Fluid Mechanics - MTF053

Lecture 16

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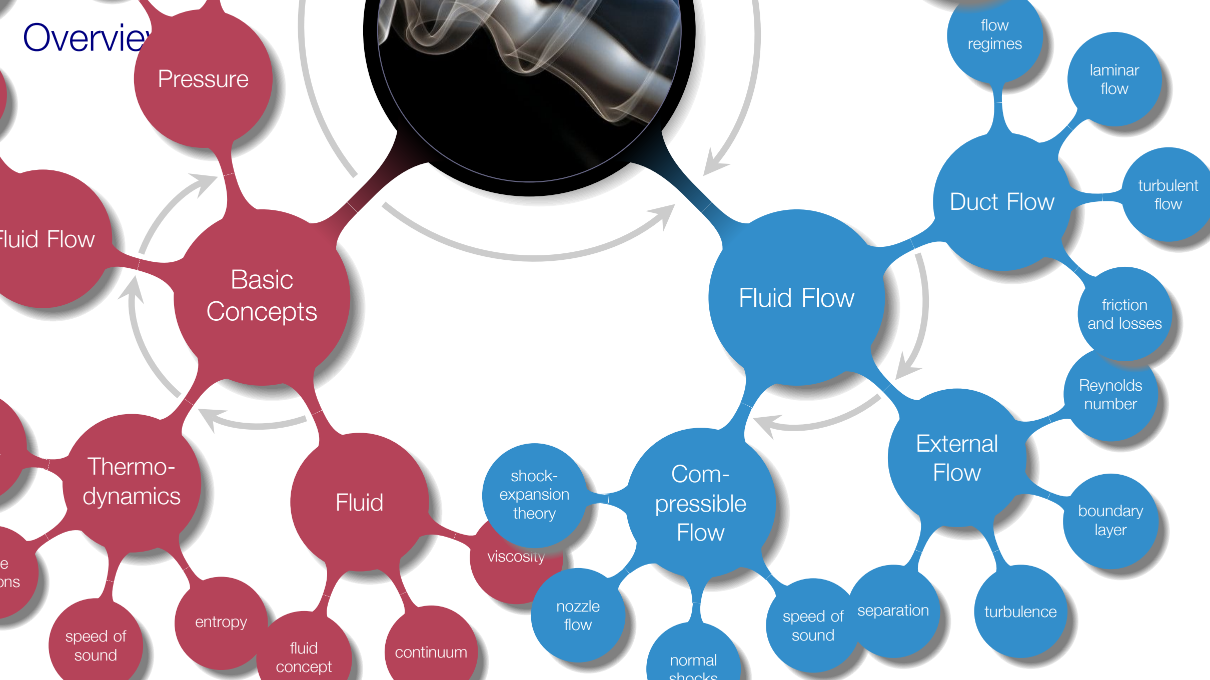
Chapter 7 - Flow Past Immersed Bodies

Overvie


Pressure
Basic Concepts
Thermo-dynamics
Fluid
viscosity
speed of sound
entropy
fluid concept
continuum
Fluid Flow
flow regimes
laminar flow
Duct Flow
turbulent flow
friction and losses
Reynolds number
External Flow
boundary layer
turbulence
separation
Com-pressible Flow
shock-expansion theory
nozzle flow
normal
speed of sound

Learning Outcomes

4 Be **able to categorize** a flow and **have knowledge about** how to select applicable methods for the analysis of a specific flow based on category

6 **Explain** what a boundary layer is and when/where/why it appears

21 **Explain** how the flat plate boundary layer is developed (transition from laminar to turbulent flow)

22 **Explain** and use the Blasius equation

23 **Define** the Reynolds number for a flat plate boundary layer

24 **Explain** what is characteristic for a turbulent flow

29 **Explain** flow separation (separated cylinder flow)

30 **Explain** how to delay or avoid separation

31 **Derive** the boundary layer formulation of the Navier-Stokes equations

32 **Understand** and explain displacement thickness and momentum thickness

33 **Understand**, **explain** and **use** the concepts drag, friction drag, pressure drag, and lift

