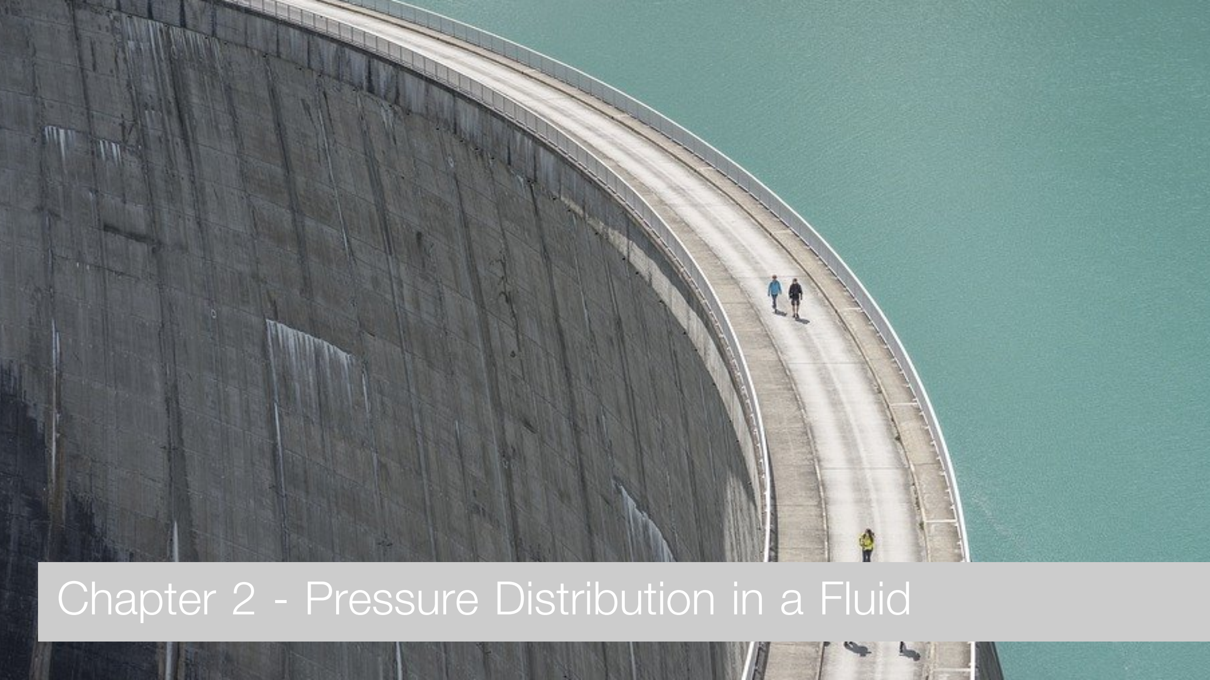
Fluid Mechanics - MTF053

Lecture 3

Niklas Andersson

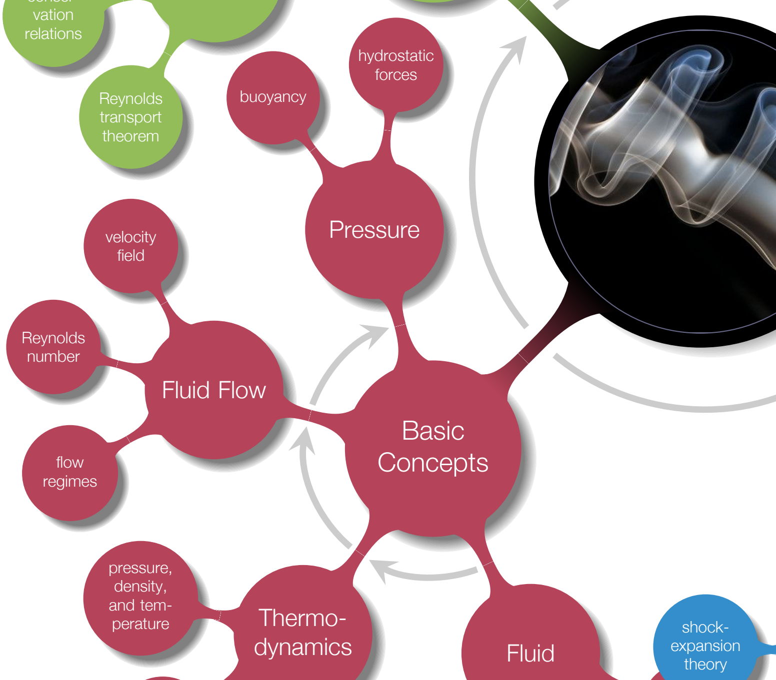
Chalmers University of Technology
Department of Mechanics and Maritime Sciences
Division of Fluid Mechanics
Gothenburg, Sweden

