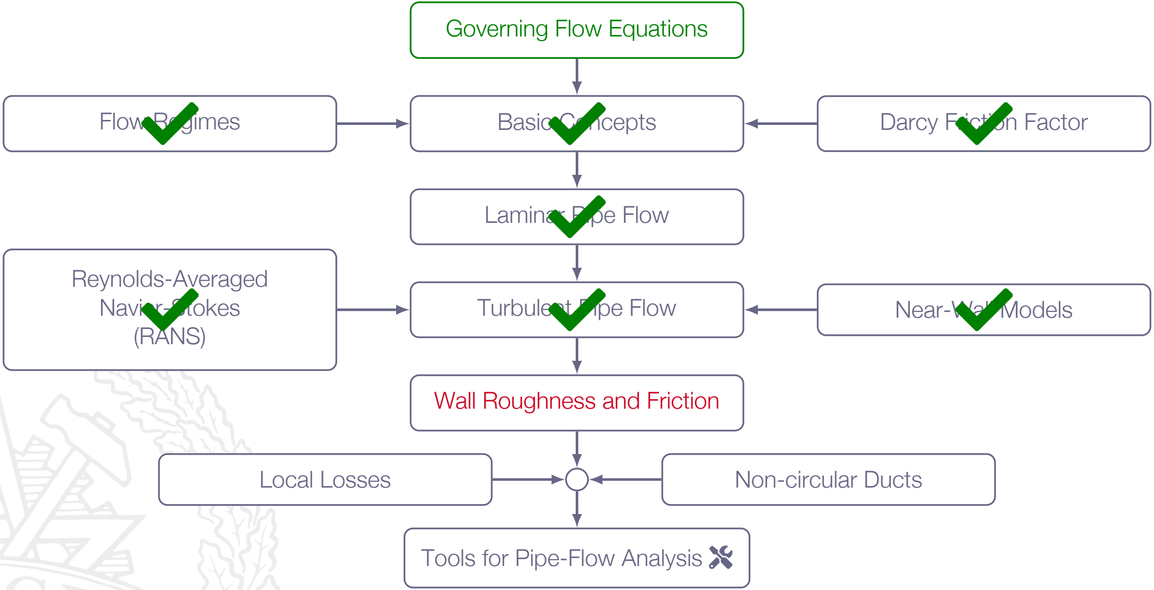
Turbulent Pipe Flow

Alternative 2:

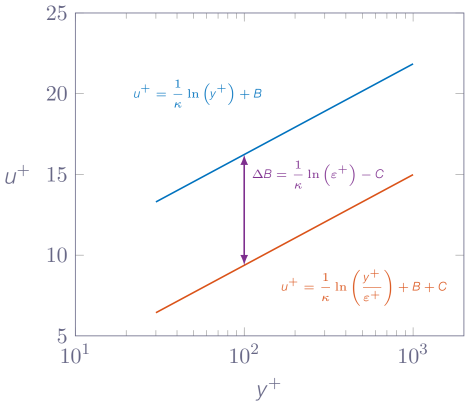
If we assume that $\frac{\bar{u}}{u^*} = 8.3 \left(\frac{u^* y}{\nu}\right)^{1/7}$ applies all over the cross section we get

$$f_D = \frac{0.3164}{Re_D^{1/4}}$$

Roadmap - Viscous Flow in Ducts



Wall Roughness



Effects of surface roughness on friction:

- Negligible for laminar pipe flow
- Significant for turbulent flow
  - breaks up the viscous sublayer
  - modifies the log law (changes the value of the integration constant B)

$$\Delta B \propto (1/\kappa) \ln \epsilon^+ \quad \text{where } \epsilon^+ = \frac{\epsilon u^*}{\nu}$$

