Fluid Mechanics - MTF053

Lecture 18

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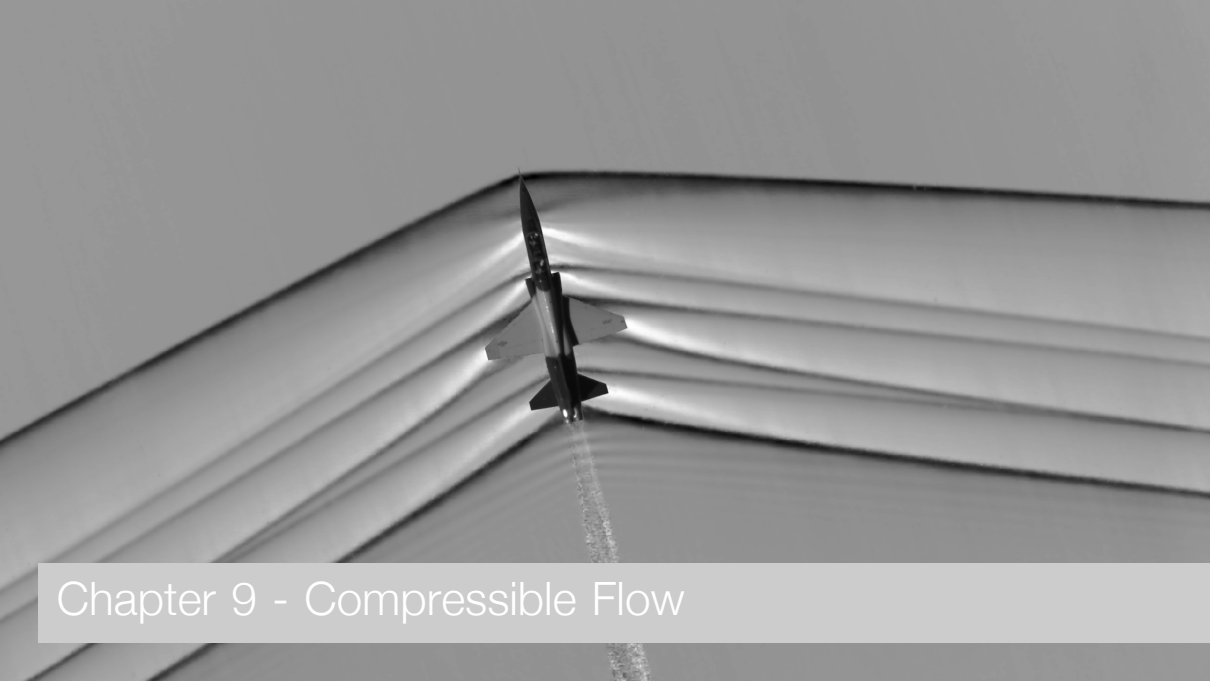
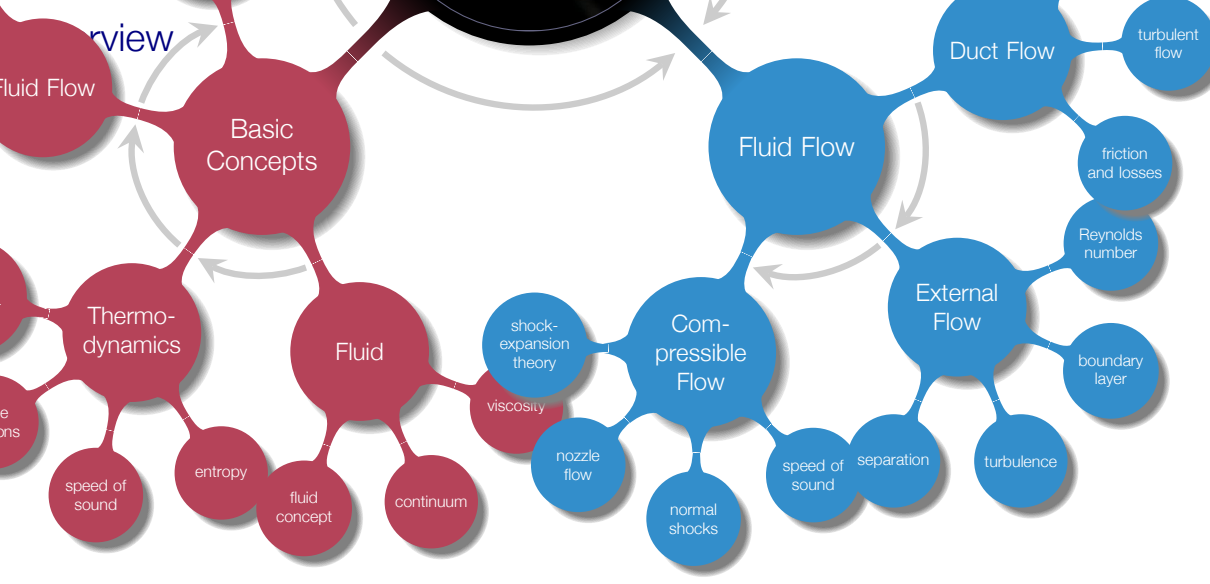