`niklas.andersson@chalmers.se`



Chapter 2 - Pressure Distribution in a Fluid

Overview


vation
relations
Reynolds transport theorem
buoyancy
hydrostatic forces
Pressure
velocity field
Reynolds number
Fluid Flow
flow regimes
Basic Concepts
pressure, density, and tem-perature
Thermo-dynamics
Fluid
shock-expansion theory

Learning Outcomes

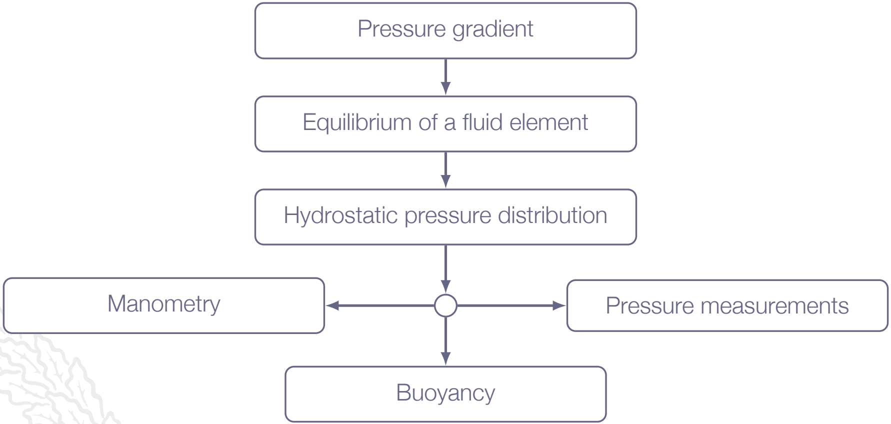
9 **Explain** how to do a force balance for fluid element (forces and pressure gradients)

10 **Understand and explain** buoyancy and cavitation

11 **Solve** problems involving hydrostatic pressure and buoyancy

we will have a look at the pressure distribution in a fluid at rest, i.e. no flow yet...

Roadmap - Pressure Distribution in a Fluid



Motivation

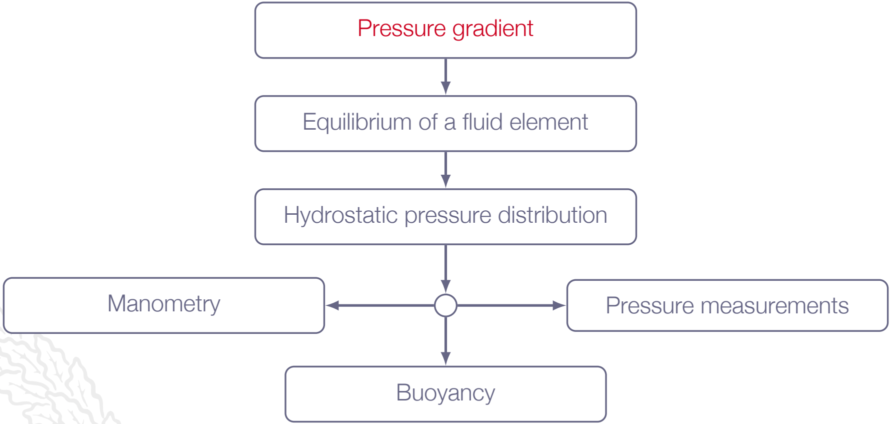
- Many problems does not include fluid motion
  - pressure distribution in a static fluid
  - pressure on solid surfaces due to presence of static fluid
  - floating and submerged bodies

Motivation

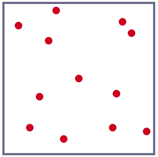
Examples:

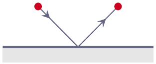
- pressure distribution in the atmosphere and in oceans
- design of pressure measurement devices
- buoyancy on a submerged body
- behavior of floating bodies

Roadmap - Pressure Distribution in a Fluid



Pressure

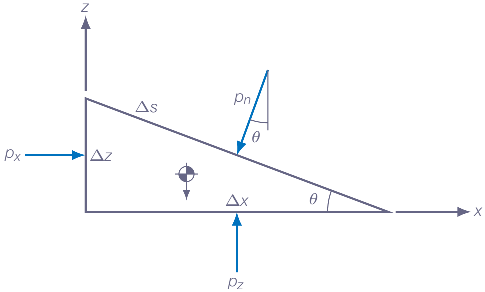




- Pressure is a thermodynamic property
- Pressure is not a force and has no direction
- Forces arise when the molecules of the fluid interacts with the surface of an immersed body
- A force in the surface-normal direction is generated due to the collision of fluid molecules and the surface

Pressure Variation in a Fluid at Rest

- Fluid at rest - no shear (by definition)
- Pressures p_x, p_z, and p_n may be different
- Small element $\Rightarrow$ constant pressure on each face