Wall Roughness

$$\frac{\epsilon u^*}{\nu} < 5$$

hydraulically smooth

no effects of roughness

$$5 \leq \frac{\epsilon u^*}{\nu} \leq 70$$

transitional

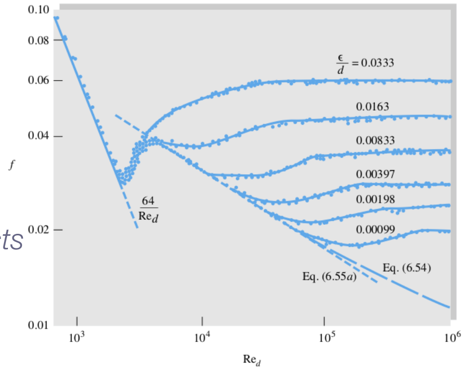
moderate Reynolds number effects

$$\frac{\epsilon u^*}{\nu} > 70$$

fully rough

sublayer totally broken up

independent of Reynolds number



Wall Roughness

$\frac{\epsilon u^*}{\nu} < 5$ hydraulically smooth

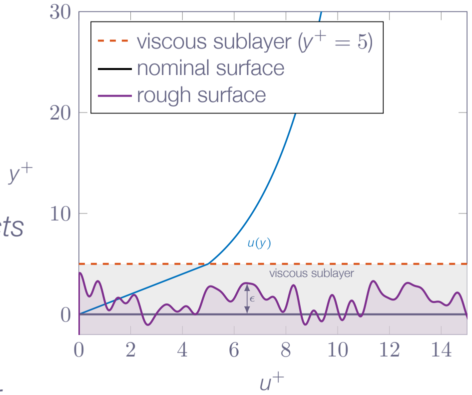
no effects of roughness

$5 \leq \frac{\epsilon u^*}{\nu} \leq 70$ transitional

moderate Reynolds number effects

$\frac{\epsilon u^*}{\nu} > 70$ fully rough

sublayer totally broken up
independent of Reynolds number



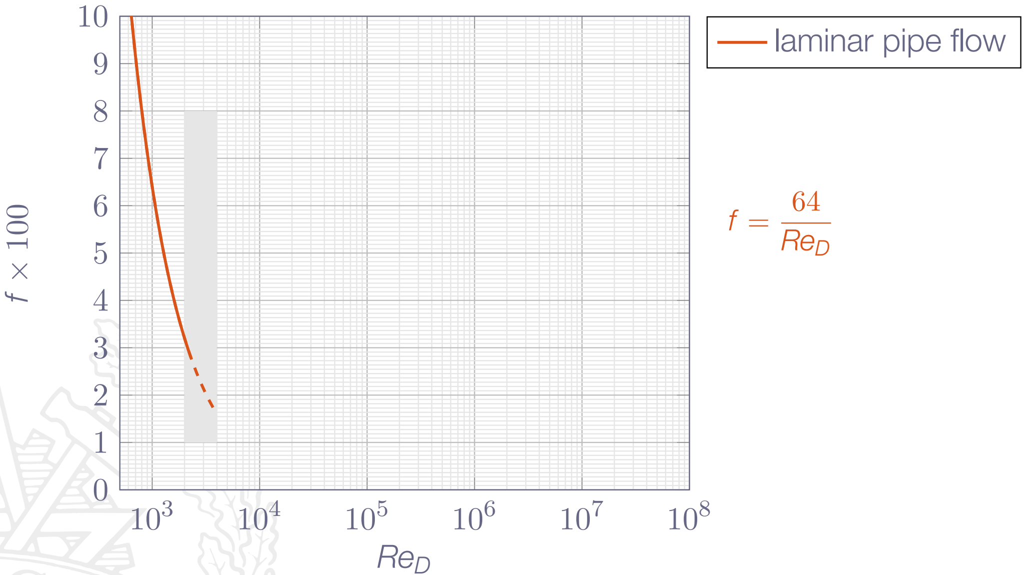
Wall Roughness

Ra	Roughness Average	arithmetic average of the absolute values of the profile heights
Rq	RMS Roughness	root mean square average of the profile heights
Rp	Maximum Profile Peak Height	distance between the highest point of the profile and the mean line
Rpm	Average Maximum Profile Peak Height	average of the successive values of **Rp**
Rv	Maximum Profile Valley Depth	distance between the deepest valley of the profile and the mean line
Rt	Maximum Height of the Profile	vertical distance between the highest and lowest points of the profile
Rz	Average Maximum Height of the Profile	average of the successive values of **Rt**

Wall Roughness

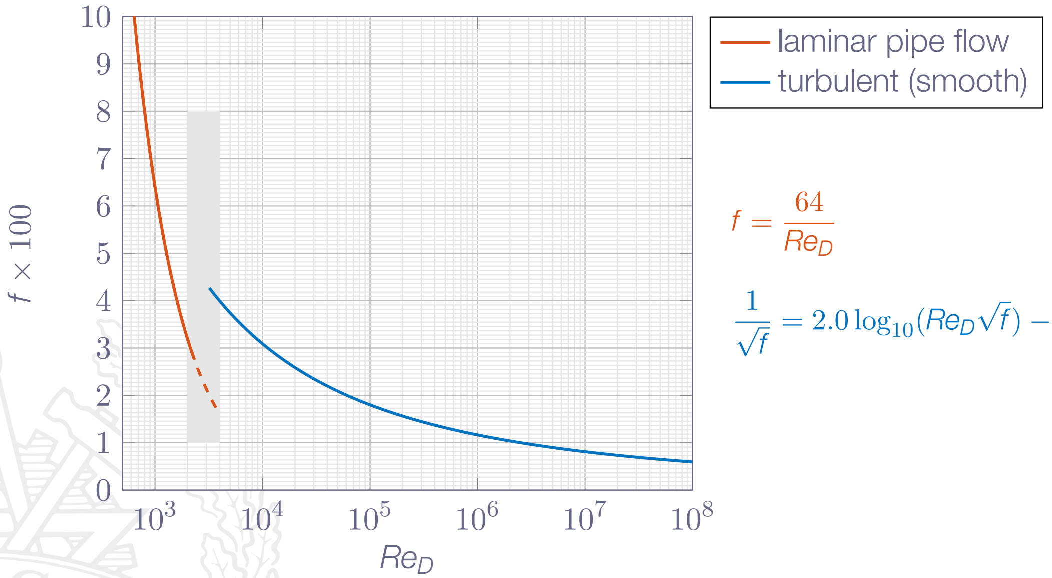
$$\frac{1}{\sqrt{f_D}} = -2.0 \log_{10} \left(\frac{\epsilon/D}{3.7} + \frac{2.5}{Re_D \sqrt{f_D}} \right)$$

