Fluid Mechanics - MTF053

Lecture 20

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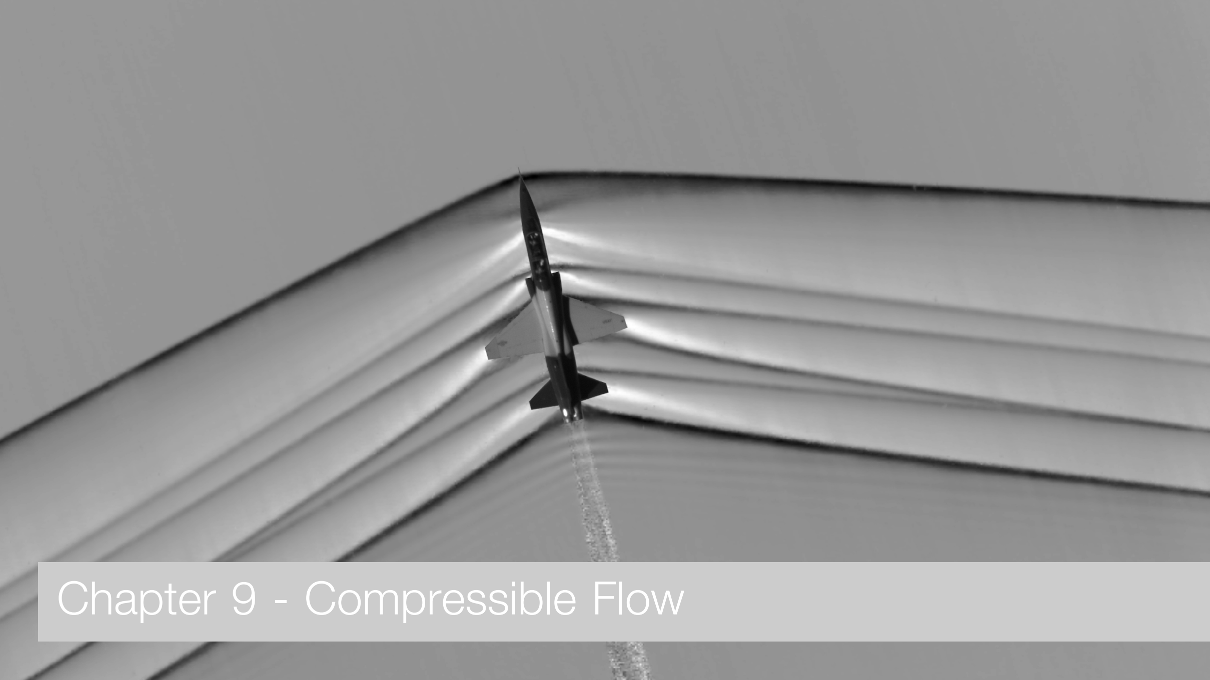
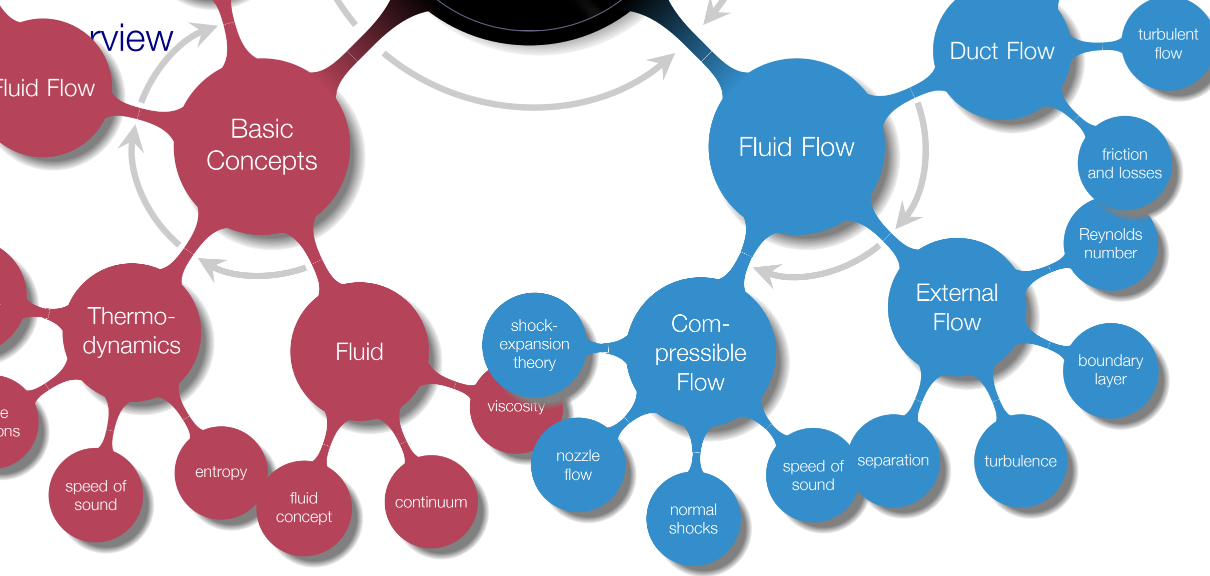