Chapter 9 - Compressible Flow


rview
Fluid Flow
Basic Concepts
Thermo-dynamics
Fluid
speed of sound
entropy
fluid concept
continuum
viscosity
shock-expansion theory
Com-pressible Flow
nozzle flow
normal shocks
speed of sound
separation
turbulence
External Flow
boundary layer
Reynolds number
friction and losses
Duct Flow
turbulent flow
Fluid Flow

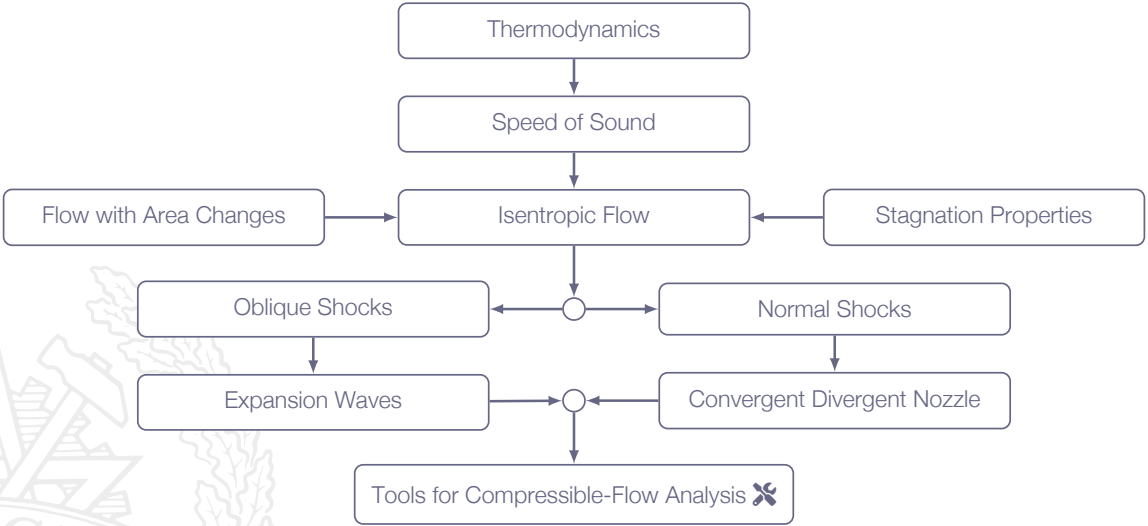
Learning Outcomes

4 Be **able to categorize** a flow and **have knowledge about** how to select applicable methods for the analysis of a specific flow based on category

37 **Understand and explain** basic concepts of compressible flows (the gas law, speed of sound, Mach number, isentropic flow with changing area, normal shocks, oblique shocks, Prandtl-Meyer expansion)

Let's go supersonic ...

Roadmap - Compressible Flow



Motivation

Compressible flow:

flows where **variations in density** are significant

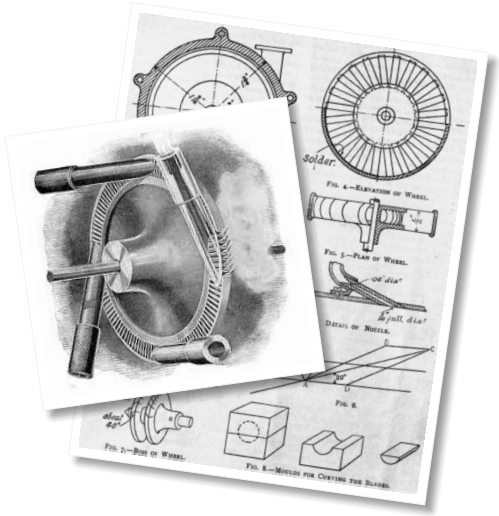
most often **high-speed gas flows** (gas dynamics)

fluids moving at speeds comparable to the **speed of sound**

not common in liquids (would require very high pressures)