The Moody Chart



$$f = \frac{64}{Re_D}$$

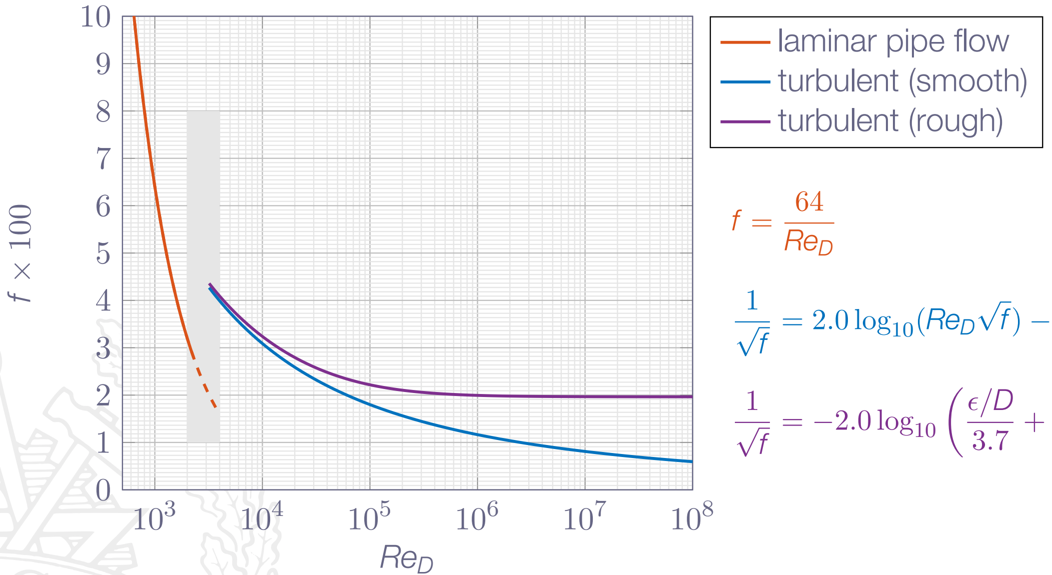
The Moody Chart



$$f = \frac{64}{Re_D}$$

$$\frac{1}{\sqrt{f}} = 2.0\log_{10}(Re_D\sqrt{f}) - 0.8$$

The Moody Chart

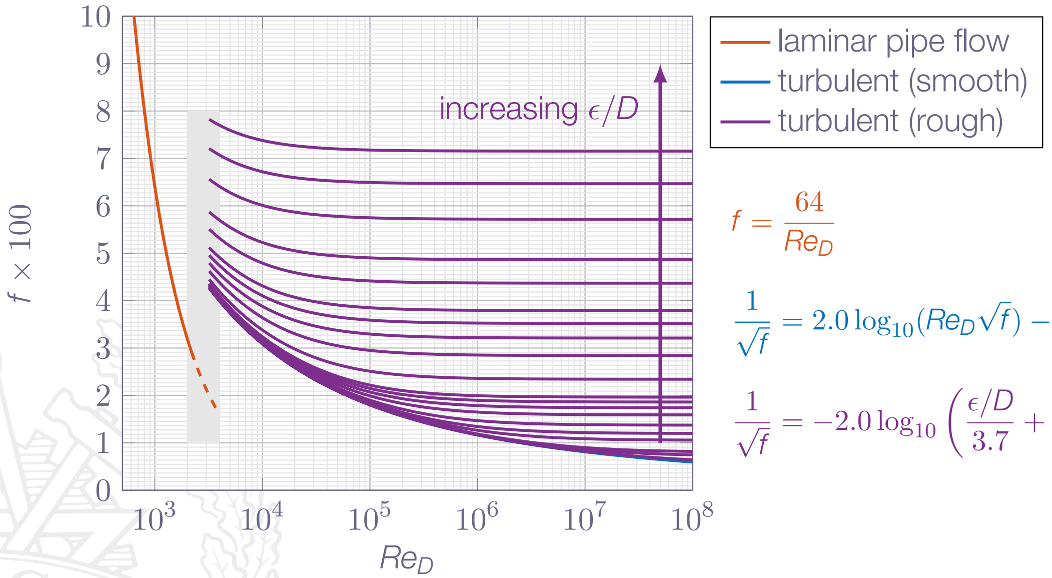


$$f = \frac{64}{Re_D}$$

$$\frac{1}{\sqrt{f}} = 2.0\log_{10}(Re_D\sqrt{f}) - 0.8$$

$$\frac{1}{\sqrt{f}} = -2.0\log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{2.5}{Re_D\sqrt{f}}\right)$$

The Moody Chart



$$f = \frac{64}{Re_D}$$

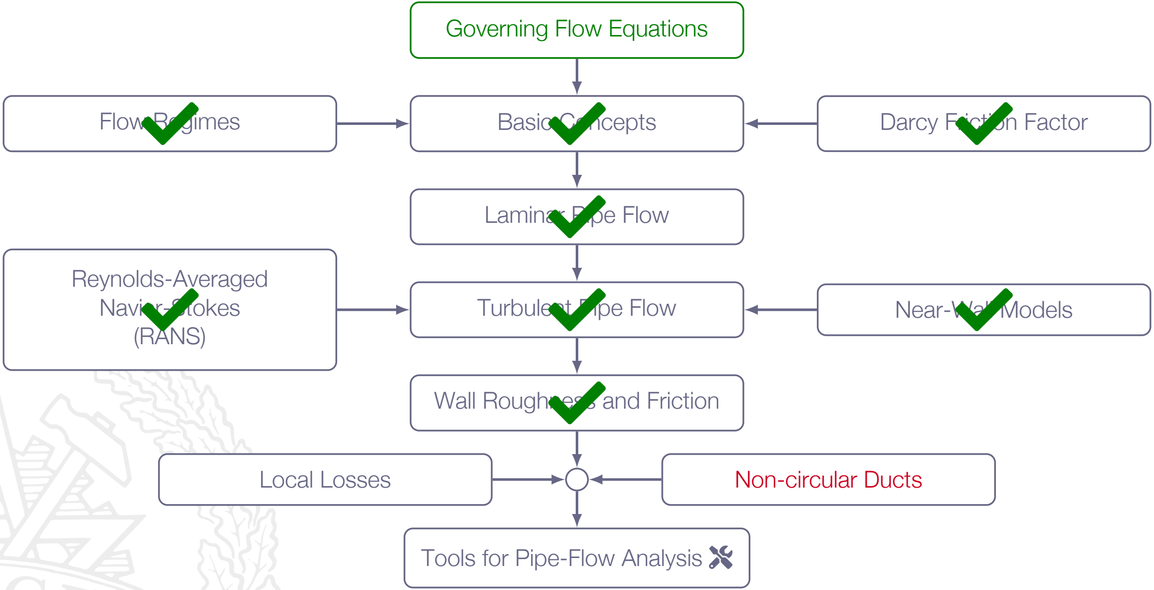
$$\frac{1}{\sqrt{f}} = 2.0 \log_{10}(Re_D\sqrt{f}) - 0.8$$

$$\frac{1}{\sqrt{f}} = -2.0 \log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{2.5}{Re_D\sqrt{f}}\right)$$