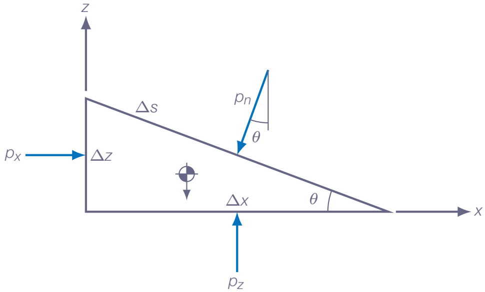


Pressure Variation in a Fluid at Rest

$$\sum F_x = 0 = p_x b \Delta z - p_n b \Delta s \sin\theta$$

$$\sum F_z = 0 = p_z b \Delta x - p_n b \Delta s \cos\theta - \frac{1}{2}\rho g b \Delta x \Delta z$$

$$\begin{cases} \Delta z = \Delta s \sin\theta \\ \Delta x = \Delta s \cos\theta \end{cases}$$

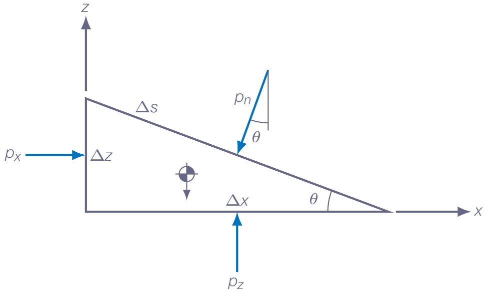


Pressure Variation in a Fluid at Rest

$$\sum F_x = 0 = p_x b\Delta z - p_n b\Delta z$$

$$\sum F_z = 0 = p_z b\Delta x - p_n b\Delta x - \frac{1}{2}\rho g b\Delta x\Delta z$$

$$\begin{cases} p_x = p_n \\ p_z = p_n + \frac{1}{2}\rho g\Delta z \end{cases}$$

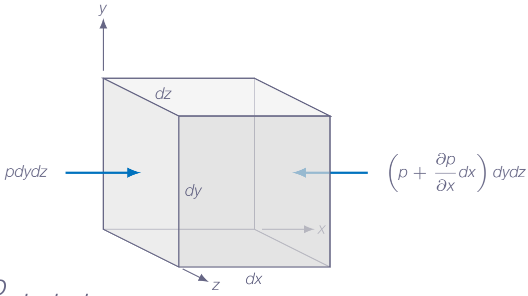


Pressure Variation in a Fluid at Rest

- Since θ is arbitrary, the result is general
- There is no pressure change in the horizontal direction
- The pressure change in the vertical direction is proportional to the depth

"The pressure in a static fluid is a point property, independent of orientation"

Pressure Forces on a Fluid Element



$$p = p(x, y, z, t)$$

$$dF_x = pdydz - \left(p + \frac{\partial p}{\partial x} dx\right) dydz = -\frac{\partial p}{\partial x} dxdydz$$

$$dF_p = -\left[\frac{\partial p}{\partial x}\mathbf{e}_x + \frac{\partial p}{\partial y}\mathbf{e}_y + \frac{\partial p}{\partial z}\mathbf{e}_z\right] dxdydz$$

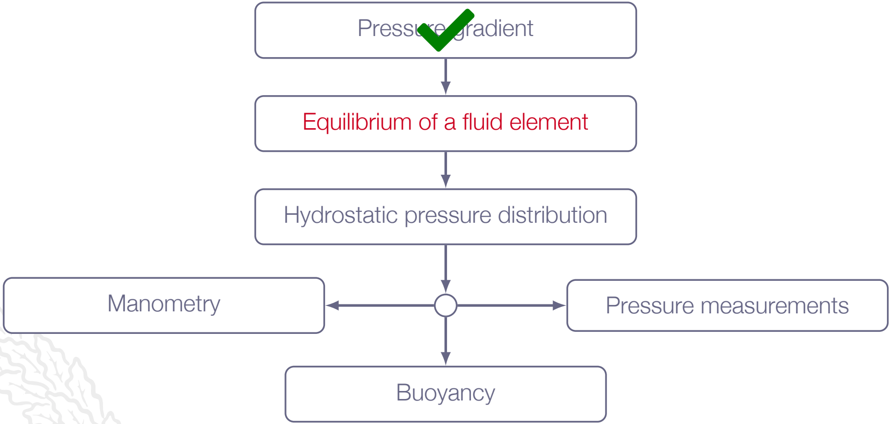
$$\mathbf{f}_p = -\nabla p$$

$\mathbf{f}$ *is the net force per unit volume*

Pressure Forces on a Fluid Element

"it is not the pressure but the pressure gradient causing a net force which must be balanced by gravity or acceleration"