Chapter 9 - Compressible Flow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Fluid Flow
Basic Concepts
Thermo-dynamics
Fluid
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ons
speed of sound
entropy
fluid concept
continuum
viscosity
Fluid Flow
Duct Flow
turbulent flow
friction and losses
Reynolds number
External Flow
boundary layer
separation
turbulence
Com-pressible Flow
shock-expansion theory
nozzle flow
normal shocks
speed of sound

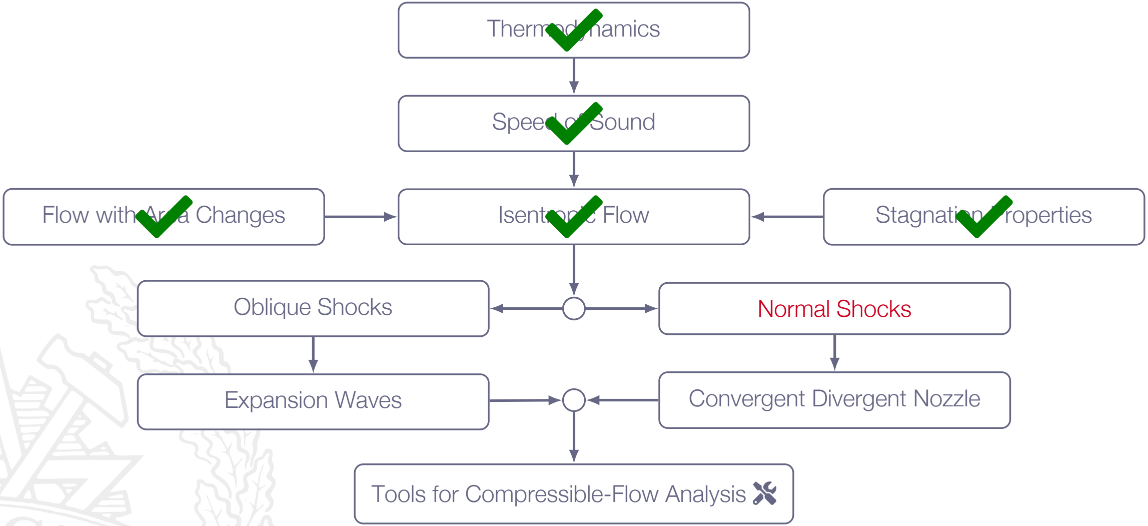
Learning Outcomes

4 Be **able to categorize** a flow and **have knowledge about** how to select applicable methods for the analysis of a specific flow based on category

37 **Understand and explain** basic concepts of compressible flows (the gas law, speed of sound, Mach number, isentropic flow with changing area, normal shocks, oblique shocks, Prandtl-Meyer expansion)

Let's go supersonic ...

Roadmap - Compressible Flow



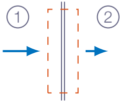
Shock Waves

"Shock waves are nearly discontinuous changes in a supersonic flow"

Reasons for the appearance of shocks in a flow can be for example:

1. higher downstream pressure
2. sudden changes in flow direction
3. blockage by a downstream body
4. explosion

Normal Shocks



Continuity:

$$\rho_1 u_1 = \rho_2 u_2$$

Momentum:

$$p_1 - p_2 = \rho_2 u_2^2 - \rho_1 u_1^2$$

Energy:

$$h_1 + \frac{1}{2}u_1^2 = h_2 + \frac{1}{2}u_2^2 = h_o$$

The Rankine-Hugoniot relation:

$$h_2 - h_1 = \frac{1}{2}(p_2 - p_1)\left(\frac{1}{\rho_2} + \frac{1}{\rho_1}\right)$$

Normal Shocks

$$h_2 - h_1 = \frac{1}{2}(p_2 - p_1)\left(\frac{1}{\rho_2} + \frac{1}{\rho_1}\right)$$

Note! The **Rankine-Hugoniot** relation **only** includes **thermodynamic properties** (no velocities) and gives a relation between the flow state upstream of the shock and the flow downstream of the shock

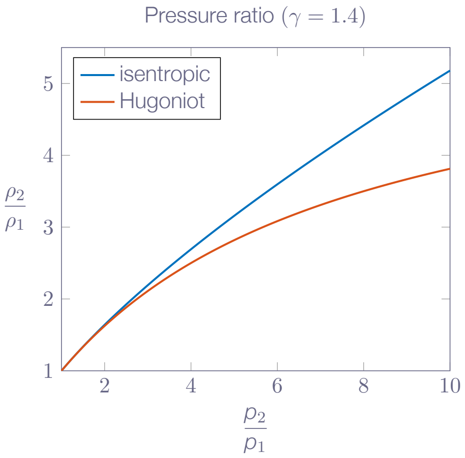
Normal Shocks

The Rankine-Hugoniot relation

$$\frac{\rho_2}{\rho_1} = \frac{1 + \left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{p_2}{p_1}\right)}{\left(\frac{\gamma+1}{\gamma-1}\right) + \left(\frac{p_2}{p_1}\right)}$$