Historical Milestones

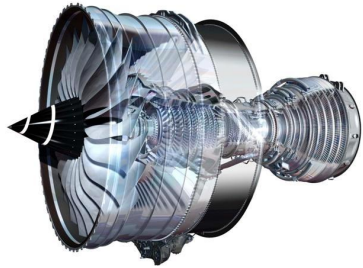
First supersonic flight - Charles Yeager 1947



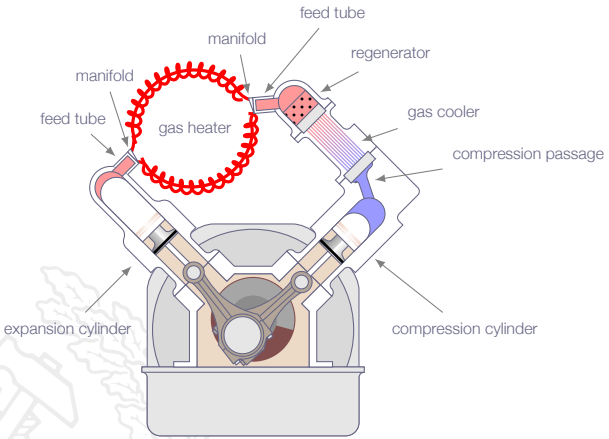
Steam turbine with convergent-divergent nozzles - Carl Gustav de Laval 1893

Compressible Flow Applications

Compressible Flow Applications



Compressible Flow Applications



Governing Equations

With significant density changes follows substantial changes in pressure and temperature

The **energy equation must be included**

Four equations:

1. Continuity
2. Momentum
3. Energy
4. Equation of state

Unknowns: ρ, p, T, and $\mathbf{V}$

The four equations must be solved simultaneously

Mach Number Regimes

Incompressible flow

insignificant density changes

Subsonic flow

local and global Mach number less than unity

Transonic flow

1. subsonic flow with regions of supersonic flow (local Mach number can be higher than one)
2. supersonic flow with regions of subsonic flow (local Mach number can be less than one)

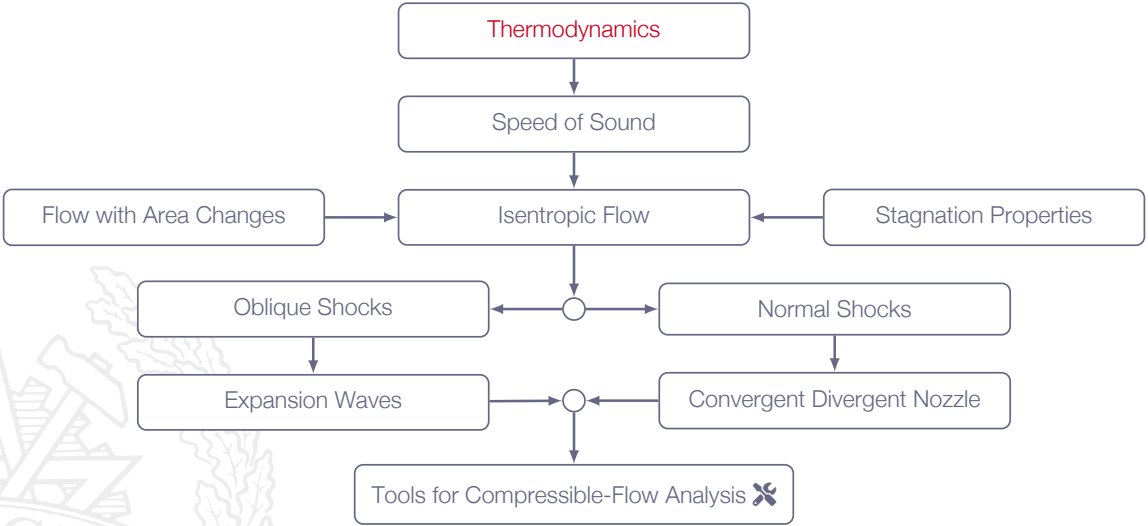
Supersonic flow

local and global Mach number higher than one

Hypersonic flow

Mach number higher than 5.0

Roadmap - Compressible Flow



Ratio of Specific Heats

The ratio of specific heats is important in compressible flow

$$\gamma = \frac{C_p}{C_v}$$

γ is a fluid property

For **moderate temperatures** γ is a **constant**

For **higher temperatures** γ **varies** with temperature

(*for air:* $\gamma = 1.4$)