Roadmap - Pressure Distribution in a Fluid



Equilibrium of a Fluid Element

- Force balance for a small element
  - pressure gradients gives surface forces
  - body forces (electromagnetic or gravitational potentials)
  - surface forces due to viscous stresses

Newton's second law:

$$\sum \mathbf{f} = \mathbf{f}_p + \mathbf{f}_g + \mathbf{f}_v = -\nabla p + \rho \mathbf{g} + \mathbf{f}_v = \rho \mathbf{a}$$

Equilibrium of a Fluid Element

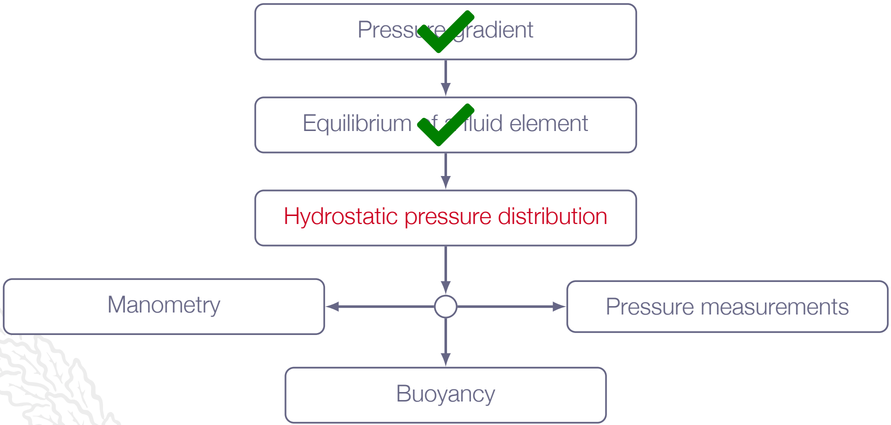
- Hydrostatic problems:
  - no viscous forces
  - no acceleration

Newton's second law reduces to:

$$\nabla p = \rho \mathbf{g}$$

(the general form of Newton's second law will be studied later)

Roadmap - Pressure Distribution in a Fluid



Hydrostatic Pressure in Liquids

$$\nabla p = \rho \mathbf{g}$$

- ∇p is perpendicular everywhere to surfaces of constant p
- The normal of constant-pressure surfaces will be aligned with $\mathbf{g}$

Hydrostatic Pressure in Liquids

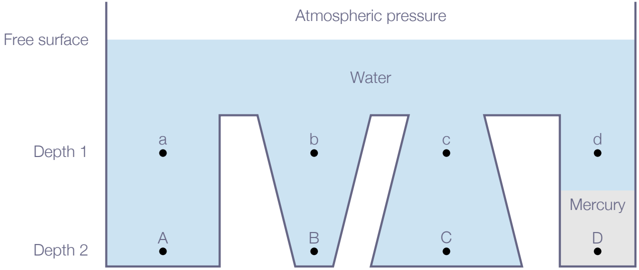
$$\mathbf{g} = -g\mathbf{e}_z$$

$$\frac{dp}{dz} = -\rho g$$

$$p_2 - p_1 = -\int_1^2 \rho g dz$$

Hydrostatic Pressure in Liquids

for liquids, we assume constant density $\Rightarrow p_2 - p_1 = -\int_1^2 \rho g dz = -\rho g(z_2 - z_1)$



$$p_a = p_b = p_c = p_d$$
$$p_A = p_B = p_C \neq p_D$$

Hydrostatic Pressure in Liquids

- Is the incompressible assumption for liquids a good assumption?

  the density is 4.6 percent higher at the deepest part of the ocean - so yes!

$$\boxed{p_2 - p_1 = -\int_1^2 \rho g dz = -\rho g(z_2 - z_1)}$$

Hydrostatic Pressure in Liquids

$$p_2 - p_1 = -\int_1^2 \rho g dz = -\rho g(z_2 - z_1)$$

Why is mercury used for pressure measurements?

Hydrostatic Pressure in Gases

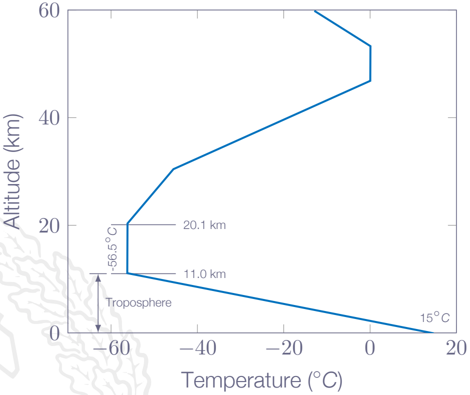
$$\frac{dp}{dz} = -\rho g = -\frac{p}{RT}g$$

