Fluid Mechanics - MTF053

Lecture 14

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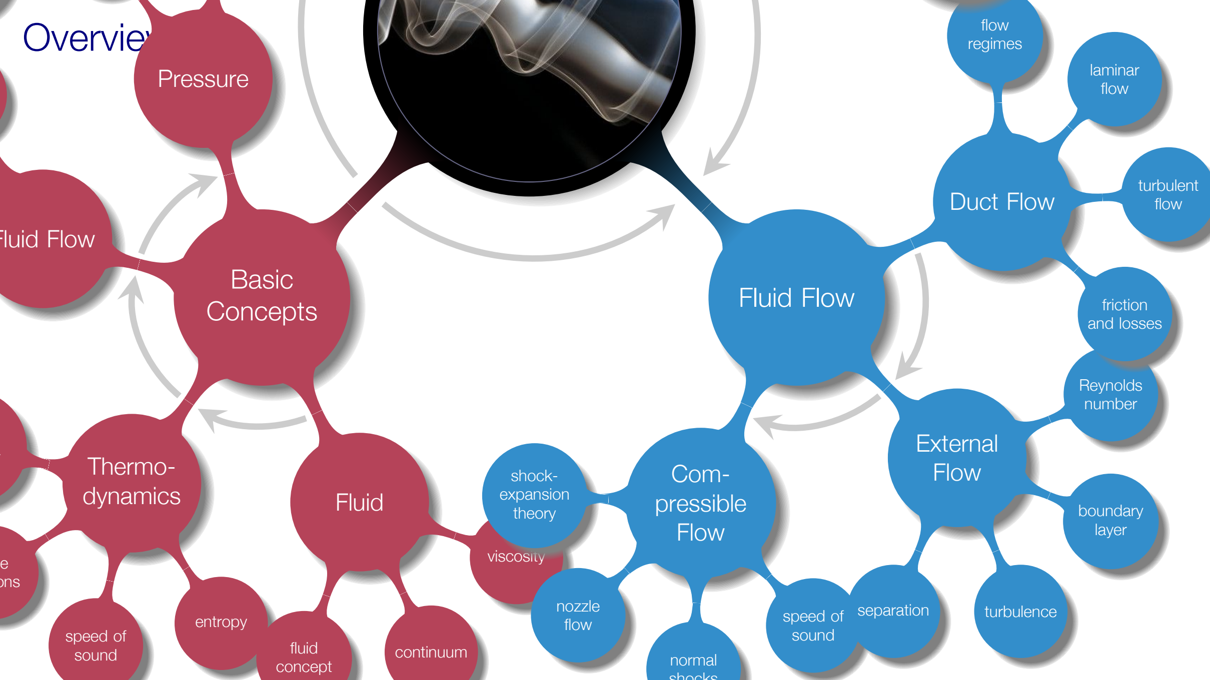
Chapter 7 - Flow Past Immersed Bodies



Overvie
Pressure
luid Flow
Basic Concepts
Thermo-dynamics
Fluid
viscosity
speed of sound
entropy
fluid concept
continuum
flow regimes
laminar flow
Duct Flow
turbulent flow
Fluid Flow
friction and losses
Reynolds number
External Flow
boundary layer
Com-pressible Flow
shock-expansion theory
nozzle flow
normal
speed of sound
separation
turbulence

Learning Outcomes

4 Be **able to categorize** a flow and **have knowledge about** how to select applicable methods for the analysis of a specific flow based on category

6 **Explain** what a boundary layer is and when/where/why it appears

21 **Explain** how the flat plate boundary layer is developed (transition from laminar to turbulent flow)

22 **Explain** and use the Blasius equation

23 **Define** the Reynolds number for a flat plate boundary layer

24 **Explain** what is characteristic for a turbulent flow

29 **Explain** flow separation (separated cylinder flow)