Let's take a deep dive into boundary-layer theory

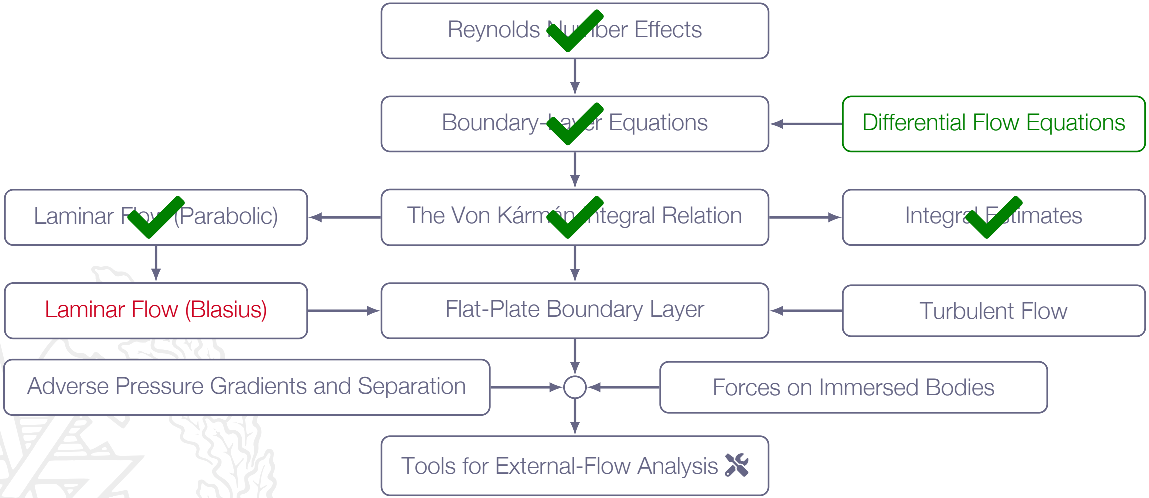
Complementary Course Material

These lecture notes covers chapter 7 in the course book and additional course material that you can find in the following documents

`MTF053_Equation-for-Boundary-Layer-Flows.pdf`

`MTF053_Turbulence.pdf`

Roadmap - Flow Past Immersed Bodies



The Blasius Velocity Profile

For laminar flow, the boundary layer equations can be solved for u and v

Blasius presented a solution 1908 where he had used a coordinate transformation and showed that $\frac{u}{U_\infty}$ is a function of a single dimensionless variable $\eta = y\sqrt{\frac{U_\infty}{\nu x}}$

The coordinate transformation corresponds to a scaling of the y coordinate with the boundary layer thickness δ

$$\frac{\delta}{x} \propto \frac{1}{\sqrt{Re_x}} \Rightarrow \frac{y}{\delta} \propto \frac{y}{x/\sqrt{Re_x}} = \frac{y}{x}\sqrt{\frac{U_\infty x}{\nu}} = y\sqrt{\frac{U_\infty}{\nu x}} = \eta$$

The Blasius Velocity Profile

1. Rewrite the boundary layer equations using the stream function (Chapter 4)
2. Rewrite the equation again $\Psi = f(\eta)\sqrt{\nu U_\infty x}$ where η is the scaled wall-normal coordinate and $f(\eta)$ is a non-dimensional stream function
3. Lots of math ....

The Navier-Stokes equations are reduced to an ordinary differential equation (ODE)

$$f''' + \frac{1}{2} f f'' = 0$$

with the boundary conditions

$$\begin{cases} f(0) = f'(0) = 0 \\ f'_{\eta\to\infty} \to 1.0 \end{cases}$$

The Blasius Velocity Profile

$$\frac{u}{U_\infty} = f'(\eta)$$

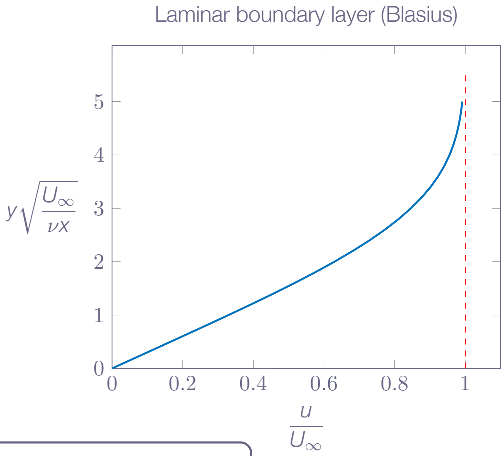
Note! $u/U_\infty \to 1$ as $y \to \infty$ and therefore δ is usually defined as the distance from the wall where $u/U_\infty = 0.99$