The isentropic relation

$$\frac{\rho_2}{\rho_1} = \left(\frac{p_2}{p_1}\right)^{1/\gamma}$$



Normal Shocks

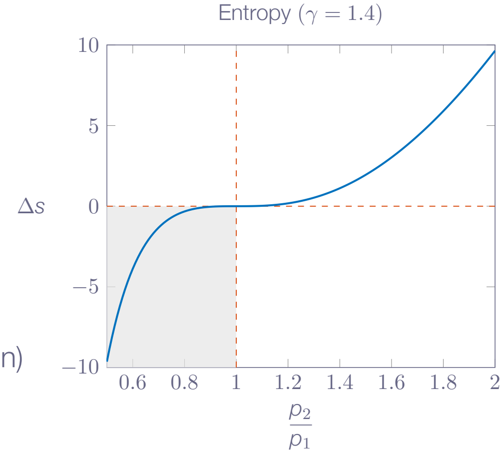
The second law of thermodynamics

$$s_2 - s_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

can be rewritten as

$$s_2 - s_1 = C_v \ln \left[\frac{p_2}{p_1} \left(\frac{\rho_1}{\rho_2} \right)^{\gamma} \right]$$

(ρ_2/ρ_1 from the Rankine-Hugoniot relation)



Note! a reduction of entropy is a violation of the second law of thermodynamics

Normal Shocks

For a perfect gas, it is possible to obtain relations for normal shocks that only include upstream variables

Momentum equation: $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$

$$p_2 - p_1 = \rho_1 u_1^2 - \rho_2 u_2^2 = \{\rho_1 u_1 = \rho_2 u_2\} = \rho_1 u_1 (u_1 - u_2) = \rho_1 u_1^2 \left(1 - \frac{u_2}{u_1}\right)$$

divide by p_1

$$\frac{p_2}{p_1} = 1 + \frac{\rho_1 u_1^2}{p_1}\left(1 - \frac{u_2}{u_1}\right)$$

$$u_1^2 = M_1^2 a_1^2 = M_1^2 \gamma R T_1 = \gamma M_1^2 \frac{p_1}{\rho_1} \Rightarrow \frac{p_2}{p_1} = 1 + \gamma M_1^2 \left(1 - \frac{u_2}{u_1}\right)$$

Normal Shocks

$$\frac{p_2}{p_1} = 1 + \gamma M_1^2 \left(1 - \frac{u_2}{u_1}\right)$$

Using the energy equation its possible obtain a relation for $\frac{u_2}{u_1}$ (the derivation is quite lengthy though)

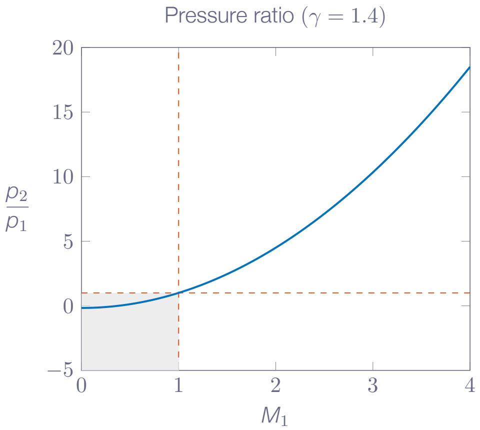
$$\frac{u_2}{u_1} = \frac{2 + (\gamma - 1)M_1^2}{(\gamma + 1)M_1^2}$$

and thus

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}\left(M_1^2 - 1\right)$$

Normal Shocks

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}\left(M_1^2 - 1\right)$$



Note! from before we know that p_2/p_1 must be greater than 1.0, which means that M_1 must be greater than 1.0

Normal Shocks

Momentum equation: $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$

$$M = \frac{u}{a} \Rightarrow p_1 + \rho_1 M_1^2 a_1^2 = p_2 + \rho_2 M_2^2 a_2^2$$

$$a = \sqrt{\gamma R T} = \sqrt{\frac{\gamma p}{\rho}} \Rightarrow p_1 + \rho_1 M_1^2 \frac{\gamma p_1}{\rho_1} = p_2 + \rho_2 M_2^2 \frac{\gamma p_2}{\rho_2}$$

$$p_1 \left(1 + \gamma M_1^2\right) = p_2 \left(1 + \gamma M_2^2\right)$$

$$\frac{p_2}{p_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}$$

Normal Shocks

Two ways to calculate the pressure ratio over the shock

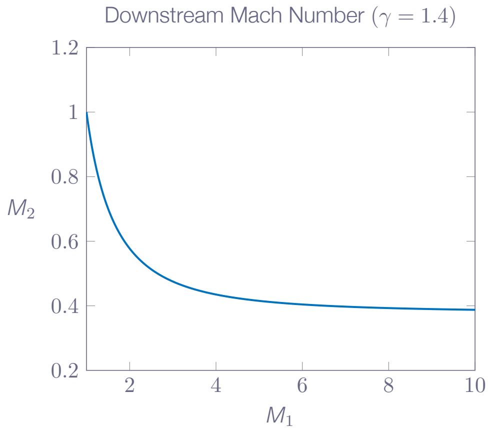
$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right) \qquad \frac{p_2}{p_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2}$$

Setting the relations equal gives a relation for the downstream Mach number

$$M_2^2 = \frac{(\gamma-1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma-1)}$$

Normal Shocks

$$M_2^2 = \frac{(\gamma - 1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma - 1)}$$