Equation of State

In the following, we will assume that the **ideal gas law** is applicable and that the **specific heats** are **constants**:

$$p = \rho RT$$

$$R = C_p - C_v = const$$

$$\gamma = \frac{C_p}{C_v} = const$$

Auxiliary relations:

$$C_v = \frac{R}{\gamma - 1}, C_p = \frac{\gamma R}{\gamma - 1}$$

Internal Energy and Enthalpy

Constant specific heats:

$$d\hat{u} = C_v dT$$

$$dh = C_p dT$$

Variable specific heats:

$$\hat{u} = \int C_v dT$$

$$h = \int C_p dT$$

Isentropic Relations

First law of thermodynamics

$$\delta q + \delta w = de$$

For reversible processes: $\delta w = -p d\upsilon$ (where $\upsilon = p/\rho$)

$$h = e + \frac{p}{\rho} = e + p\upsilon \Rightarrow dh = de + pd\upsilon + \upsilon dp$$

$$\delta q = dh - \upsilon dp$$

Second law of thermodynamics

$$ds = \frac{\delta q_{rev}}{T} = \frac{\delta q}{T} + ds_{irev} \Rightarrow ds \geq \frac{\delta q}{T}$$

Isentropic Relations

compute entropy change from the first and second law of thermodynamics (assuming reversible heat addition)

$$Tds = dh - \frac{dp}{\rho}$$

for perfect gases, $dh = C_p dT$

$$\int_1^2 ds = \int_1^2 C_p \frac{dT}{T} - R \int_1^2 \frac{dp}{p}$$

for constant specific heats (calorically perfect)

$$s_2 - s_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = C_v \ln \frac{T_2}{T_1} - R \ln \frac{\rho_2}{\rho_1}$$

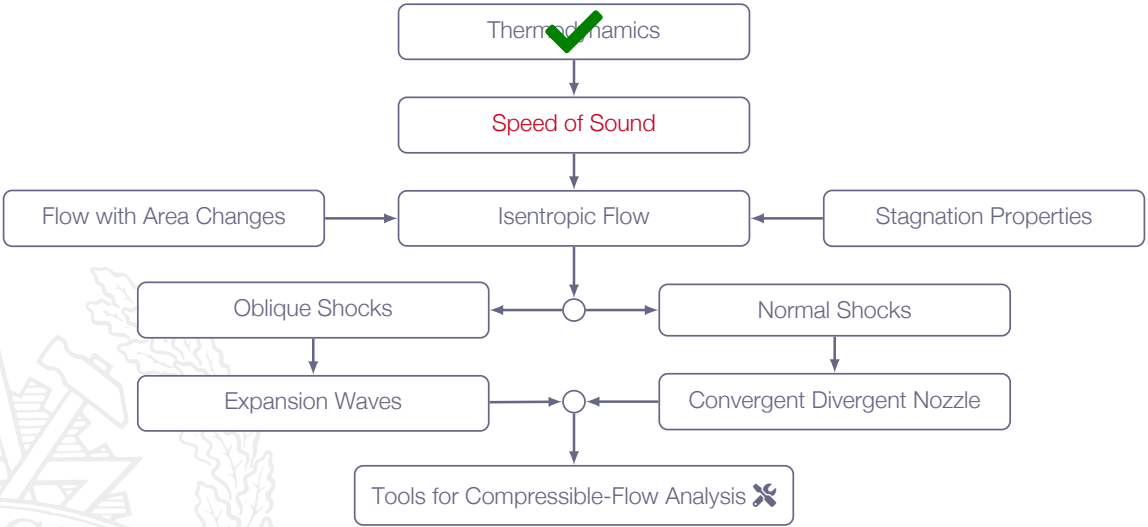
Isentropic Relations

$$s_2 - s_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = C_v \ln \frac{T_2}{T_1} - R \ln \frac{\rho_2}{\rho_1}$$