The Blasius Velocity Profile

$$\frac{u}{U_\infty} = f'(\eta)$$

$$\eta = y\sqrt{\frac{U_\infty}{\nu x}}$$

$$\delta_{99\%}\sqrt{\frac{U_\infty}{\nu x}} \approx 5.0$$



$$\boxed{\delta_{99\%}\sqrt{\frac{U_\infty}{\nu x}} \approx 5.0 \quad \text{or} \quad \frac{\delta}{x} \approx \frac{5.0}{\sqrt{Re_x}}}$$

The Blasius Velocity Profile

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu \left[\frac{du}{d\eta}\frac{\partial \eta}{\partial y}\right]_{\eta=0} = \mu U_\infty \left[\frac{d}{d\eta}\left(\frac{u}{U_\infty}\right)\frac{\partial \eta}{\partial y}\right]_{\eta=0}$$

$$\eta = y\sqrt{\frac{U_\infty}{\nu x}} \Rightarrow \frac{\partial \eta}{\partial y} = \sqrt{\frac{U_\infty}{\nu x}} \Rightarrow \tau_w = \mu U_\infty \sqrt{\frac{U_\infty}{\nu x}}\frac{d}{d\eta}\left(\frac{u}{U_\infty}\right)_{\eta=0} = \frac{\rho U_\infty^2}{\sqrt{Re_x}}\frac{d}{d\eta}\left(\frac{u}{U_\infty}\right)_{\eta=0}$$

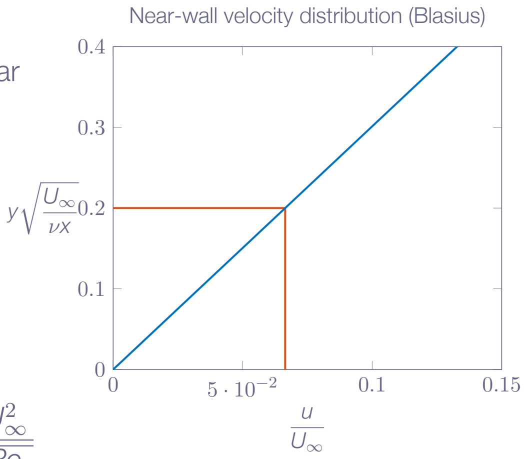
The Blasius Velocity Profile

close the the wall the velocity profile is linear

$$\eta = 0.2 \Rightarrow \frac{u}{U_\infty} \approx 0.0664$$

$$\frac{d}{d\eta}\left(\frac{u}{U_\infty}\right)_{\eta=0} \approx \frac{0.0664}{0.2} = 0.332$$

$$\tau_w = \frac{\rho U_\infty^2}{\sqrt{Re_x}} \frac{d}{d\eta}\left(\frac{u}{U_\infty}\right)_{\eta=0} \approx 0.332 \frac{\rho U_\infty^2}{\sqrt{Re_x}}$$



Laminar Boundary Layer - Blasius

$$\boxed{\tau_w(x) \approx \frac{0.332\rho^{1/2}\mu^{1/2}U_\infty^{3/2}}{x^{1/2}}}$$

Note! the wall shear stress drops off with increasing distance due to the boundary layer growth

Recall *for pipe flow, the wall shear stress is independent of x – pipe flow is confined and the boundary layer height is restricted*

Laminar Boundary Layer - Blasius

wall shear stress:

$$\tau_w(x) \approx \frac{0.332\rho^{1/2}\mu^{1/2}U_\infty^{3/2}}{x^{1/2}}$$

drag force:

$$D(x) = b\int_0^x \tau_w(x)dx \approx 0.664b\rho^{1/2}\mu^{1/2}U_\infty^{3/2}x^{1/2}$$

drag coefficient:

$$C_D = \frac{2D(L)}{\rho U_\infty^2 bL} \approx \frac{1.328}{\sqrt{Re_L}}$$

Laminar Boundary Layer - Blasius

From before we have $D(x) = \rho b \int_0^{\delta(x)} u(U_\infty - u)dy$

$$D(x) = \rho b U_\infty^2 \underbrace{\int_0^{\delta(x)} \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy}_{\theta(x)} = \rho b U_\infty^2 \theta(x)$$

$$b \int_0^x \tau_w(x)dx = \rho b U_\infty^2 \theta(x) \approx 0.664 b \rho^{1/2} \mu^{1/2} U_\infty^{3/2} x^{1/2} \Rightarrow \theta(x) \approx \frac{0.664 \mu^{1/2} x^{1/2}}{\rho^{1/2} U_\infty^{1/2}}$$

