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Learning Outcomes

- 4 Be **able to categorize** a flow and **have knowledge about** how to select applicable methods for the analysis of a specific flow based on category
- 6 Explain what a boundary layer is and when/where/why it appears
- **Explain** how the flat plate boundary layer is developed (transition from laminar to turbulent flow)
- **Explain** and use the Blasius equation
- **Define** the Reynolds number for a flat plate boundary layer
- **Explain** what is characteristic for a turbulent flow
- **Explain** flow separation (separated cylinder flow)
- **Explain** how to delay or avoid separation
- **Derive** the boundary layer formulation of the Navier-Stokes equations
- 32 Understand and explain displacement thickness and momentum thickness
- **Understand**, **explain** and **use** the concepts drag, friction drag, pressure drag, and lift

Let's take a deep dive into boundary-layer theory

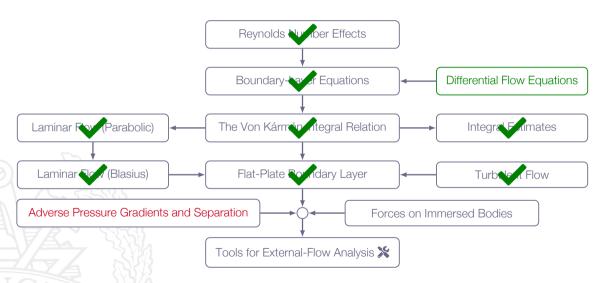
Complementary Course Material

These lecture notes covers chapter 7 in the course book and additional course material that you can find in the following documents

MTF053_Equation-for-Boundary-Layer-Flows.pdf

 ${\tt MTF053_Turbulence.pdf}$

Roadmap - Flow Past Immersed Bodies



Adverse pressure gradient

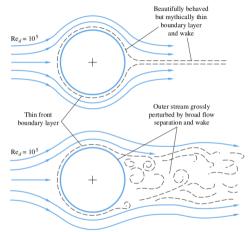
pressure increases in the flow direction may lead to separation

Favorable pressure gradient

pressure decreases in the flow direction the flow will not separate

Separation mechanism

loss of momentum near the wall adverse pressure gradient decelerated fluid will force flow to separate from the body



Boundary layer formulation of the momentum equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{d\rho}{dx} + \frac{1}{\rho}\frac{\partial \tau}{\partial y}$$

with u = v = 0 close at the wall, we get

$$\left. \frac{\partial \tau}{\partial y} \right|_{wall} = \left. \mu \frac{\partial^2 u}{\partial y^2} \right|_{wall} \Rightarrow \left. \frac{\partial^2 u}{\partial y^2} \right|_{wall} = \frac{1}{\mu} \frac{d\rho}{dx}$$

Note! applies both for laminar and turbulent flow

$$\left. \frac{\partial^2 u}{\partial y^2} \right|_{wall} = \frac{1}{\mu} \frac{dp}{dx}$$

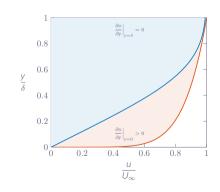
$$\frac{\partial u^2}{\partial y^2} \sim \frac{\left. \frac{\partial u}{\partial y} \right|_{y=\delta} - \left. \frac{\partial u}{\partial y} \right|_{y=0}}{\delta} < 0$$

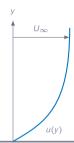
Adverse pressure gradient $(\frac{dp}{dx} > 0)$:

$$\frac{\partial^2 u}{\partial v^2} > 0$$
 at the wall $(y = 0)$

$$\frac{\partial^2 u}{\partial y^2} < 0 \text{ in the outer layer } (y \to \delta)$$

thus $\frac{\partial^2 u}{\partial v^2} = 0$ somewhere in the boundary layer

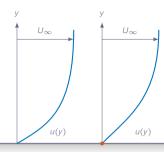




Favorable gradient (dp/dx < 0)

Point of inflection: inside wall

No separation



Favorable gradient (dp/dx < 0)

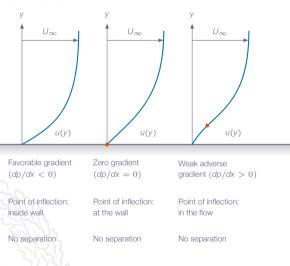
Zero gradient (dp/dx = 0)