for isentropic flow ($s_2 = s_1$) we get

$$\boxed{\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}}$$

Roadmap - Compressible Flow



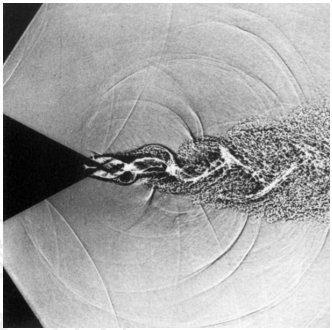
Speed of Sound

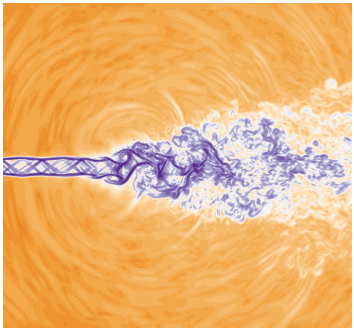
The rate of propagation of a **pressure pulse of infinitesimal strength** through a **fluid at rest**

Related to the molecular activity of the fluid

A thermodynamic property

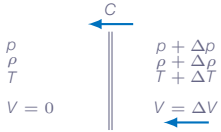
Speed of Sound



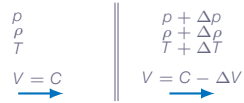


Speed of Sound

frame of reference fixed to fluid



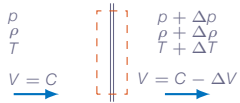
frame of reference following the wave



Speed of Sound

frame of reference following the wave

continuity:



$$\rho AC = (\rho + \Delta\rho)A(C - \Delta V)$$

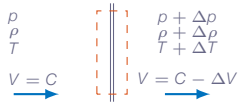
$$\Delta V = C\frac{\Delta\rho}{\rho + \Delta\rho}$$

Note! there are no gradients in the flow so viscous effects are confined to the interior of the wave

Speed of Sound

frame of reference following the wave

momentum:



$$pA - (p + \Delta p)A = (\rho AC)(C - \Delta V - C) \Rightarrow \Delta p = \rho C \Delta V$$

with ΔV from the continuity equation we get

$$C^2 = \frac{\Delta p}{\Delta \rho}\left(1 + \frac{\Delta \rho}{\rho}\right)$$

Note! the larger $\Delta\rho/\rho$, the higher the propagation velocity

Speed of Sound

In the limit of infinitesimal strength $\Delta\rho \to 0$ and thus

$$C^2 = a^2 = \frac{\partial p}{\partial \rho}$$

There is no added heat and thus the process adiabatic

For weak waves the process can also be assumed to be reversible

$$a^2 = \left.\frac{\partial p}{\partial \rho}\right|_s$$

Speed of Sound

$$a^2 = \left.\frac{\partial p}{\partial \rho}\right|_s$$

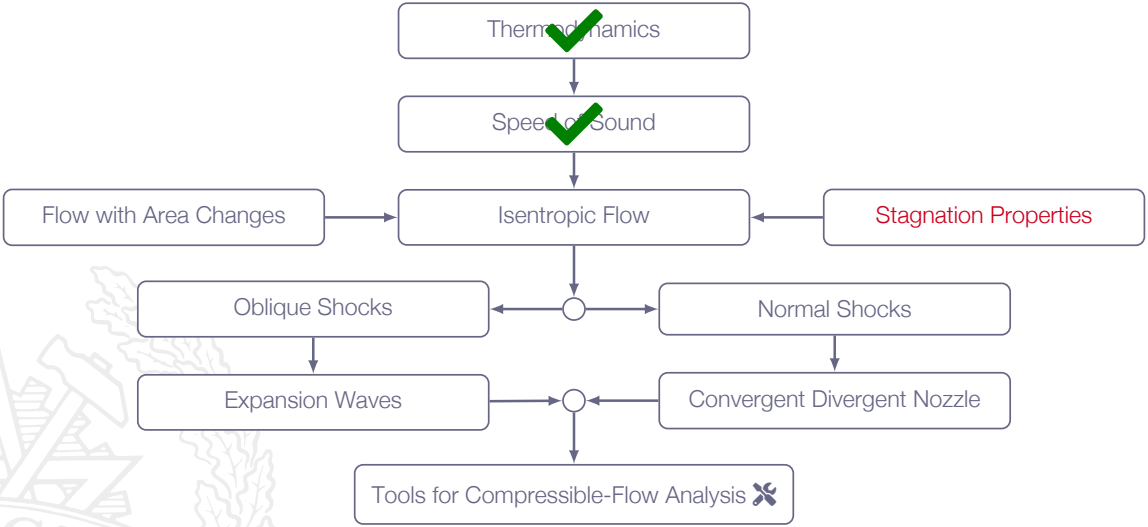
The isentropic relation gives

$$p = \rho^\gamma \Rightarrow \frac{\partial p}{\partial \rho} = \gamma \rho^{\gamma-1} = \gamma \frac{p}{\rho} = \gamma RT$$