$\Rightarrow \theta(x) \approx \dfrac{0.664 \mu^{1/2} x}{\rho^{1/2} U_\infty^{1/2} x^{1/2}}$ and thus $\dfrac{\theta(x)}{x} \approx \dfrac{0.664}{\sqrt{Re_x}}$

Laminar Boundary Layer - Blasius

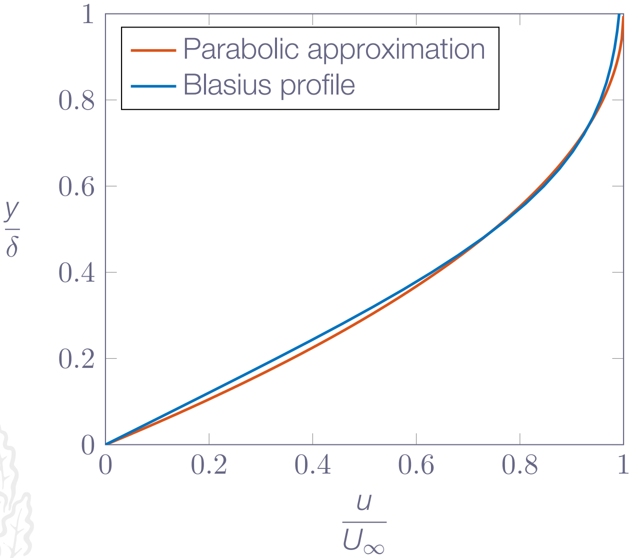
Displacement thickness:

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U_\infty}\right) dy$$

$$\frac{\delta^*}{x} \approx \frac{1.721}{Re_x^{1/2}}$$

Note! *since δ^* is much smaller than x for large values of Re_x, the velocity component in the wall-normal direction will be much smaller than the velocity parallel to the plate*

Laminar Boundary Layer



Laminar Boundary Layer

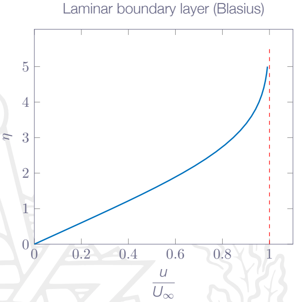
description	variable	laminar flow (Blasius)	turbulent flow (Prandtl)
boundary layer thickness	$\frac{\delta}{x}$	$\frac{5.0}{\sqrt{Re_x}}$	
displacement thickness	$\frac{\delta^*}{x}$	$\frac{1.721}{\sqrt{Re_x}}$	
momentum thickness	$\frac{\theta}{x}$	$\frac{0.664}{\sqrt{Re_x}}$	
shape factor	$H = \frac{\delta^*}{\theta}$	2.59	
wall shear stress	τ_w	$0.332\frac{\rho U_\infty^2}{\sqrt{Re_x}}$	
local skin friction coefficient	$c_f = \frac{2\tau_w}{\rho U_\infty^2}$	$\frac{0.664}{\sqrt{Re_x}}$	
drag coefficient	C_D	$\frac{1.328}{\sqrt{Re_L}}$	

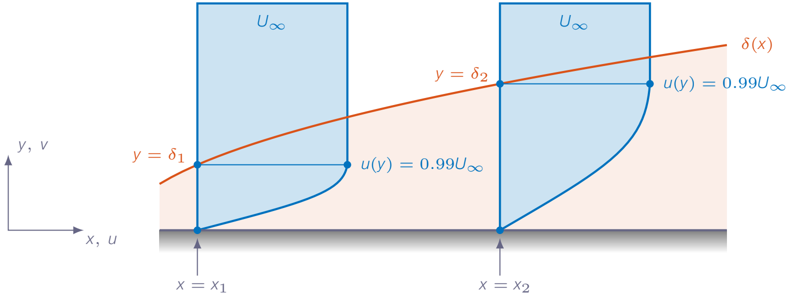
The Blasius Velocity Profile - Self Similarity