30 **Explain** how to delay or avoid separation

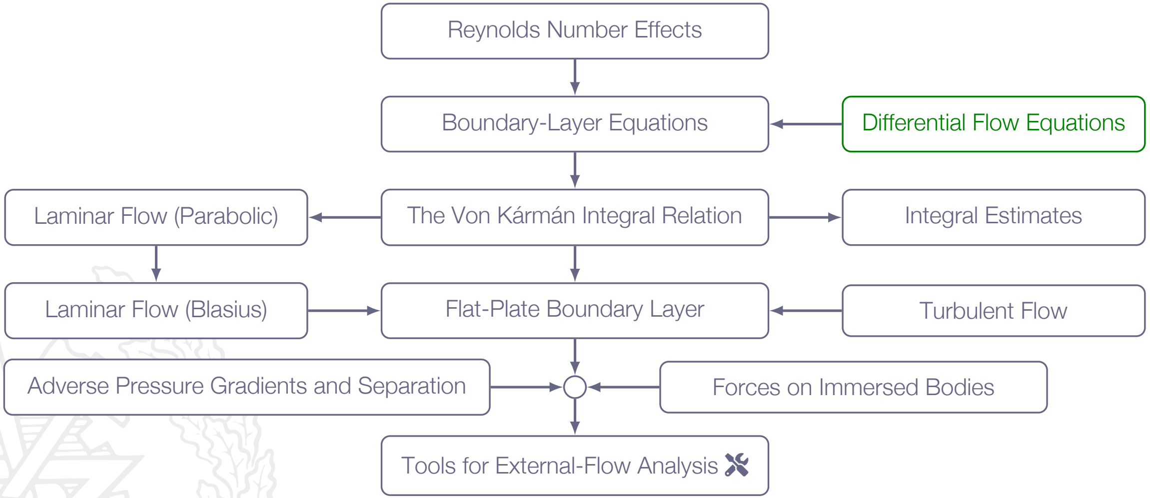
31 **Derive** the boundary layer formulation of the Navier-Stokes equations

32 **Understand** and explain displacement thickness and momentum thickness

33 **Understand**, **explain** and **use** the concepts drag, friction drag, pressure drag, and lift

Let's take a deep dive into boundary-layer theory

Roadmap - Flow Past Immersed Bodies



Complementary Course Material

These lecture notes covers chapter 7 in the course book and additional course material that you can find in the following documents

`MTF053_Equation-for-Boundary-Layer-Flows.pdf`

`MTF053_Turbulence.pdf`


"Understanding the mechanisms behind flow-related forces is a key factor to success in many engineering applications"
S MB 7177

External Flow

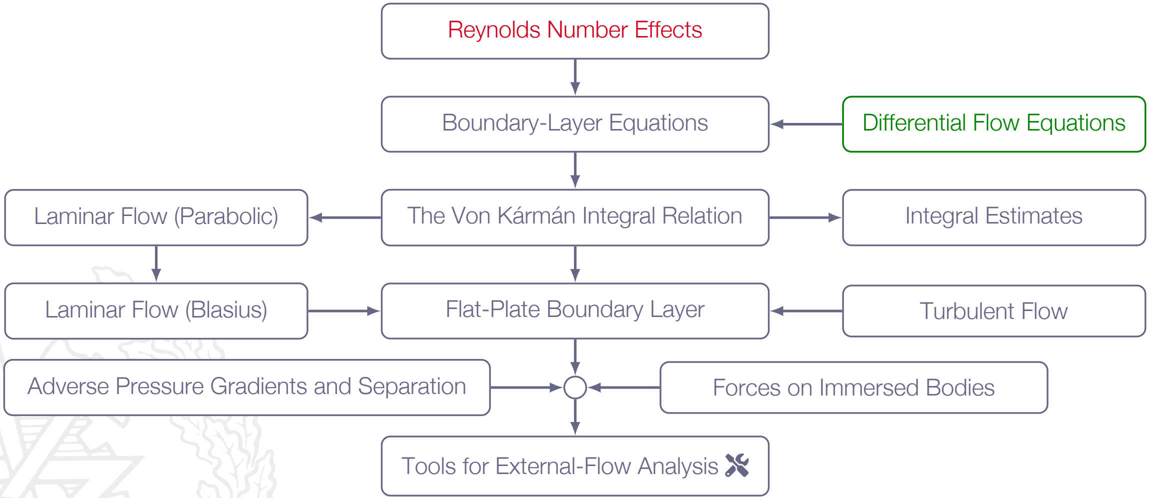
Significant viscous effects near the surface of an **immersed body**