Wall Roughness

Material	Condition	ϵ [mm]	**Uncertainty** [%]
Steel	Sheet metal (new)	0.05	$\pm$ 60
	Stainless (new)	0.002	$\pm$ 50
	Commercial (new)	0.046	$\pm$ 30
	Riveted	3.0	$\pm$ 70
	Rusted	2.0	$\pm$ 50
Iron	Cast (new)	0.26	$\pm$ 50
	Wrought (new)	0.046	$\pm$ 20
	Galvanized (new)	0.15	$\pm$ 40
	Asphalted cast	0.12	$\pm$ 50
Brass	Drawn (new)	0.002	$\pm$ 50
Plastic	Drawn tubing	0.0015	$\pm$ 60
Glass	-	smooth	
Concrete	Smoothed	0.04	$\pm$ 60
	Rough	2.0	$\pm$ 50
Rubber	Smoothed	0.01	$\pm$ 60
Wood	Stave	0.5	$\pm$ 40

Roadmap - Viscous Flow in Ducts



Non-circular Ducts

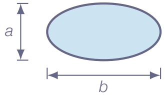
Use the same formulas of the Moody chart but replace the pipe diameter D with the hydraulic diameter D_h

$$D_h = \frac{4A}{\mathcal{P}}$$

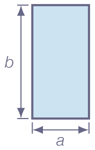
where A is the cross section area and $\mathcal{P}$ is the wetter perimeter

$$\Delta p_f = f_D \frac{L}{D_h} \frac{\rho V^2}{2}, \; Re_{D_h} = \frac{V D_h}{\nu}, \; \frac{\epsilon}{D_h}$$

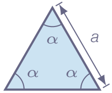
Non-circular Ducts



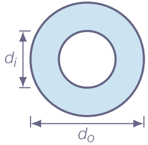
a/b	D_h	C
0.7	$1.17a$	65.0
0.5	$1.30a$	68.0
0.3	$1.44a$	73.0
0.2	$1.50a$	78.0
0.1	$1.55a$	79.0



b/a	D_h	C
1.0	$1.00a$	57.0
1.25	$1.11a$	57.6
2.0	$1.33a$	62.0
3.0	$1.50a$	69.0
4.0	$1.60a$	73.0
5.0	$1.67a$	78.0
8.0	$1.78a$	83.0
10.0	$1.82a$	85.0

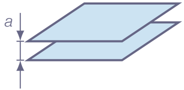


D_h	C
$0.58a$	53.0



d_i/d_o	C
$\frac{d_i}{d_o} = 0.10$	89.2
$\frac{d_i}{d_o} = 0.25$	94.0
$0.5 < \frac{d_i}{d_o} < 1.0$	96.0

$$D_h = d_o - d_i$$



D_h	C
$2.0a$	96.0

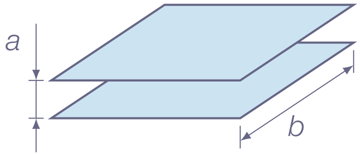
Non-circular Ducts

Laminar flow:

$$f_D = \frac{C}{Re_{D_h}}$$

(for circular pipes: $C = 64$ *and* $D_h = D$*)*

Non-circular Ducts

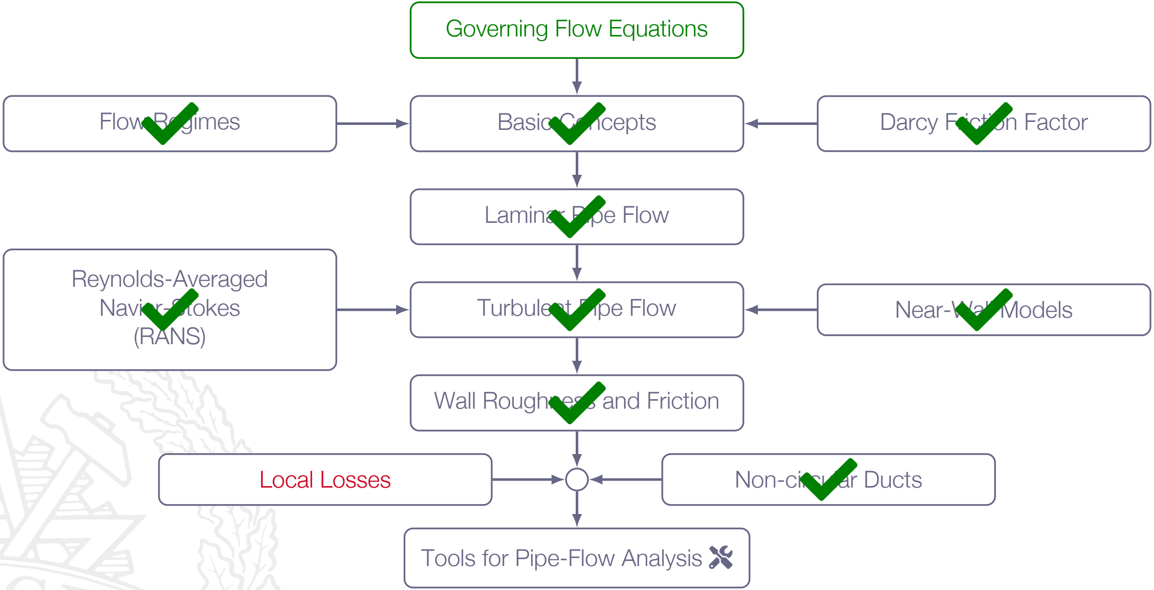


Flow between parallel plates

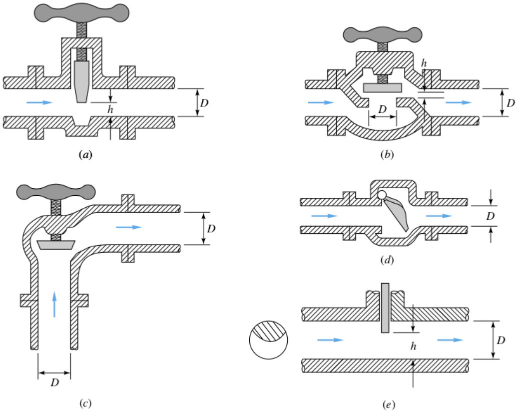
- vertical distance between plates: a
- plate width: b

$$D_h = \frac{4A}{\mathcal{P}} = \left.\frac{4ab}{2a+2b}\right|_{b\to\infty} = \frac{4ab}{2b} = 2a$$

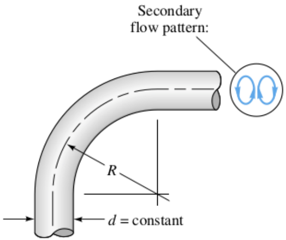
Roadmap - Viscous Flow in Ducts



Local Losses



Local Losses



Swirl generated by:

- Inlets or outlets
- Sudden area changes
- Bends
- Valves
- Gradual expansions or contractions

$$\Delta p_f = K\frac{\rho V^2}{2}$$

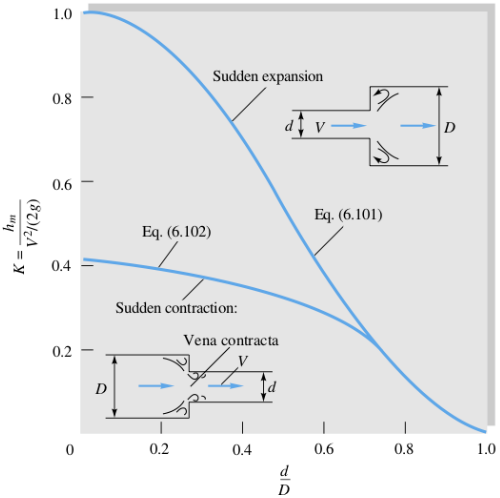
$$\Delta p_{f_{tot}} = \sum_i f_{D_i}\frac{L_i}{D_i}\frac{\rho V_i^2}{2} + \sum_j K_j\frac{\rho V_j^2}{2}$$

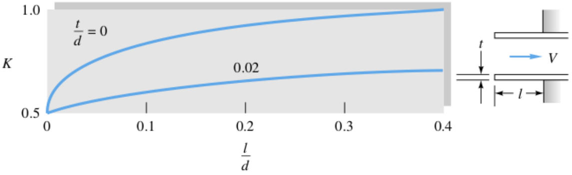
Local Losses

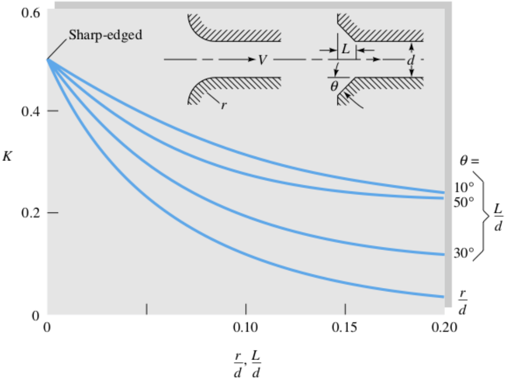
Generated swirl will be damped out by inner friction