From before:

$$\eta(x,y) = y\sqrt{\frac{U_\infty}{\nu x}}$$

$$\frac{u}{U_\infty} = 0.99 \Rightarrow \eta \approx 5.0$$

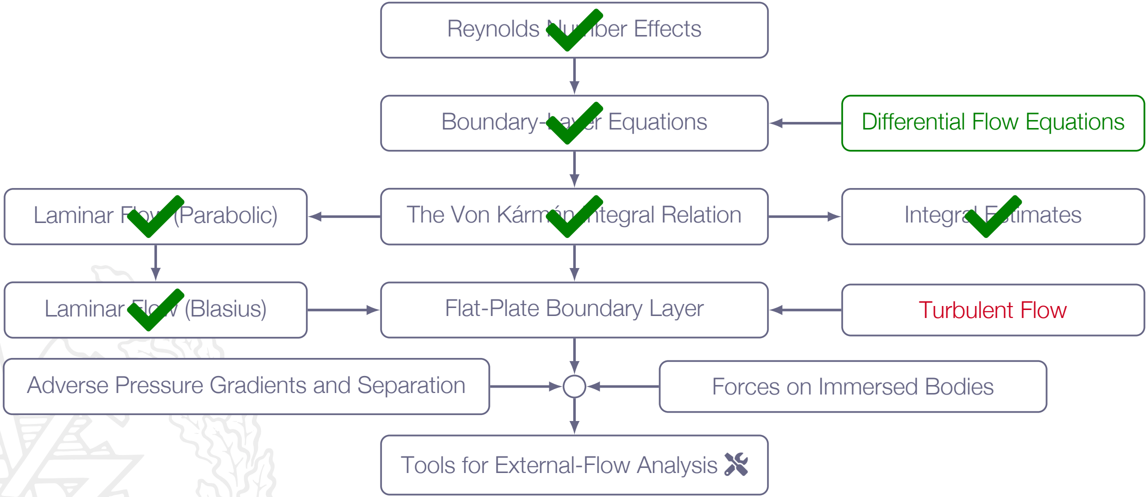




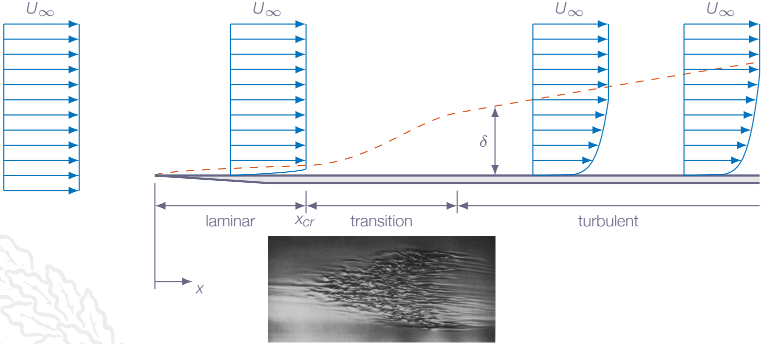
$$\eta(x_1,\delta_1) = \eta(x_2,\delta_2) \Rightarrow \delta_1\sqrt{\frac{U_\infty}{\nu x_1}} = \delta_2\sqrt{\frac{U_\infty}{\nu x_2}}$$

$$x_1 < x_2 \Rightarrow \sqrt{\frac{U_\infty}{\nu x_1}} > \sqrt{\frac{U_\infty}{\nu x_2}} \Rightarrow \delta_1 < \delta_2$$

Roadmap - Flow Past Immersed Bodies



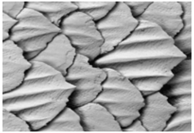
Boundary Layer Transition



$u = 0.99U_\infty$

$$Re_x = \frac{U_\infty x}{\nu}$$

$$Re_{x_{cr}} = \frac{U_\infty x_{cr}}{\nu} \approx 5.0 \times 10^5$$



Boundary Layer Transition

For low Re_x, disturbances in the flow are damped out by viscous forces

For somewhat higher Reynolds numbers, friction forces are less important and the flow becomes unstable

The transition region is short - can be treated as a point (the transition point)

Boundary Layer Transition

The onset of transition from laminar to turbulent is affected by a number of factors such as:

- **Turbulence in the freestream**
- **Surface roughness**
- **Pressure gradient**

With a smooth surface, no turbulence in the freestream, and zero pressure gradient, the onset of transition can be pushed up to $Re_x \approx 3.0 \times 10^6$

As a **rule of thumb**, we can assume $Re_{x_{cr}} \approx 5.0 \times 10^5$

Boundary Layer Transition

Freestream turbulence:

- frestream turbulence reduces the critical Reynolds number
- with high turbulence intensity in the freestream, the transition can start already at $Re_x \approx 3.0 \times 10^5$ or lower

Boundary Layer Transition

Surface roughness:

surface roughness does not affect transition significantly if $Re_\epsilon = \frac{U_\infty \epsilon}{\nu} < 680$