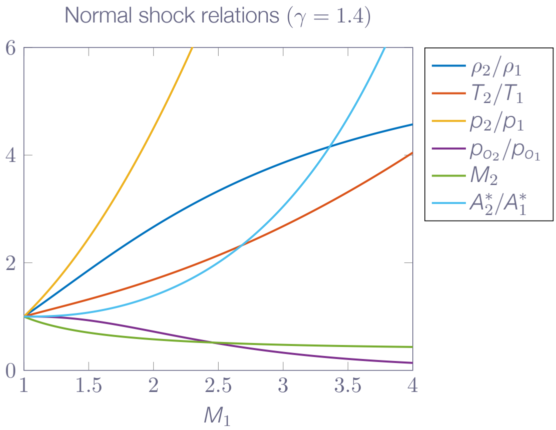


Note! for $\gamma > 1$ and $M_1 > 1$, the downstream Mach number **must** be less than 1.0, i.e we will **always** have subsonic flow downstream of a normal shock

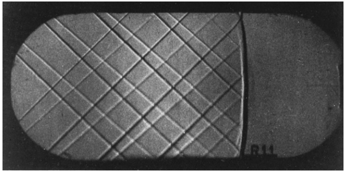
Normal Shocks - Summary

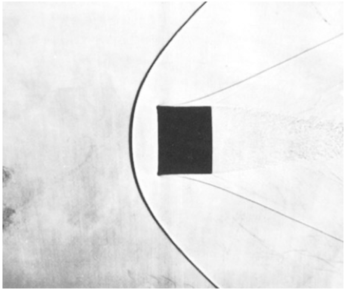
1. **Supersonic** flow **upstream** of normal shock
2. **Subsonic** flow **downstream** of normal shock
3. **Entropy increases** over the shock and consequently **total pressure decreases**
4. Sonic throat area increases
5. Very weak shock waves are nearly isentropic

Normal Shocks - Trends

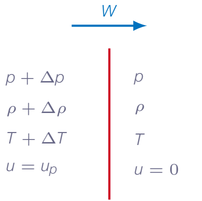


Normal Shocks - Examples





Moving Normal Shocks

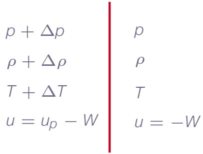


frame of reference:
fixed to the fluid

Change **frame of reference**:

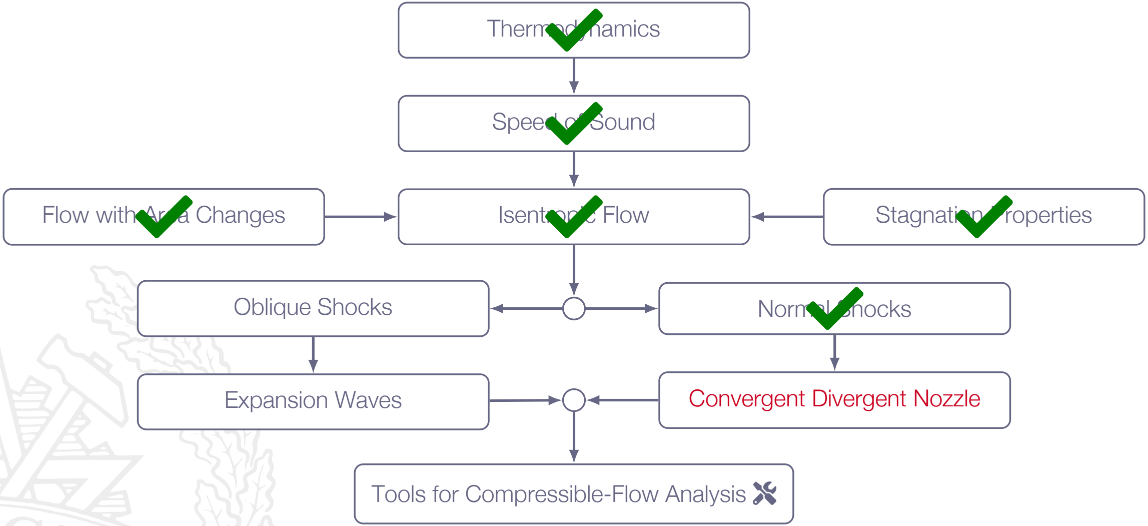
1. coordinate system moving with the shock
2. thermodynamic properties does not change

$$h_2 - h_1 = \frac{1}{2}(p_2 - p_1)\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$$



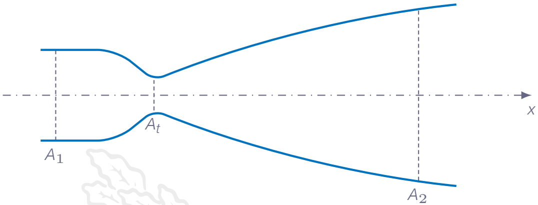
frame of reference:
following the shock

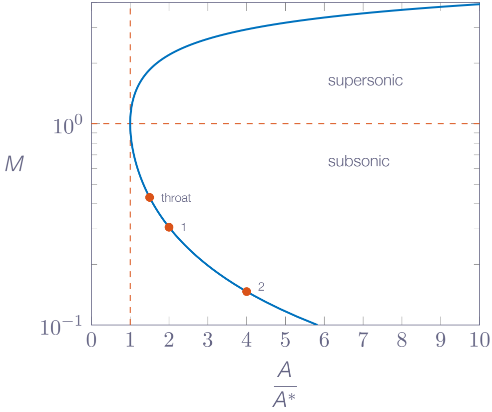
Roadmap - Compressible Flow



The Area-Mach-Number Relation

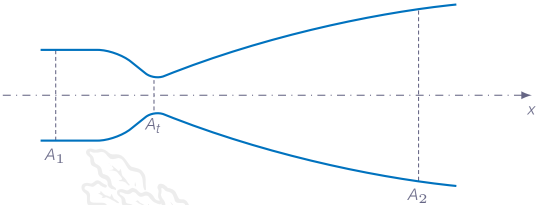
Sub-critical (**non-choked**) nozzle flow