Kinetic energy is converted to internal energy, which results in a pressure loss

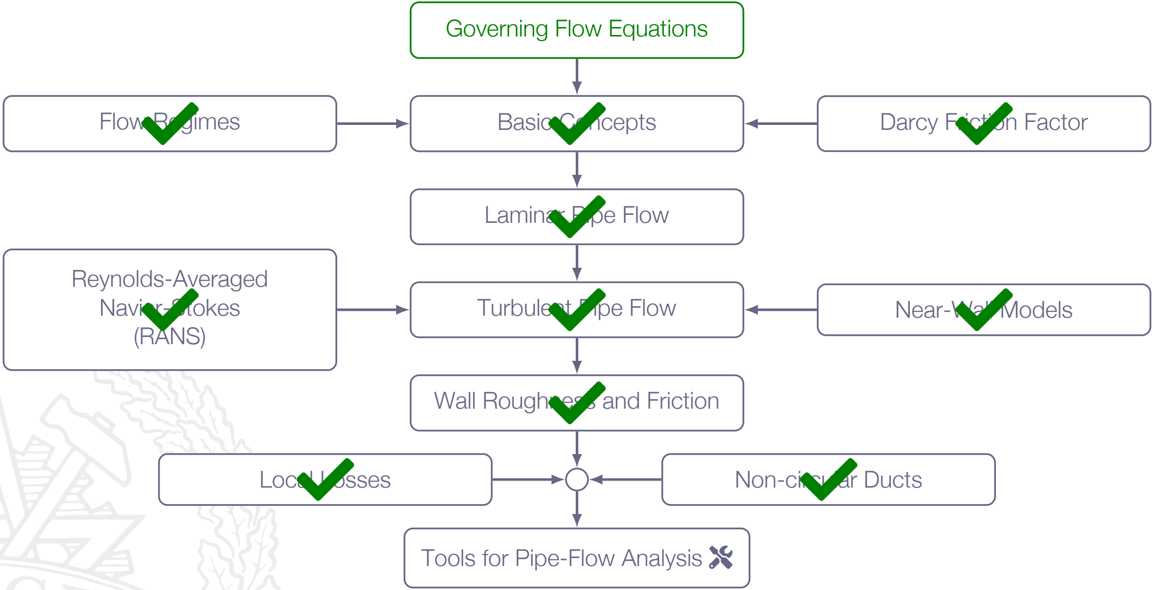
Local Losses







Roadmap - Viscous Flow in Ducts



Pipe Flow Example 1: Find Flow Rate (Rough Pipe)

Given data:
Oil with the density $\rho = 950.0 \, kg/m^3$ and viscosity $\nu = 2.0 \times 10^{-5} \, m^2/s$ flows through a $L = 100 \, m$ long pipe with the diameter $D = 0.3 \, m$. The roughness ratio is $\varepsilon/D = 2.0 \times 10^{-4}$ and the head loss is $h_f = 8.0 \, m$.

Assumptions:
steady-state, fully developed, turbulent, incompressible pipe flow

Task:
Find the average flow velocity (V) and the flow rate (Q)

Pipe Flow Example 1: Find Flow Rate (Rough Pipe)

We are given a measure of the head loss (h_f) for the pipe

The definition of the **Darcy friction factor** gives a relation between head loss (h_f) and the average velocity (V)

$$h_f = f\frac{V^2}{2g}\frac{L}{D}$$

To be able to calculate the average velocity (V), we need the friction factor (f)

Pipe Flow Example 1: Find Flow Rate (Rough Pipe)

The flow in the pipe is assumed to be turbulent and fully developed

For turbulent flows in rough pipes, **Colebrook's formula** gives a relation between friction factor (f) and average flow velocity (V)

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re_D \sqrt{f}} \right)$$

Use an iterative approach to find the friction factor (f) using Colebrook's relation and

$$Re_D = \frac{VD}{\nu}, \text{ where } V = \sqrt{\frac{2h_f gD}{L}}$$

Pipe Flow Example 1: Find Flow Rate (Rough Pipe)

```python
import numpy as np

def GetVelocity(hf,f,D,L):
  return np.sqrt((2.*9.81*hf*D)/(f*L))

def GetReynoldsNumber(D,V,nu):
  return D*V/nu

def Colebrook(f,D,nu,eps,V):
  # Colebrook friction factor
  return -2.0*np.log10(((eps/D)/3.7)+(2.51/(GetReynoldsNumber(D,V,nu)*np.
    sqrt(f))))-1./np.sqrt(f)

def GetFlowRate(V,D):
  return (V*np.pi*D**2)/4.
```

Pipe Flow Example 1: Find Flow Rate (Rough Pipe)

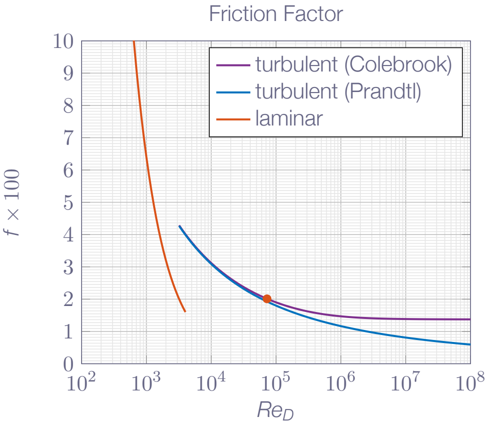
```
nu    = 2.0e-5   # fluid viscosity [m^2/s]
D     = 3.0e-1   # pipe diameter [m]
L     = 1.0e2    # pipe length [m]
hf    = 8.0      # head loss [m]
eps   = 2.0e-4*D # surface roughness [m]
f     = 1.5e-2   # friction factor (inital guess)

# Newton-Raphson solver
f_old = 1.0e3
df     = 1.0e-6
while np.abs(f-f_old)>1.0e-6*f:
  f_old = f
  V    = GetVelocity(hf,f,D,L)
  ff   = Colebrook(f,D,nu,eps,V)
  dff  = (Colebrook(f+df,D,nu,eps,V)-Colebrook(f-df,D,nu,eps,V))/(2.*df)
  f    = f_old-(ff/dff)
```

Pipe Flow Example 1: Find Flow Rate (Rough Pipe)

Result:

Average flow velocity	V	4.84	m/s
Flow rate	Q	0.342	m^3/s
Reynolds number	Re_D	72585	
Friction factor	f	0.0201	



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Pipe Flow Example 2: Find Pipe Diameter (Rough Pipe)

Given data:
Oil with the density $\rho = 950.0\ kg/m^3$ and viscosity $\nu = 2.0 \times 10^{-5}\ m^2/s$ flows through a $L = 100\ m$ long pipe at a flow rate of $Q = 0.342\ m^3/s$. The surface roughness is $\varepsilon = 0.06\ mm$ and the head loss is $h_f = 8.0\ m$.