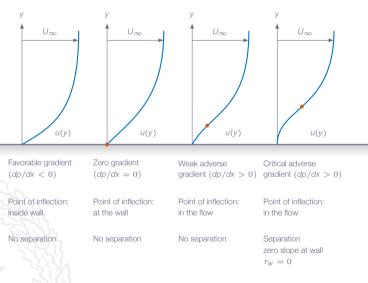
Point of inflection: inside wall

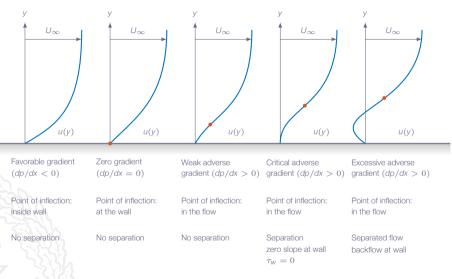
Point of inflection: at the wall

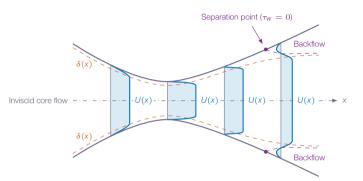
No separation

No separation





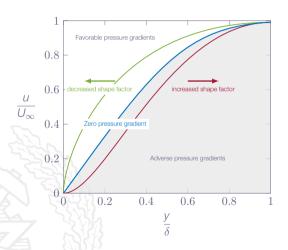




Nozzle	Throat	Diffuser
decreasing area	minimum area	increasing area
favorable pressure gradient	zero pressure gradient	adverse pressure gradient
dp/dx < 0	dp/dx = 0	dp/dx > 0
dU/dx > 0	dU/dx = 0	dU/dx < 0

Shape Factor





Laminar flow:

No pressure gradient: $H \approx 2.6$

Separation: $H \approx 3.5$

Turbulent flow:

No pressure gradient: $H \approx 1.3$

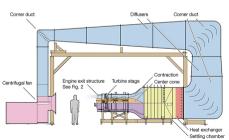
Separation: $H \approx 2.4$

Avoid or Delay Separation

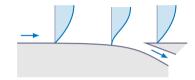


Decrease magnitude of adverse pressure gradient

Guide vanes



Avoid or Delay Separation



Remove decelerated fluid

Boundary layer suction

Avoid or Delay Separation

Increase near-wall momentum



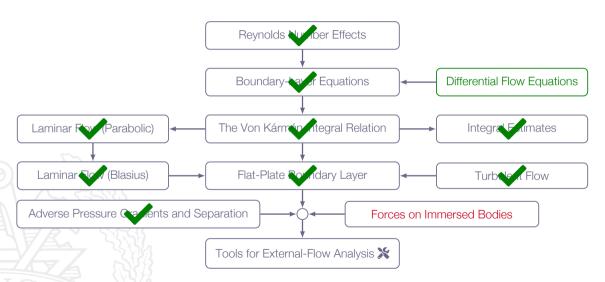
Injection of high-velocity fluid

Forced transition to turbulence

surface roughness surface irregularities (dimples on the surface of a golf ball) trip wires

Negative consequence: comes with increased friction

Roadmap - Flow Past Immersed Bodies



Drag of Immersed Bodies

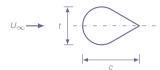
$$C_D = \frac{drag}{\frac{1}{2}\rho U_{cs}^2 A} = f\left(\frac{U_{\infty}L}{\nu}\right)$$

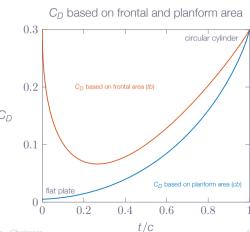
Characteristic area A:

- 1. Frontal area
 - blunt objects: cylinders, cars
- 2. Planform area

wide flat bodies: wings, hydrofoils

3. Wetted area ships





Drag of Immersed Bodies

$$C_D = C_{D_{pressure}} + C_{D_{friction}}$$

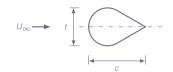
Pressure drag:

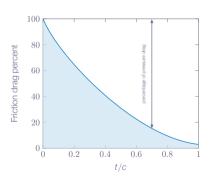
"the difference between the high front stagnation pressure and the low wake pressure on the backside of the body"

"often larger than the friction drag"

The relative importance of friction and pressure drag depends on:

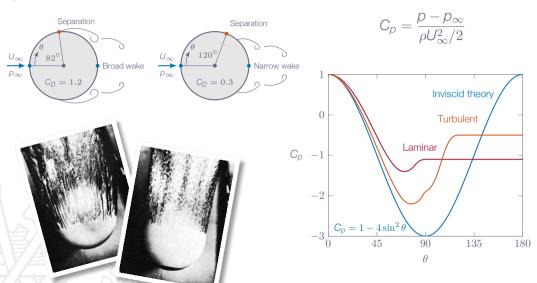
- 1. body shape
- 2. surface roughness



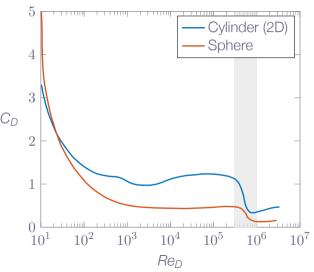


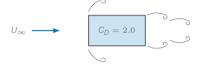
Note! for a cylinder, friction drag can be as low as a few percent of the total drag