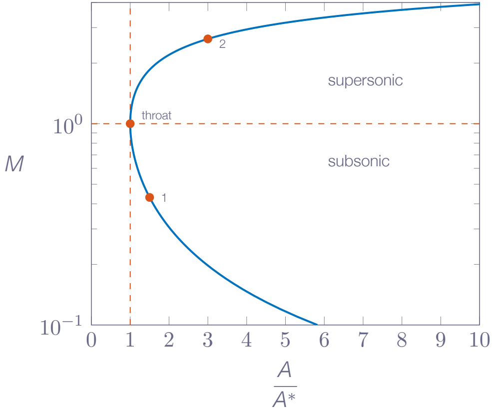




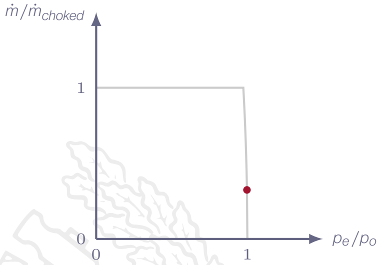
The Area-Mach-Number Relation

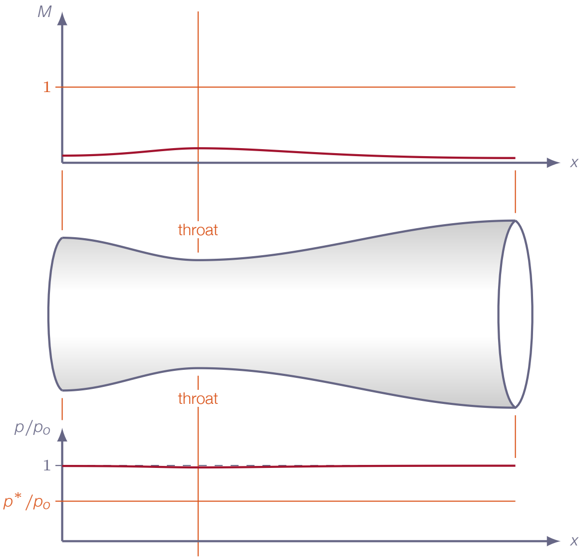
Critical (**choked**) nozzle flow





Convergent-Divergent Nozzle





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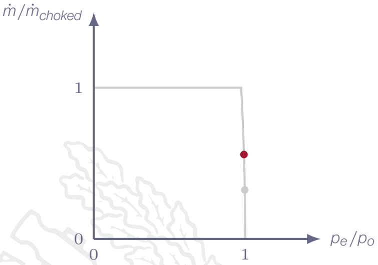

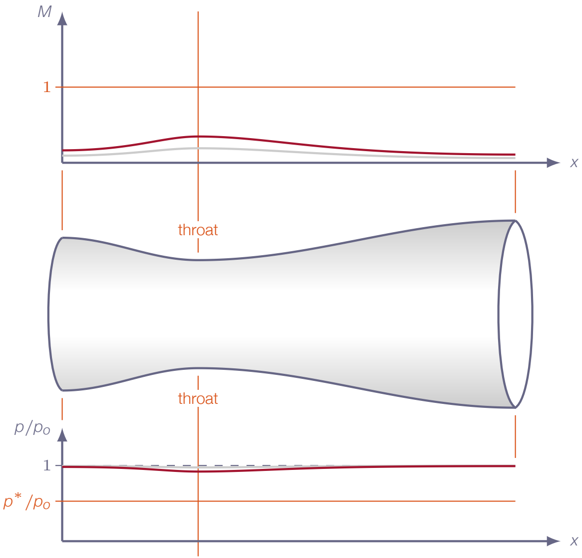

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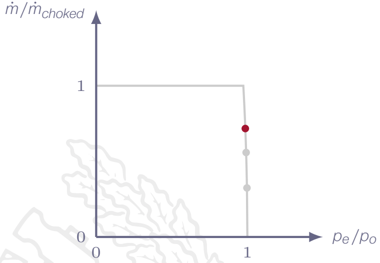

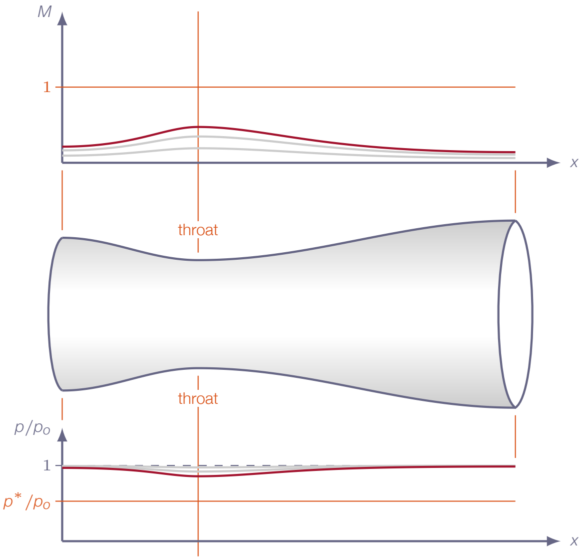

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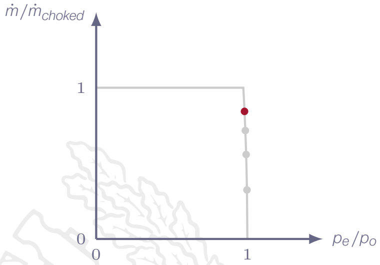