Assumptions:
steady-state, fully developed, turbulent, incompressible pipe flow

Task:
Find the pipe diameter (D)

Pipe Flow Example 2: Find Pipe Diameter (Rough Pipe)

We are given a measure of the head loss (h_f) for the pipe

The definition of the **Darcy friction factor** gives a relation between head loss (h_f) and the pipe diameter (D)

$$h_f = f\frac{V^2}{2g}\frac{L}{D} = \left\{Q = V\frac{\pi D^2}{4}\right\} = f\frac{8Q^2L}{\pi^2 g D^5}$$

To be able to calculate the pipe diameter (D), we need the friction factor (f)

Pipe Flow Example 2: Find Pipe Diameter (Rough Pipe)

The flow in the pipe is assumed to be turbulent and fully developed

For turbulent flows in rough pipes, **Colebrook's formula** gives a relation between friction factor (f) and pipe diameter (D)

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{Re_D \sqrt{f}} \right)$$

Use an iterative approach to find the friction factor (f) using Colebrook's relation and

$$Re_D = \frac{VD}{\nu}, \text{ where } V = \frac{4Q}{\pi D^2}$$

Pipe Flow Example 2: Find Pipe Diameter (Rough Pipe)

```python
import numpy as np

def GetDiameter(hf,f,L,Q):
  return ((8.*f*Q**2*L)/(9.81*np.pi**2*hf))**(1./5.)

def GetReynoldsNumber(D,V,nu):
  return D*V/nu

def Colebrook(f,D,nu,eps,V):
  # Colebrook friction factor
  return -2.0*np.log10(((eps/D)/3.7)+(2.51/(GetReynoldsNumber(D,V,nu)*np.
    sqrt(f))))-1./np.sqrt(f)

def GetVelocity(Q,D):
  return 4.*Q/(np.pi*D**2)
```

Pipe Flow Example 2: Find Pipe Diameter (Rough Pipe)

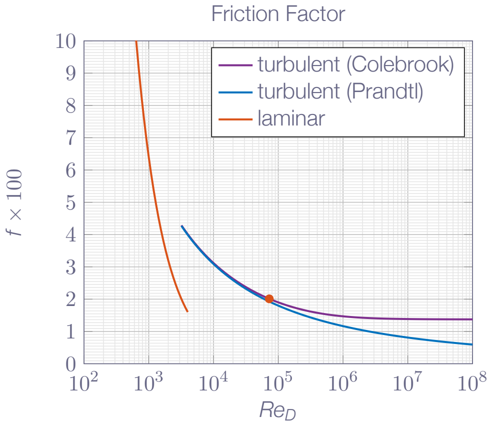
```
nu    = 2.0e-5    # fluid viscosity [m^2/s]
L     = 1.0e2     # pipe length [m]
hf    = 8.0       # head loss [m]
eps   = 6.0e-5    # surface roughness [m]
Q     = 3.42e-1   # flow rate [m^3/s]
f     = 1.5e-2    # friction factor (inital guess)

# Newton-Raphson solver
f_old = 1.0e3
df    = 1.0e-6
while np.abs(f-f_old)>1.0e-6*f:
  f_old = f
  D   = GetDiameter(hf,f,L,Q)
  V   = GetVelocity(Q,D)
  ff  = Colebrook(f,D,nu,eps,V)
  dff = (Colebrook(f+df,D,nu,eps,V)-Colebrook(f-df,D,nu,eps,V))/(2.*df)
  f   = f_old-(ff/dff)
```

Pipe Flow Example 2: Find Pipe Diameter (Rough Pipe)

Result:

Pipe diameter	D	0.299	m
Average flow velocity	V	4.84	m/s
Reynolds number	Re_D	72579	
Friction factor	f	0.0201	

IFLOW



Pipe Flow Example 3: Find Pipe Diameter (Smooth Pipe)

Given data:
A smooth plastic pipe is to be designed to carry $Q = 0.25\ m^3/s$ of water at $20°C$ through a $L = 300\ m$ horizontal pipe with the exit at atmospheric pressure. The pressure drop is approximated to be $\Delta p = 1.7\ MPa$.

Water @ $20°C$: $\rho = 998\ kg/m^3$ and $\mu = 0.001\ kg/(ms)$ $(\nu = 1.002 \times 10^{-6}\ m^2/s)$

Assumptions:
steady-state, fully developed, turbulent, incompressible pipe flow

Task:
Find a suitable pipe diameter (D)

Pipe Flow Example 3: Find Pipe Diameter (Smooth Pipe)

The energy equation on integral form gives us a relation between the pressure drop Δp and the pipe head loss h_f

$$\left(\frac{p}{\rho g}+\frac{\alpha V^2}{2g}+z\right)_1=\left(\frac{p}{\rho g}+\frac{\alpha V^2}{2g}+z\right)_2+h_t-h_p+h_f$$

1. Steady-state, incompressible flow ($Q_1=Q_2=Q$) in a constant-diameter pipe ($D_1=D_2=D$) $\Rightarrow V_1=V_2=V$
2. Fully-developed turbulent pipe flow with constant average velocity $\Rightarrow$ $\alpha_1=\alpha_2=\alpha$
3. No information about elevation change is given so we will assume that $z_1=z_2=z$
4. There are no turbines or pumps in the pipe $\Rightarrow h_t=h_p=0$.

$$\frac{p_1-p_2}{\rho g}=\frac{\Delta p}{\rho g}=h_f$$

Pipe Flow Example 3: Find Pipe Diameter (Smooth Pipe)

Again, we will use the definition of the **Darcy friction factor** (f) to get a relation between the losses and the pipe diameter

$$h_f = f\frac{V^2}{2g}\frac{L}{D} \Rightarrow \left\{h_f = \frac{\Delta p}{\rho g}, Q = V\frac{\pi D^2}{4}\right\} \Rightarrow f = \frac{\pi^2 \Delta p}{8Q^2 L\rho}D^5$$

To be able to calculate the pipe diameter (D), we need the friction factor (f)