Cylinder Surface Pressure



Cylinder Drag

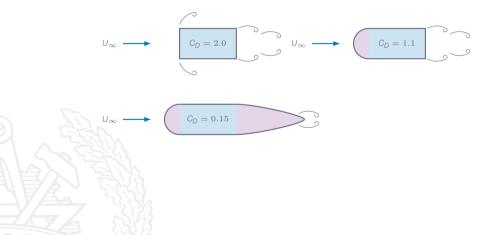


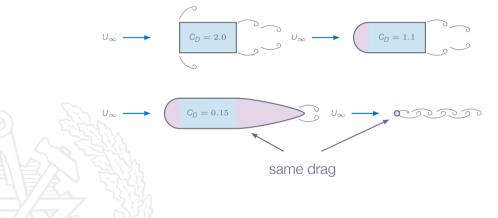












Drag Prediction

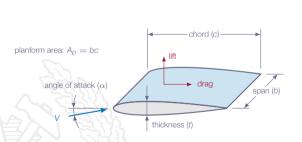
No reliable theory for drag prediction (with the exception of flat plates)

The separation point can be predicted with some accuracy but not the wake flow

CFD or experiments needed

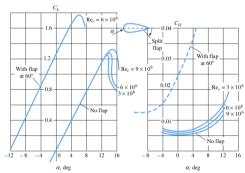
Wing Lift and Drag

$$C_D = \frac{F_D}{\frac{1}{2}\rho U_\infty^2 A_\rho}$$

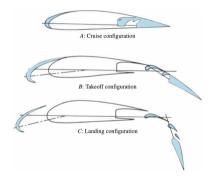


$$C_L = \frac{F_L}{\frac{1}{2}\rho U_\infty^2 A_\rho}$$

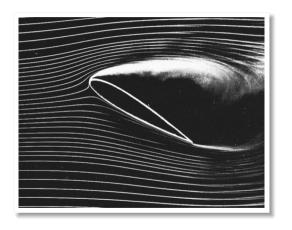




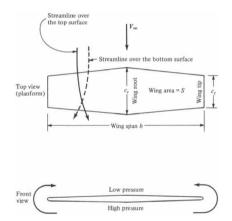
Wing Lift and Drag - High-Lift Devices

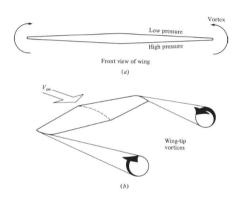


Wing Lift and Drag - Wing Stall



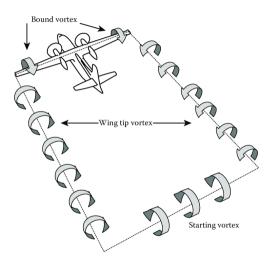
Wing Lift and Drag - Induced Drag





Wing Lift and Drag - Induced Drag



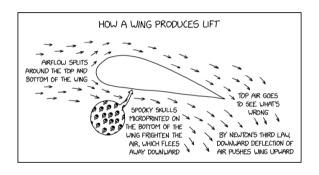


Wing Lift and Drag - Induced Drag

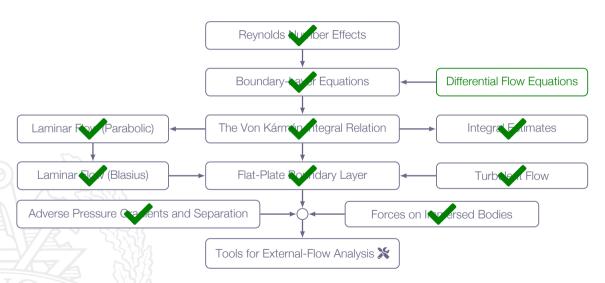




Wing Lift and Drag



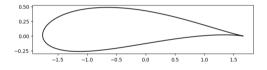
Roadmap - Flow Past Immersed Bodies



Joukowsky Transform



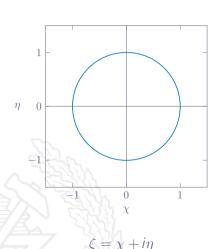
A Joukowsky wing is generated in the complex plane by applying the Joukowsky transform to a cylinder

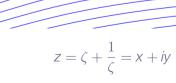


Since the potential flow around a cylinder is well known it is by using so-called conformal mapping possible to get the flow around the wing profile from the cylinder solution

Joukowsky Transform







Complex Conjugate