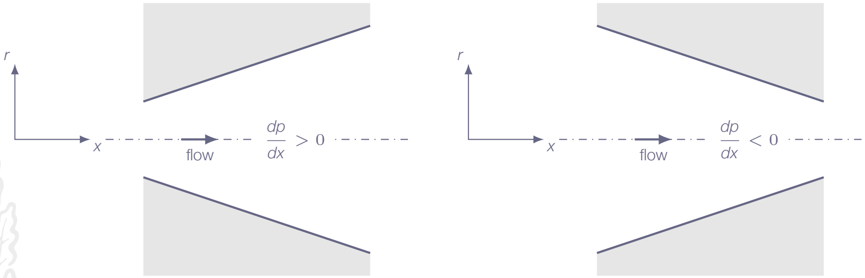
if $Re_\epsilon > 680$, the extent of the laminar region can be shortened significantly ($Re_x \approx 3.0 \times 10^5$)

Note! rule of thumb

Boundary Layer Transition

Negative pressure gradient:

decreasing pressure in the flow direction has a **stabilizing** effect on the flow and can delay transition from laminar to turbulent flow



Boundary Layer Transition

Forced transition:

a **trip wire** or **added surface roughness** can make the transition to turbulence really fast

the critical Reynolds number is not meaningful if the boundary layer is forced to transition

Flat Plate - Turbulent Boundary Layer

A turbulent boundary layer grows faster than a laminar boundary layer

- the velocity fluctuations (u', v', w') leads to **increased exchange of momentum**
- increased shear stress compared to the laminar case where we only have forces related to molecular viscosity
- larger portion of the fluid will be decelerated close to the wall

Flat Plate - Turbulent Boundary Layer

The Von Kármán integral relation and the integral estimates are valid for both laminar and turbulent boundary layers

$$\frac{\tau_w}{\rho} = \frac{d}{dx}\int_0^\delta u(U_\infty - u)dy$$

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) d$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$$

We need a velocity profile $u(y)$ for turbulent boundary layers to be able to calculate τ_w, θ, and δ^*

Approach 1: the log law 🎓

Approach 2: Prandtl's power law approximation

Flat Plate - Turbulent Boundary Layer

Approach 1: the log law

$$\frac{u}{u^*} \approx \frac{1}{\kappa}\ln\left(\frac{yu^*}{\nu}\right) + B \quad \text{where } \kappa = 0.41 \text{ and } B = 5.0$$

u^* is the **friction velocity** defined as $u^* = \sqrt{\dfrac{\tau_w}{\rho}}$

at the edge of the boundary layer $u = U_\infty$ and $y = \delta$ and thus

$$\frac{U_\infty}{u^*} \approx \frac{1}{\kappa}\ln\left(\frac{\delta u^*}{\nu}\right) + B$$

Flat Plate - Turbulent Boundary Layer

Approach 1: the log law

The **skin friction coefficient** c_f is defined as $c_f = \dfrac{2\tau_w}{\rho U_\infty^2} \Rightarrow \tau_w = c_f \dfrac{1}{2}\rho U_\infty^2$

the **friction velocity** can be expressed as $u^* = \sqrt{\dfrac{\tau_w}{\rho}} = U_\infty\sqrt{\dfrac{c_f}{2}}$

insert in the **log-law** and we get

$$\sqrt{\frac{2}{c_f}} \approx \frac{1}{\kappa}\ln\left(Re_\delta\sqrt{\frac{c_f}{2}}\right) + B$$

rather difficult to work with ...

Flat Plate - Turbulent Boundary Layer

Approach 2: Prandtl's power law approximation

Prandtl suggested the following relations:

$$c_f \approx 0.02 Re_\delta^{-1/6}$$

$$\frac{u}{U_\infty} \approx \left(\frac{y}{\delta}\right)^{1/7}$$

from before we have the following relation: $\tau_w = \rho U_\infty^2 \dfrac{d\theta}{dx} \Rightarrow c_f = 2\dfrac{d\theta}{dx}$

calculate the **momentum thickness** $\theta = \displaystyle\int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy = \frac{7}{72}\delta$

Flat Plate - Turbulent Boundary Layer

Approach 2: Prandtl's power law approximation

Now, combining the two **skin friction coefficient** relations we see that

$$0.02Re_\delta^{-1/6} = 2\frac{d}{dx}\left(\frac{7}{72}\delta\right)$$

and thus $Re_\delta^{-1/6} \approx 9.72\dfrac{d\delta}{dx} = 9.72\dfrac{d(Re_\delta)}{d(Re_x)}$

integration gives $Re_\delta \approx 0.16 Re_x^{6/7}$ or $\dfrac{\delta}{x} \approx \dfrac{0.16}{Re_x^{1/7}}$