Nearly inviscid far from the body

Unconfined - boundary layers are free to grow

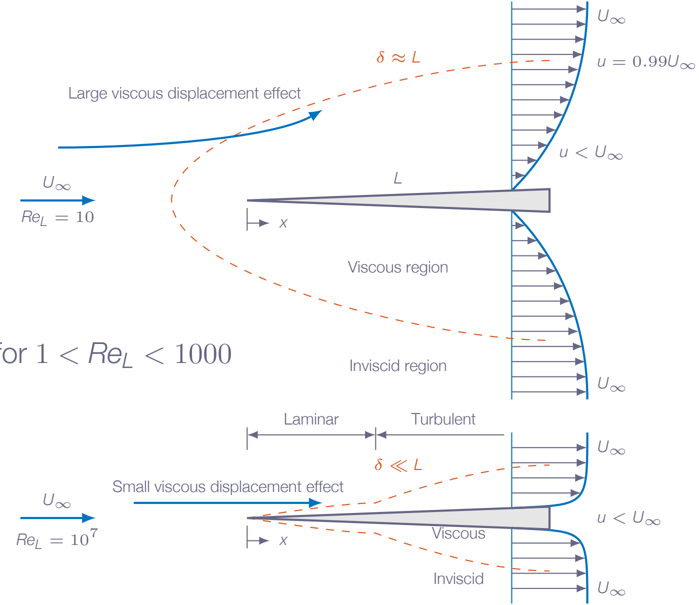
Most often CFD or experiments are needed to analyze an external flow unless the geometry is very simple

Roadmap - Flow Past Immersed Bodies



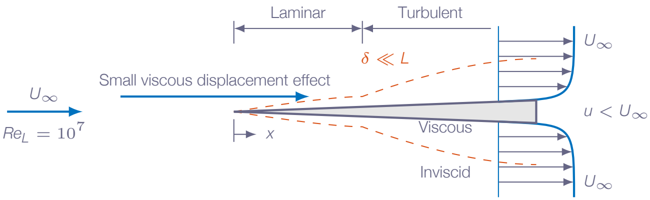
Reynolds Number Effects

$$Re_L = \frac{U_\infty L}{\nu}$$



Note: no simple theory exists for $1 < Re_L < 1000$

Reynolds Number Effects



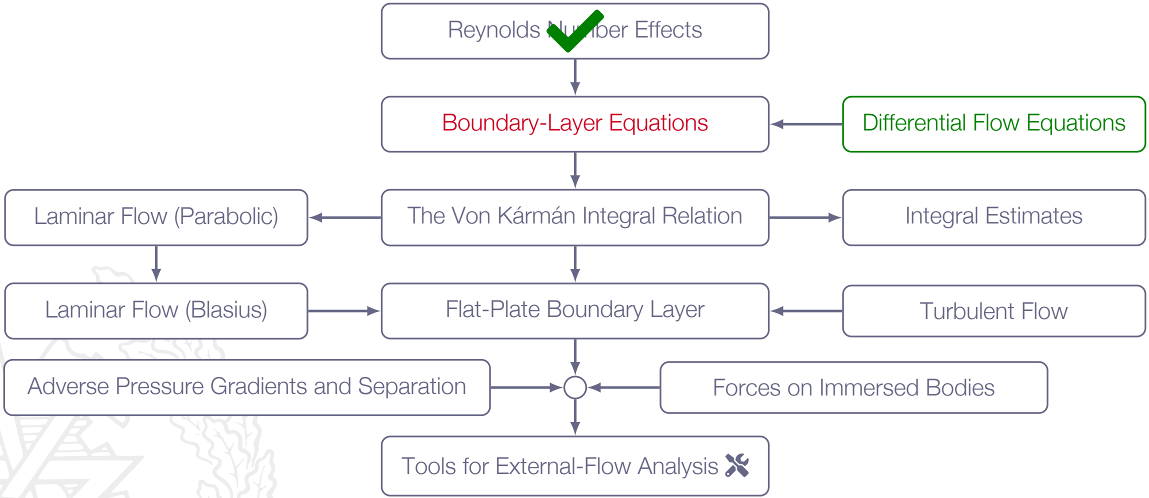
$$\frac{\delta}{x} \approx \begin{cases} \dfrac{5.0}{Re_x{}^{1/2}} & \textit{laminar} \quad 10^3 < Re_x < 10^6 \\ \dfrac{0.16}{Re_x{}^{1/7}} & \textit{turbulent} \quad 10^6 < Re_x \end{cases}$$