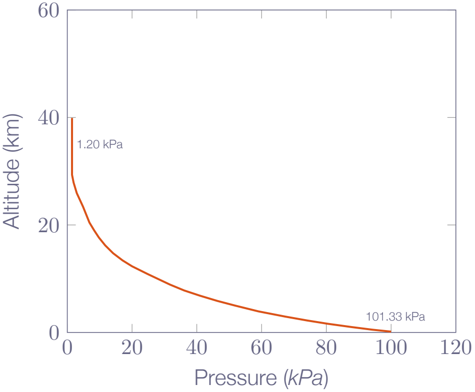
both pressure and temperature varies with altitude

$$\int_1^2 \frac{dp}{p} = \ln\frac{p_2}{p_1} = -\frac{g}{R}\int_1^2 \frac{dz}{T}$$

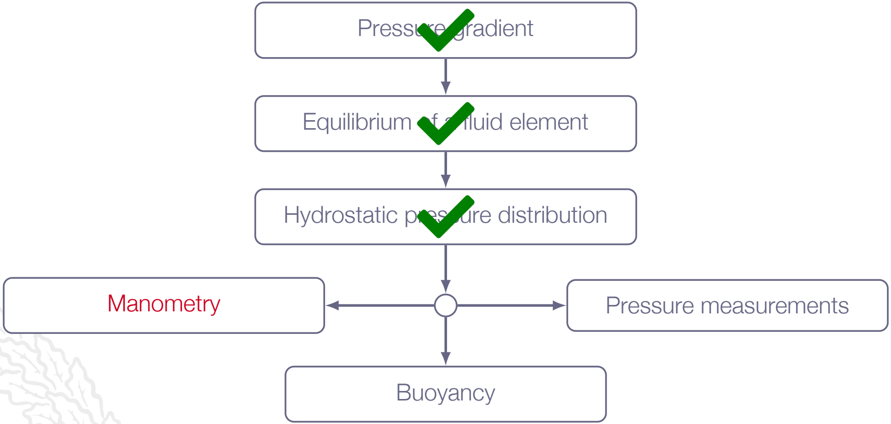
Temperature variation $T(z)$ needed

Hydrostatic Pressure in Gases

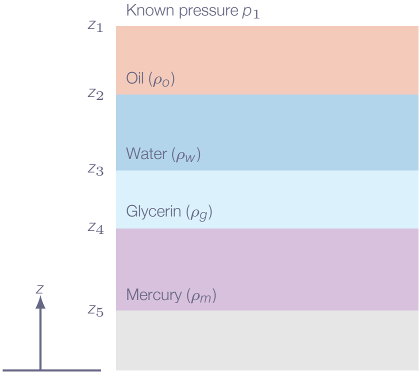




Roadmap - Pressure Distribution in a Fluid



Manometry



$$p_2 - p_1 = -\rho_o g(z_2 - z_1)$$

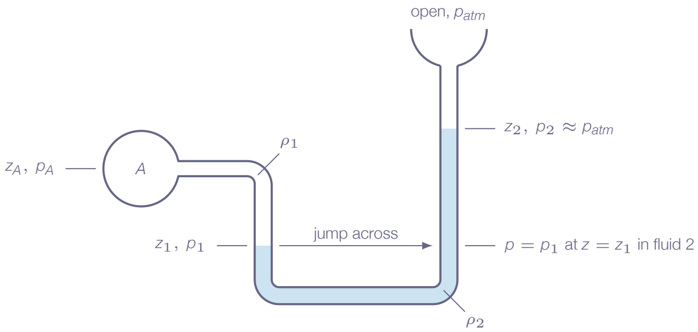
$$p_3 - p_2 = -\rho_w g(z_3 - z_2)$$

$$p_4 - p_3 = -\rho_g g(z_4 - z_3)$$

$$p_5 - p_4 = -\rho_m g(z_5 - z_4)$$

$$p_5 - p_1 = -\rho_m g(z_5 - z_4) - \rho_g g(z_4 - z_3) - \rho_w g(z_3 - z_2) - \rho_o g(z_2 - z_1)$$

Manometry

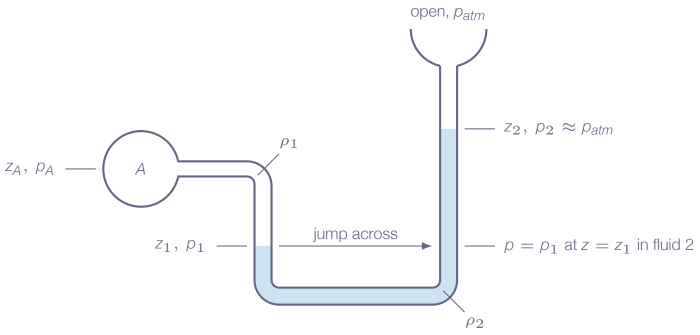


Pascal's law:

"Any two points at the same elevation in a continuous mass of the same static fluid will be at the same pressure"

Manometry

$$p_A + \rho_1 g(z_A - z_1) - \rho_2 g(z_2 - z_1) = p_2 \approx p_{atm}$$



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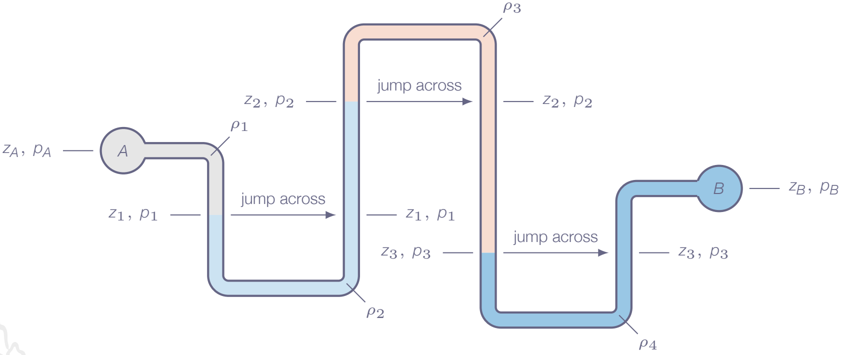
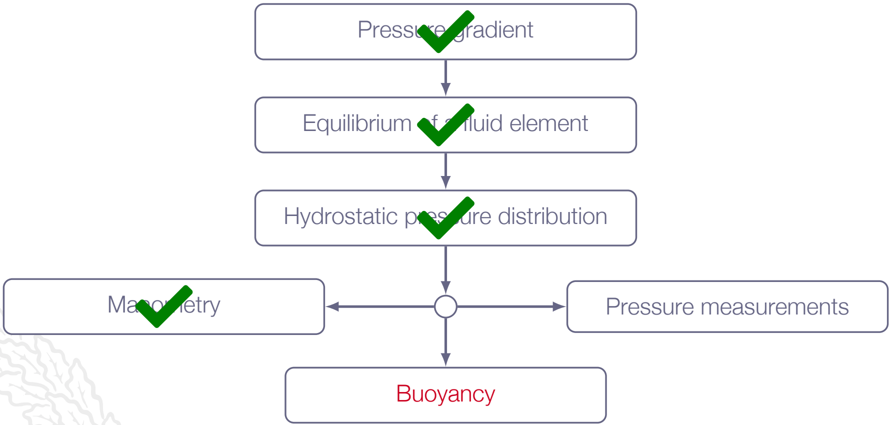
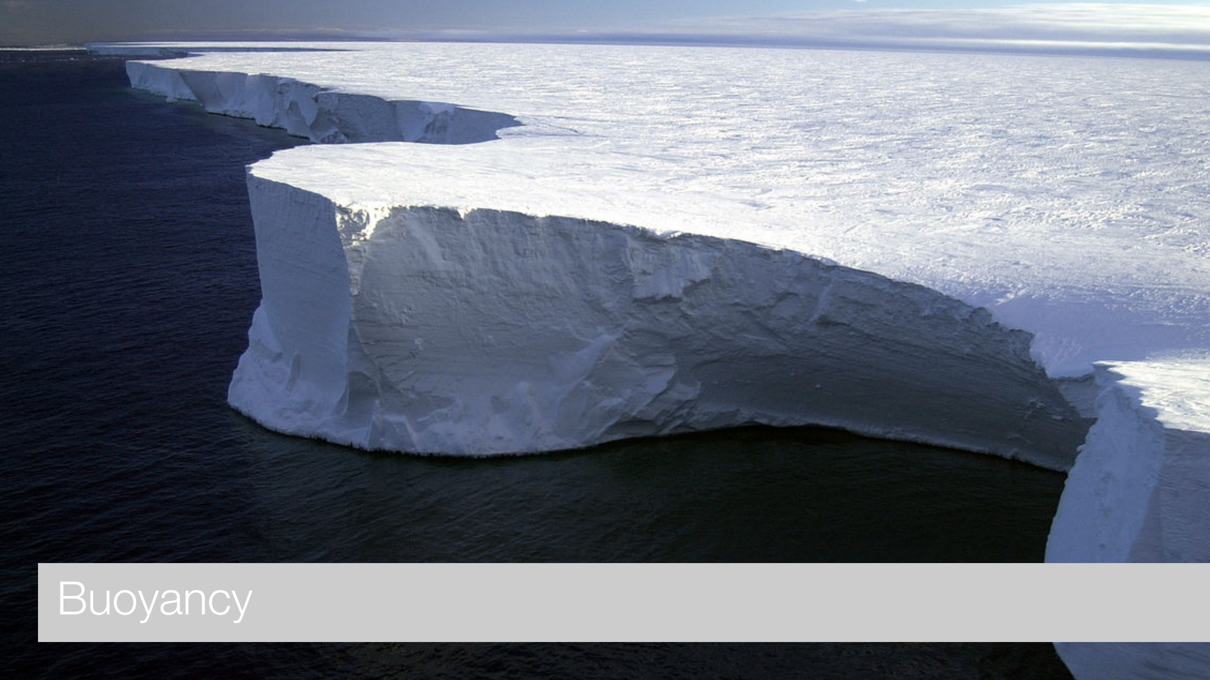
Manometry