Note! the turbulent boundary layer grows significantly faster than the laminar $\delta_{turb} \propto x^{6/7}$ vs $\delta_{lam} \propto x^{1/2}$

Flat Plate - Turbulent Boundary Layer

Approach 2: Prandtl's power law approximation

$$c_f \approx \frac{0.027}{Re_x^{1/7}}$$

$$\tau_{w_{turb}} \approx \frac{0.0135\mu^{1/7}\rho^{6/7}U_\infty^{13/7}}{x^{1/7}}$$

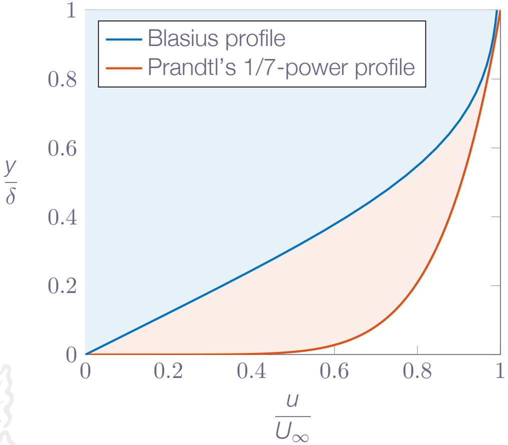
Note! friction drops slowly with x, increases nearly as ρ and U_∞^2, and is rather insensitive to viscosity

Flat Plate - Turbulent Boundary Layer

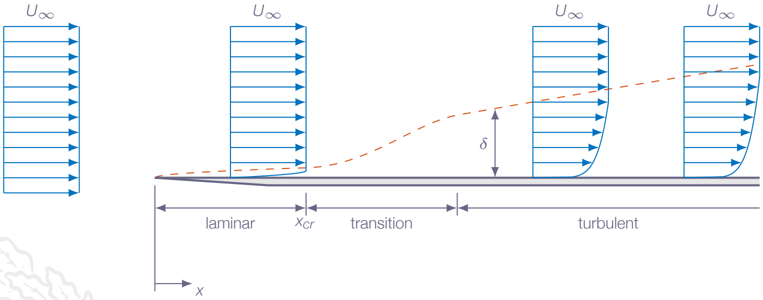
description	variable	laminar flow (Blasius)	turbulent flow (Prandtl)
boundary layer thickness	$\frac{\delta}{x}$	$\frac{5.0}{\sqrt{Re_x}}$	$\frac{0.16}{Re_x^{1/7}}$
displacement thickness	$\frac{\delta^*}{x}$	$\frac{1.721}{\sqrt{Re_x}}$	$\frac{0.02}{Re_x^{1/7}}$
momentum thickness	$\frac{\theta}{x}$	$\frac{0.664}{\sqrt{Re_x}}$	$\frac{0.016}{Re_x^{1/7}}$
shape factor	$H = \frac{\delta^*}{\theta}$	2.59	1.29
wall shear stress	τ_w	$0.332\frac{\rho U_\infty^2}{\sqrt{Re_x}}$	$0.0135\frac{\rho U_\infty^2}{Re_x^{1/7}}$
local skin friction coefficient	$c_f = \frac{2\tau_w}{\rho U_\infty^2}$	$\frac{0.664}{\sqrt{Re_x}}$	$\frac{0.027}{Re_x^{1/7}}$
drag coefficient	C_D	$\frac{1.328}{\sqrt{Re_L}}$	$\frac{0.031}{Re_L^{1/7}}$

Flat Plate - Turbulent Boundary Layer

The velocity profile in a turbulent boundary layer is quite far from the Blasius profile used for laminar boundary layers