$$Re_L = \frac{U_\infty L}{\nu}$$

$$Re_x = \frac{U_\infty x}{\nu}$$

Note! Re_L and the **local Reynolds number** Re_x are not the same if $L \neq x$

Roadmap - Flow Past Immersed Bodies



Boundary Layer Equations

We will derive a set of equations suitable for **boundary-layer flow analysis**

Starting point: the **non-dimensional equations** derived in Chapter 5

We will assume two-dimensional, incompressible, steady-state flow

We will do an **order-of-magnitude** comparison of all the terms in the governing equations on non-dimensional form and identify terms that can be neglected in a **thin-boundary-layer** flow

Non-dimensional Flow Equations

The governing equations for two-dimensional, laminar, incompressible and steady-state flow with negligible body forces:

continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

x-momentum:

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

y-momentum:

$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$

Non-dimensional Flow Equations

$$x^* = \frac{x}{L} \qquad y^* = \frac{y}{L} \qquad u^* = \frac{u}{U_\infty} \qquad v^* = \frac{v}{U_\infty} \qquad p^* = \frac{p}{\rho U_\infty^2}$$

continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial (u^* U_\infty)}{\partial (x^* L)} + \frac{\partial (v^* U_\infty)}{\partial (y^* L)} = \frac{U_\infty}{L} \left(\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} \right) = 0$$

Non-dimensional Flow Equations

$$x^* = \frac{x}{L} \qquad y^* = \frac{y}{L} \qquad u^* = \frac{u}{U_\infty} \qquad v^* = \frac{v}{U_\infty} \qquad p^* = \frac{p}{\rho U_\infty^2}$$

x-momentum:

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\rho\left(u^* U_\infty \frac{\partial(u^* U_\infty)}{\partial(x^* L)} + v^* U_\infty \frac{\partial(u^* U_\infty)}{\partial(y^* L)}\right) = -\frac{\partial(p^* \rho U_\infty^2)}{\partial(x^* L)} + \mu\left(\frac{\partial^2(u^* U_\infty)}{\partial(x^* L)^2} + \frac{\partial^2(u^* U_\infty)}{\partial(y^* L)^2}\right)$$

Non-dimensional Flow Equations

$$\frac{\rho U_\infty{}^2}{L}\left(u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}\right)=-\frac{\rho U_\infty^2}{L}\frac{\partial p^*}{\partial x^*}+\frac{\mu U_\infty}{L^2}\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{\partial p^*}{\partial x^*}+\frac{\mu}{\rho U_\infty L}\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{\partial p^*}{\partial x^*}+\frac{1}{Re_L}\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

Non-dimensional Flow Equations - Summary

continuity:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

x-momentum:

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{Re_L} \left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} \right)$$

y-momentum:

$$u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = -\frac{\partial p^*}{\partial y^*} + \frac{1}{Re_L} \left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right)$$

$$x^* = \frac{x}{L}$$

$$y^* = \frac{y}{L}$$

$$u^* = \frac{u}{U_\infty}$$

$$v^* = \frac{v}{U_\infty}$$

$$p^* = \frac{p}{\rho U_\infty^2}$$

$$Re_L = \frac{U_\infty L}{\nu}$$

Boundary Layer Equations

To be able to find the relative sizes of different terms in the equations, we will first have a look at the flow parameters and operators

$$u^* = u/U_\infty \sim 1$$

$$x^* = x/L \sim 1$$

$$y^* = y/L \sim \delta^*$$

δ *denotes boundary layer thickness and* $\delta^* = \delta/L$

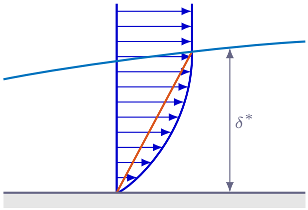
Note! here, u^* is **not** the friction velocity and δ^* is **not** the displacement thickness

Boundary Layer Equations

What about derivatives?

$$\frac{\partial u^*}{\partial y^*} \sim \frac{1-0}{\delta^*} = \frac{1}{\delta^*}$$

$y^* \to \delta^* \Rightarrow u^* \to 1$



$y^* \to 0 \Rightarrow u^* \to 0$

Note! The sign of terms is not important here, we are only interested in the **order of magnitude**

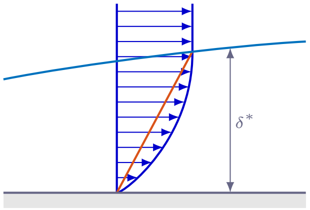
Boundary Layer Equations

What about derivatives?

$$\frac{\partial u^*}{\partial y^*} \sim \frac{1-0}{\delta^*} = \frac{1}{\delta^*}$$

$$\frac{\partial^2 u^*}{\partial y^{*2}} = \frac{\partial}{\partial y^*}\frac{\partial u^*}{\partial y^*} \sim \frac{|0-1/\delta^*|}{\delta^*} = \frac{1}{\delta^{*2}}$$

$$y^* \rightarrow \delta^* \Rightarrow u^* \rightarrow 1,\ \frac{\partial u^*}{\partial y^*} \rightarrow 0$$



$$y^* \rightarrow 0 \Rightarrow u^* \rightarrow 0,\ \frac{\partial u^*}{\partial y^*} \rightarrow \frac{1}{\delta^*}$$

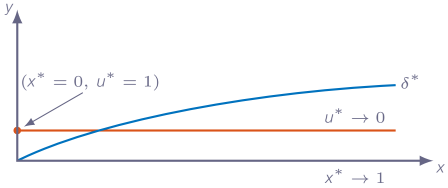
Note! The sign of terms is not important here, we are only interested in the **order of magnitude**

Boundary Layer Equations

$$\frac{\partial u^*}{\partial x^*} \sim \frac{|0-1|}{1-0} = 1$$

$x^* \to 0 \Rightarrow u^* \to 0$

$x^* \to 1 \Rightarrow u^* \to 1$

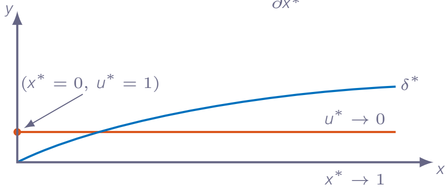


Note! The sign of terms is not important here, we are only interested in the **order of magnitude**

Boundary Layer Equations

$$x^* \to 0 \Rightarrow u^* \to 0,\ \frac{\partial u^*}{\partial x^*} \to 0$$

$$x^* \to 1 \Rightarrow u^* \to 1,\ \frac{\partial u^*}{\partial x^*} \to 1$$



$$\frac{\partial u^*}{\partial x^*} \sim \frac{|0-1|}{1-0} = 1$$

$$\frac{\partial^2 u^*}{\partial x^{*2}} = \frac{\partial}{\partial x^*}\frac{\partial u^*}{\partial x^*} \sim \frac{1-0}{1-0} = 1$$

Note! The sign of terms is not important here, we are only interested in the **order of magnitude**

Boundary Layer Equations

continuity:

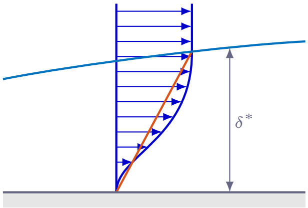
$$\underbrace{\frac{\partial u^*}{\partial x^*}}_{\sim\frac{1}{1}} + \underbrace{\frac{\partial v^*}{\partial y^*}}_{\sim\frac{?}{\delta^*}} = 0 \Rightarrow v^* \sim \delta^*$$