Pipe Flow Example 3: Find Pipe Diameter (Smooth Pipe)

The flow in the pipe is assumed to be turbulent and fully developed

For turbulent flows in smooth pipes, **Prandtl's formula** gives a relation between friction factor (f) and pipe diameter (D)

$$\frac{1}{\sqrt{f}} = 2.0 \log \left(Re_D \sqrt{f} \right) - 0.8$$

Use an iterative approach to find the friction factor (f) using Prandtl's relation and

$$Re_D = \frac{VD}{\nu}, \text{ where } V = \frac{4Q}{\pi D^2}$$

Pipe Flow Example 3: Find Pipe Diameter (Smooth Pipe)

```python
import numpy as np

def GetDiameter(Dp,rho,f,L,Q):
  return ((8.*f*Q**2*L*rho)/(np.pi**2*Dp))**(1./5.)

def GetReynoldsNumber(D,V,nu):
  return D*V/nu

def Prandtl(f,D,nu,V):
  # Prandtl friction factor
  return 2.0*np.log10(GetReynoldsNumber(D,V,nu)*np.sqrt(f))-0.8-(1./np.
    sqrt(f));

def GetVelocity(Q,D):
  return 4.*Q/(np.pi*D**2)
```

Pipe Flow Example 3: Find Pipe Diameter (Smooth Pipe)

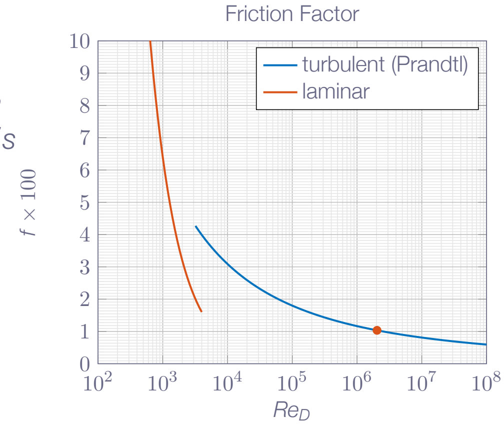
```
rho   = 998.0     # fluid density [kg/m^3]
mu    = 1.0e-3    # fluid viscosity [kg/ms]
nu    = mu/rho    # fluid viscosity [m^2/s]
L     = 3.0e2     # pipe length [m]
Dp    = 1.7e6     # pressure drop [Pa]
Q     = 2.5e-1    # flow rate [m^3/s]
f     = 1.5e-2    # friction factor (inital guess)

# Newton-Raphson solver
f_old = 1.0e3
df    = 1.0e-6
while np.abs(f-f_old)>1.0e-6*f:
  f_old = f
  D   = GetDiameter(Dp,rho,f,L,Q)
  V   = GetVelocity(Q,D)
  ff  = Prandtl(f,D,nu,V)
  dff = (Prandtl(f+df,D,nu,V)-Prandtl(f-df,D,nu,V))/(2.*df)
  f   = f_old-(ff/dff)
```

Pipe Flow Example 3: Find Pipe Diameter (Smooth Pipe)

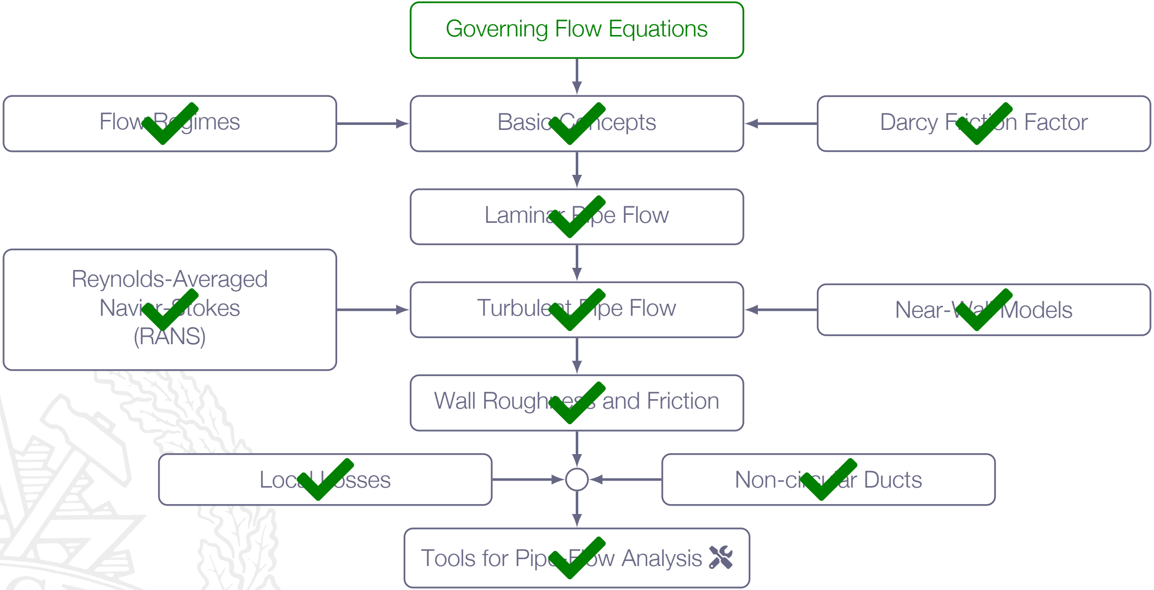
Result:

Pipe diameter	D	0.156	m
Average flow velocity	V	13.1	m/s
Reynolds number	Re_D	2036821	
Friction factor	f	0.01034	

IFLOW



Roadmap - Viscous Flow in Ducts



On-Demand Hyperloop-Style Water Delivery