Flat Plate Boundary Layer



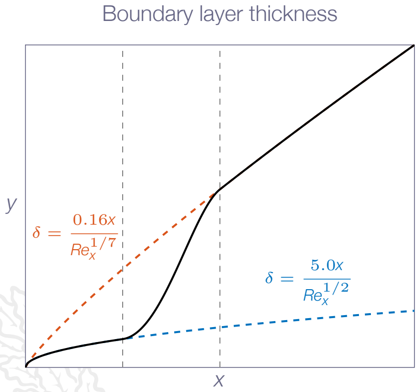
$$u = 0.99U_\infty$$

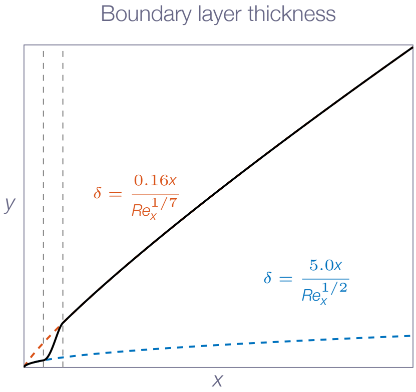
$$Re_x = \frac{U_\infty x}{\nu}$$

$$Re_{x_{cr}} = \frac{U_\infty x_{cr}}{\nu} \approx 5.0 \times 10^5$$

$$D = b\frac{1}{2}\rho U^2 \left[\int_{>0}^{x_{cr}} \frac{0.664}{Re_x^{1/2}} dx + \int_{x_{cr}}^{L} \frac{0.027}{Re_x^{1/7}} dx \right]$$

Flat Plate Boundary Layer





For a long boundary layer the length of the laminar region becomes relatively short in comparison with the length of the turbulent region

Wall Roughness

laminar:

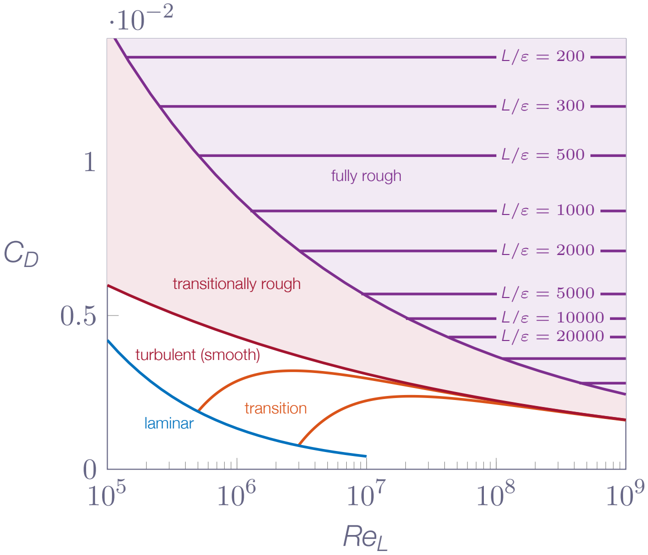
$$C_D = \frac{1.328}{Re_L^{1/2}}$$

turbulent (smooth):

$$C_D = \frac{0.031}{Re_L^{1/7}}$$

turbulent (fully rough):

$$C_D = (1.89 + 1.62\log(L/\varepsilon))^{-2.5}$$



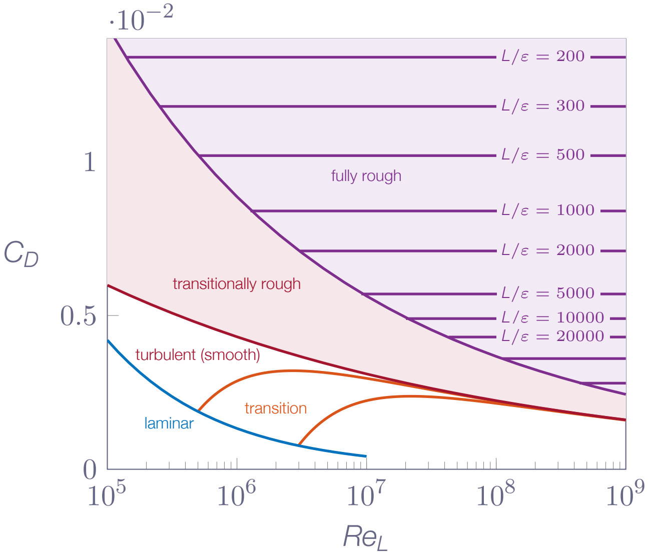
Wall Roughness

transition ($Re_{trans} = 5.0 \times 10^5$):

$$C_D = \frac{0.031}{Re_L^{1/7}} - \frac{1440}{Re_L}$$

transition ($Re_{trans} = 3.0 \times 10^6$):

$$C_D = \frac{0.031}{Re_L^{1/7}} - \frac{8700}{Re_L}$$



Wall Roughness

Recall: smooth surface:

Surface roughness (ϵ) within the viscous sublayer