and thus

$$\boxed{a = \sqrt{\gamma RT}}$$

Roadmap - Compressible Flow



Stagnation Enthalpy

Consider high-speed gas flow past an insulated wall

$$h_1 + \frac{1}{2}V_1^2 + gz_1 = h_2 + \frac{1}{2}V_2^2 + gz_2 - q + w_\nu$$

1. differences in potential energy extremely small
2. outside of the boundary layer, heat transfer and viscous work are zero

$$h_1 + \frac{1}{2}V_1^2 = h_2 + \frac{1}{2}V_2^2 = const$$

Stagnation Enthalpy

$$h + \frac{1}{2}V^2 = h_o$$

"The maximum enthalpy that the fluid would achieve if brought to rest adiabatically"

Stagnation Temperature

For a calorically perfect gas $h = C_p T$

$$h + \frac{1}{2}V^2 = h_o$$

$$C_p T + \frac{1}{2}V^2 = C_p T_o$$

Where T_o is the stagnation temperature

Mach Number Relations

$$C_pT + \frac{1}{2}V^2 = C_pT_o \Rightarrow 1 + \frac{V^2}{2C_pT} = \frac{T_o}{T}$$

$$C_pT = \frac{\gamma R}{\gamma - 1}T = \frac{\gamma RT}{\gamma - 1} = \frac{a^2}{\gamma - 1}$$

$$\boxed{\frac{T_o}{T} = 1 + \left(\frac{\gamma - 1}{2}\right)M^2}$$

Mach Number Relations

Since $a \propto T^{1/2}$ we get

$$\frac{a_0}{a} = \left(\frac{T_0}{T}\right)^{1/2} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]^{1/2}$$

Mach Number Relations

If the flow is adiabatic and reversible (isentropic), we may use the isentropic relations

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\gamma/(\gamma-1)} = \left[1 + \left(\frac{\gamma-1}{2}\right) M^2\right]^{\gamma/(\gamma-1)}$$

$$\frac{\rho_o}{\rho} = \left(\frac{T_o}{T}\right)^{1/(\gamma-1)} = \left[1 + \left(\frac{\gamma-1}{2}\right) M^2\right]^{1/(\gamma-1)}$$

Stagnation Properties

p_o and ρ_o - the pressure and density that the flow would achieve if brought to rest isentropically

All stagnation properties are constants in an isentropic flow

h_o, T_o, and a_o are constants in an adiabatic flow but not necessarily p_o and ρ_o

p_o and ρ_o will vary throughout an adiabatic flow as the entropy changes due to friction or shocks

Critical Properties

Another useful set of reference variables is the critical properties (sonic conditions)

$$\frac{T_o}{T} = 1 + \left(\frac{\gamma - 1}{2}\right) M^2 = \{M = 1.0\} = 1 + \left(\frac{\gamma - 1}{2}\right) = \left(\frac{2 + \gamma - 1}{2}\right) = \left(\frac{\gamma + 1}{2}\right)$$

Critical Properties

$$\frac{T^*}{T_o} = \left(\frac{2}{\gamma+1}\right)$$

$$\frac{a^*}{a_o} = \left(\frac{2}{\gamma+1}\right)^{1/2}$$

$$\frac{p^*}{p_o} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)}$$

$$\frac{\rho^*}{\rho_o} = \left(\frac{2}{\gamma+1}\right)^{1/(\gamma-1)}$$

Critical Properties

Air $\gamma = 1.4$

$$\frac{T^*}{T_o} = \left(\frac{2}{\gamma+1}\right) = 0.8333$$

$$\frac{a^*}{a_o} = \left(\frac{2}{\gamma+1}\right)^{1/2} = 0.9129$$

$$\frac{p^*}{p_o} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)} = 0.5283$$

$$\frac{\rho^*}{\rho_o} = \left(\frac{2}{\gamma+1}\right)^{1/(\gamma-1)} = 0.6339$$