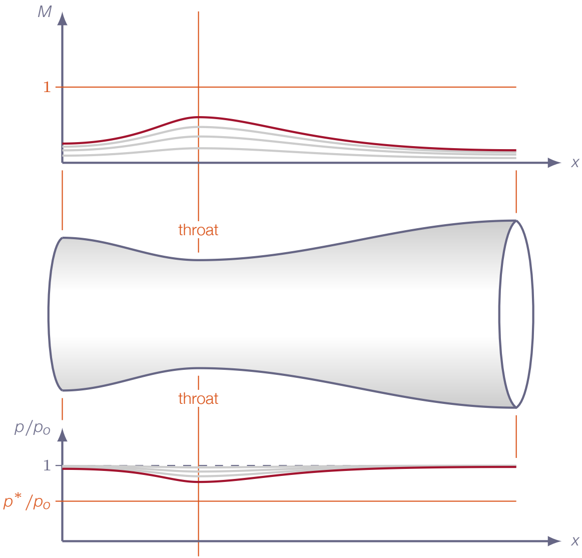

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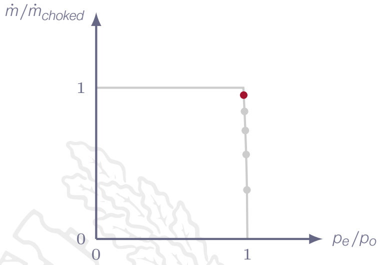

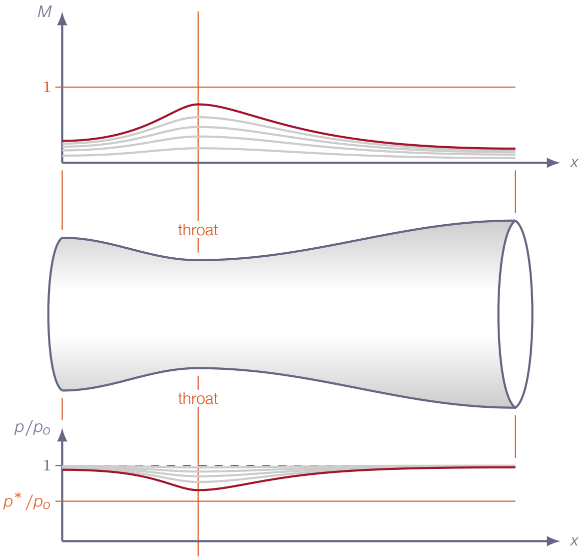

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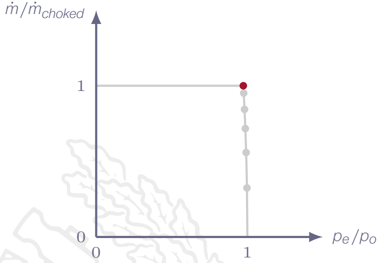

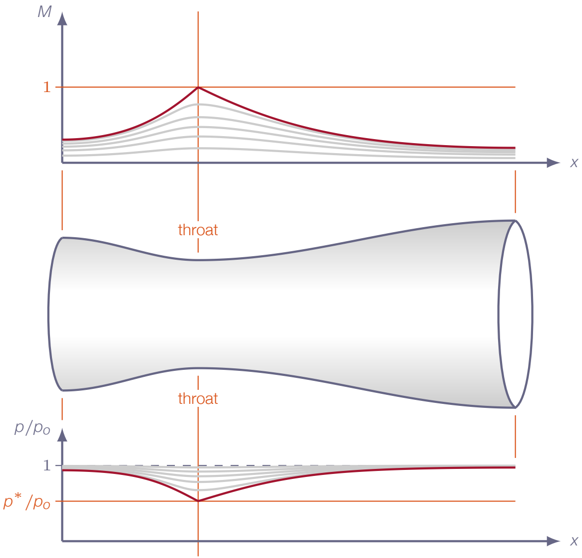

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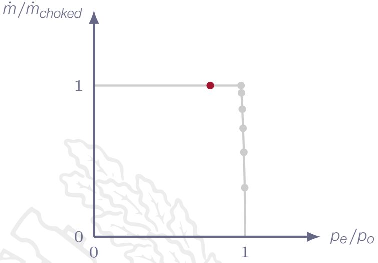