ρ_3
jump across
$z_2,\ p_2$
$z_2,\ p_2$
ρ_1
$z_A,\ p_A$
A
B
$z_B,\ p_B$
jump across
$z_1,\ p_1$
$z_1,\ p_1$
jump across
$z_3,\ p_3$
$z_3,\ p_3$
ρ_2
ρ_4

Roadmap - Pressure Distribution in a Fluid





Buoyancy

Buoyancy

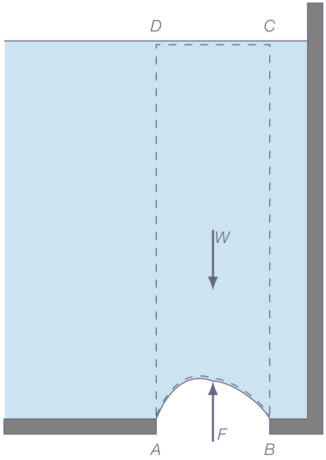
Archimedes:

- A body immersed in a fluid experiences a vertical buoyant force equal to the weight of the fluid it displaces

- A floating body displaces its own weight in the fluid in which it floats

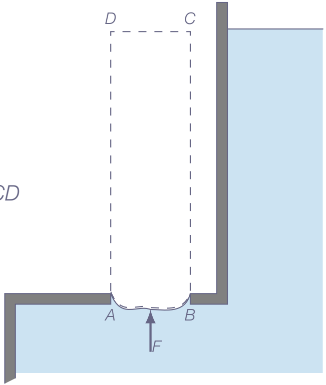
Buoyancy

$$F = \rho g \mathcal{V}_{ABCD}$$

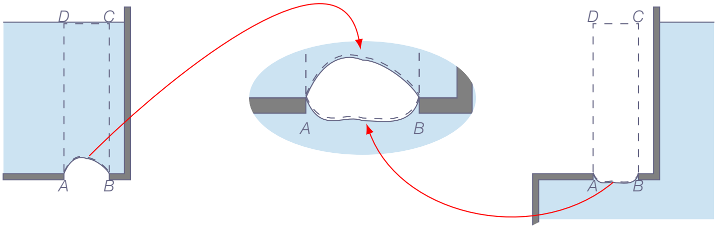


Buoyancy

$$F = \rho g \mathcal{V}_{ABCD}$$



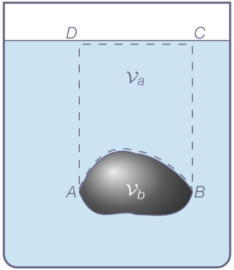
Buoyancy



Buoyancy

$$F_{up} = \rho g(\mathcal{V}_a + \mathcal{V}_b)$$

$$F_{down} = \rho g \mathcal{V}_a$$



Buoyancy

In general

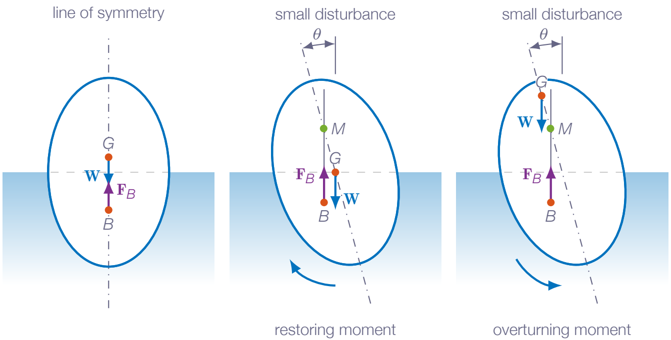
$$\mathbf{F}_B = \sum \rho_i g(displacement\ volume)_i$$