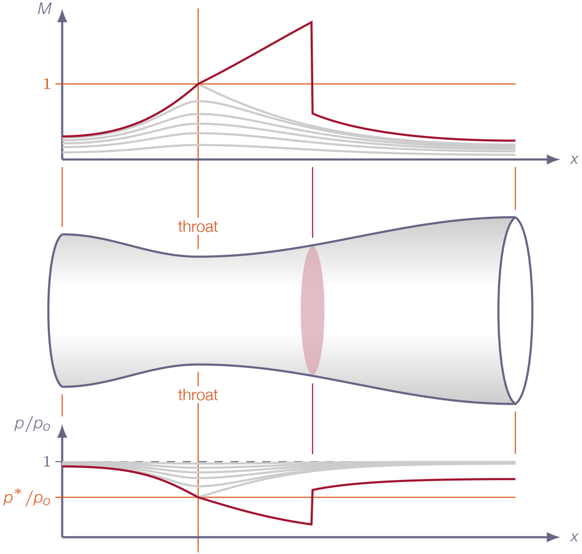

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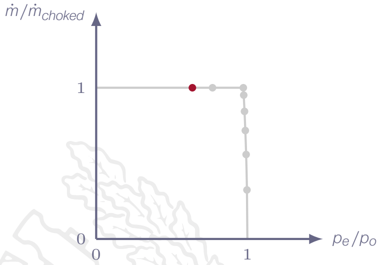

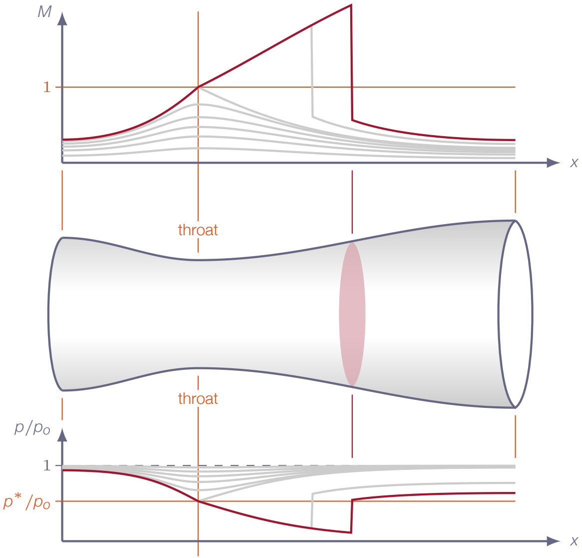

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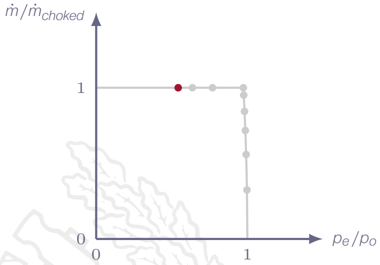

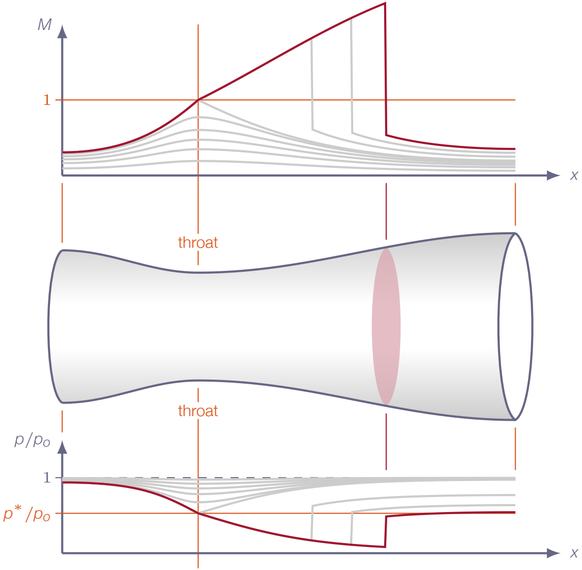

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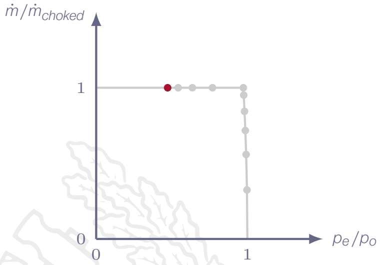

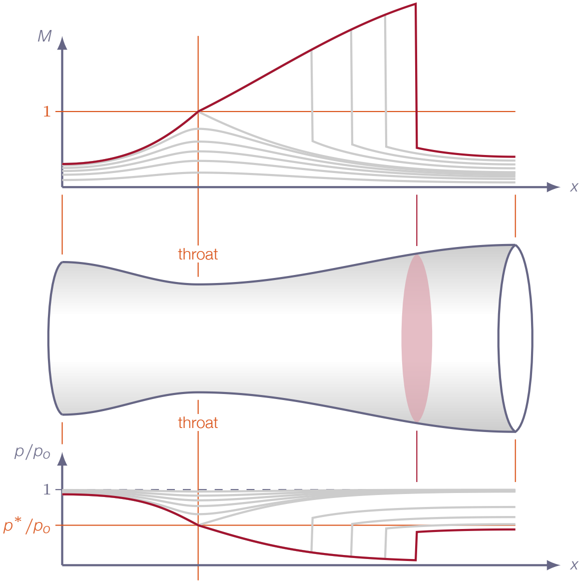

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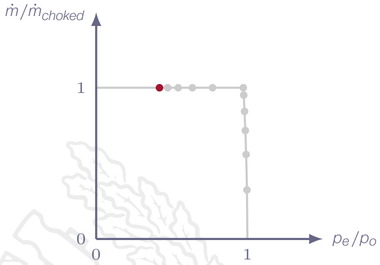

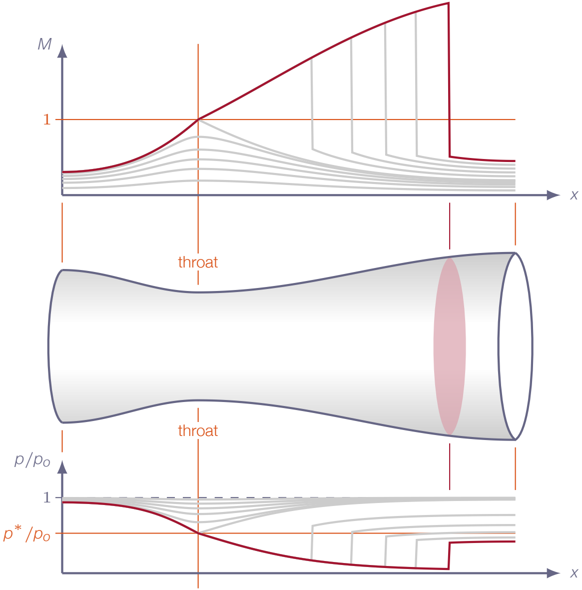

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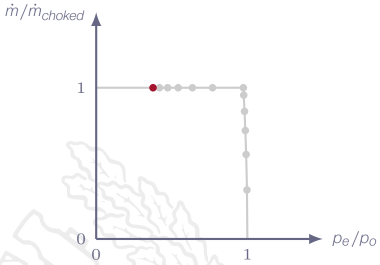
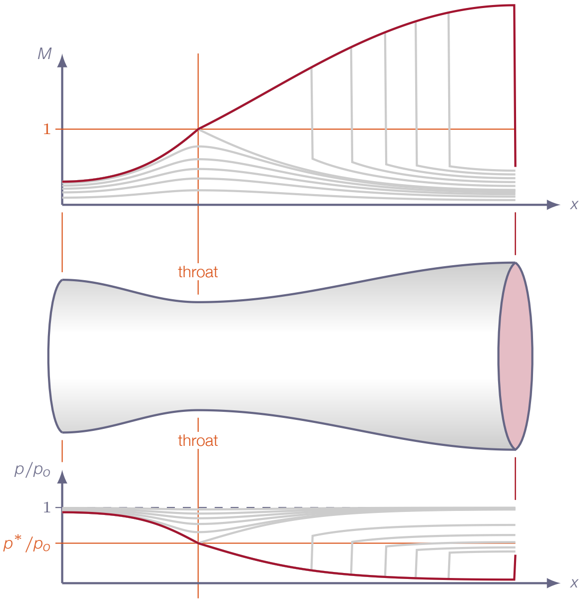
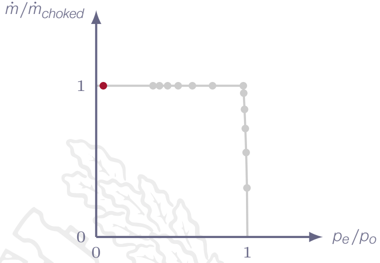
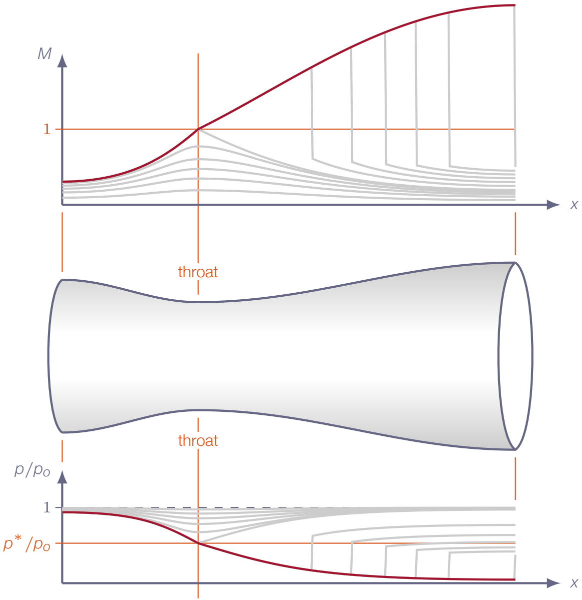


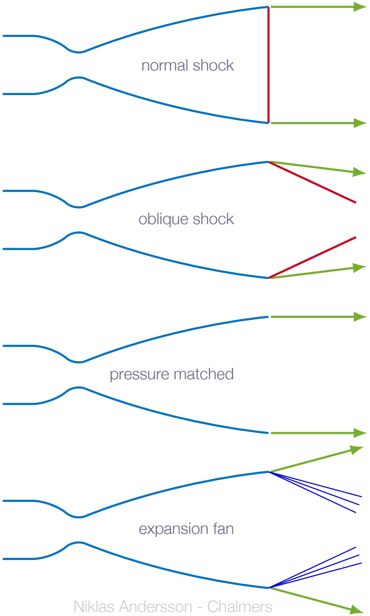


Convergent-Divergent Nozzle





Convergent-Divergent Nozzle



$p_o/p_e = (p_o/p_e)_{ne}$
normal shock at nozzle exit

$(p_o/p_e)_{ne} < p_o/p_e < (p_o/p_e)_{sc}$
overexpanded nozzle flow

$p_o/p_e = (p_o/p_e)_{sc}$
pressure matched nozzle flow

$p_o/p_e > (p_o/p_e)_{sc}$
underexpanded nozzle flow

Convergent-Divergent Nozzle