Floating bodies

$$\mathbf{F}_B = body\ weight$$

Buoyancy - Stability

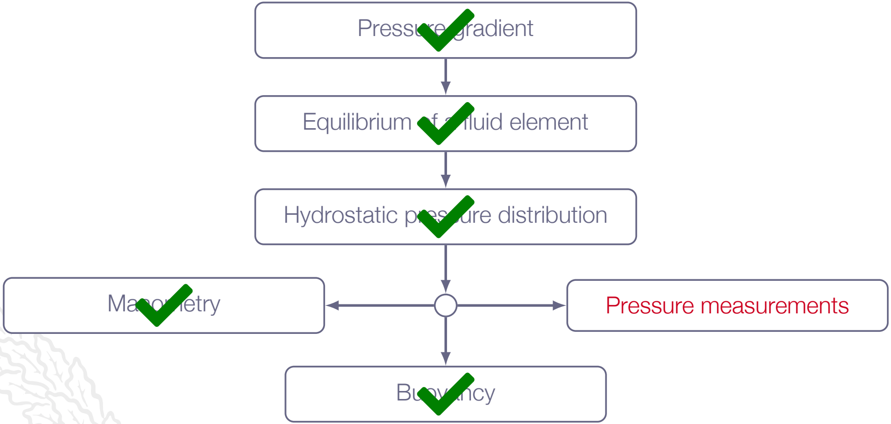
- Center of gravity G
- Center of buoyancy B
- Symmetry line
- Metacenter M



Note! the center of buoyancy (B) is, in this case, the centroid of the displaced volume of liquid

Buoyancy - Stability

Roadmap - Pressure Distribution in a Fluid

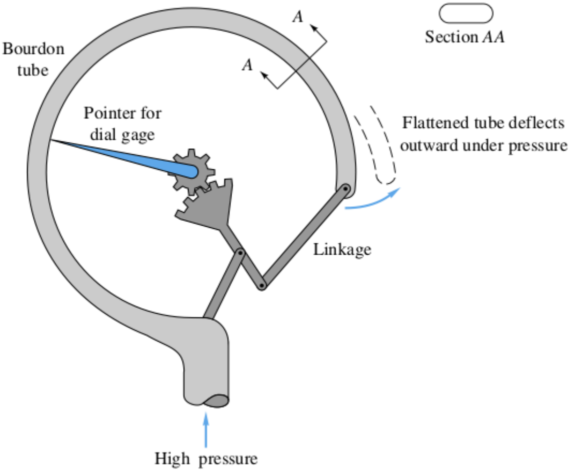


Pressure measurement

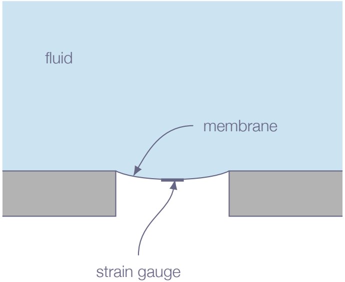
Pressure is a derived property

The force per unit area related to fluid molecular bombardment of a surface

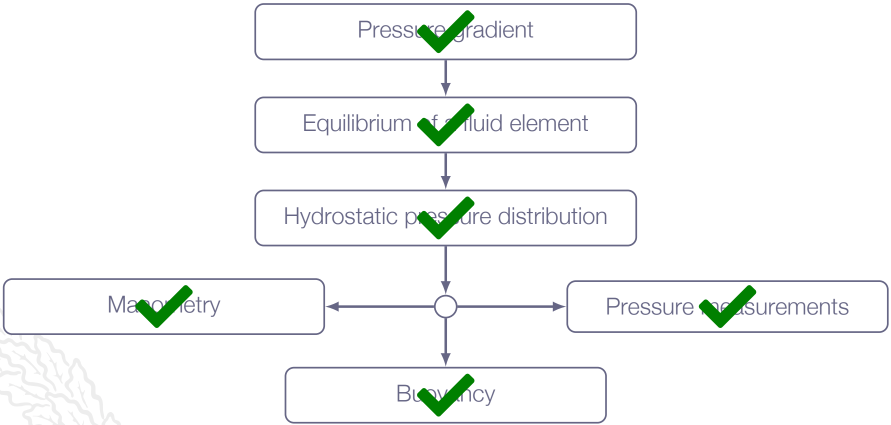
Pressure measurement



Pressure measurement



Roadmap - Pressure Distribution in a Fluid



Manometer Example

$$p_1 + \sum_{down} \rho_i g \Delta_i - \sum_{up} \rho_i g \Delta_i = p_2$$

$$p_1 + (\Delta_2 + \Delta_1)\rho_A g - \Delta_1 \rho_B g - (\Delta_2 + \Delta_3)\rho_A g = p_2$$

$$p_1 + (\Delta_2 + \Delta_1)\rho_A g - \Delta_1 \rho_B g - (\Delta_2 + z_2 - z_1)\rho_A g = p_2$$

$$\left(\frac{p_1}{\rho_A g} + z_1\right) - \left(\frac{p_2}{\rho_A g} + z_2\right) = \Delta_1\left(\frac{\rho_B}{\rho_A} - 1\right)$$