$\frac{\partial v^*}{\partial y^*}$ must be of the same order of magnitude as $\frac{\partial u^*}{\partial x^*}$ in order to fulfill the continuity equation

Boundary Layer Equations

$y^* \to \delta^* \Rightarrow v^* \to \delta^*$

$$\frac{\partial v^*}{\partial y^*} \sim \frac{\delta^*}{\delta^*} = 1$$



$y^* \to 0 \Rightarrow v^* \to 0$

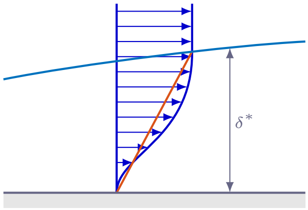
Note! The sign of terms is not important here, we are only interested in the **order of magnitude**

Boundary Layer Equations

$$\frac{\partial v^*}{\partial y^*} \sim \frac{\delta^*}{\delta^*} = 1$$

$$\frac{\partial^2 v^*}{\partial {y^*}^2} = \frac{\partial}{\partial y^*}\frac{\partial v^*}{\partial y^*} \sim \frac{|0-1|}{\delta^*} = \frac{1}{\delta^*}$$

$$y^* \rightarrow \delta^* \Rightarrow v^* \rightarrow \delta^*,\ \frac{\partial v^*}{\partial y^*} \rightarrow 0$$

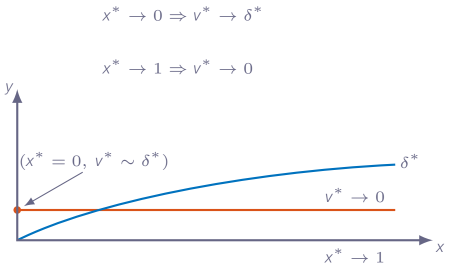


$$y^* \rightarrow 0 \Rightarrow v^* \rightarrow 0,\ \frac{\partial v^*}{\partial y^*} \rightarrow 1$$

Note! The sign of terms is not important here, we are only interested in the **order of magnitude**

Boundary Layer Equations

$$\frac{\partial v^*}{\partial x^*} \sim \frac{|0 - \delta^*|}{1 - 0} = \delta^*$$



Note! The sign of terms is not important here, we are only interested in the **order of magnitude**

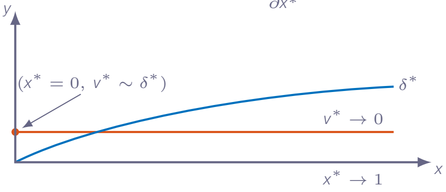
Boundary Layer Equations

$$\frac{\partial v^*}{\partial x^*} \sim \frac{|0 - \delta^*|}{1 - 0} = \delta^*$$

$$\frac{\partial^2 v^*}{\partial x^{*2}} = \frac{\partial}{\partial x^*}\frac{\partial v^*}{\partial x^*} \sim \frac{\delta^* - 0}{1 - 0} = \delta^*$$

$$x^* \to 0 \Rightarrow v^* \to \delta^*, \ \frac{\partial v^*}{\partial x^*} \to 0$$

$$x^* \to 1 \Rightarrow v^* \to 0, \ \frac{\partial v^*}{\partial x^*} \to \delta^*$$



Note! The sign of terms is not important here, we are only interested in the **order of magnitude**

Boundary Layer Equations

x-momentum:

$$\underbrace{u^* \frac{\partial u^*}{\partial x^*}}_{\sim 1 \times 1 = 1} + \underbrace{v^* \frac{\partial u^*}{\partial y^*}}_{\sim \delta^* \frac{1}{\delta^*} = 1} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{Re_L} \left(\underbrace{\frac{\partial^2 u^*}{\partial x^{*2}}}_{\sim 1} + \underbrace{\frac{\partial^2 u^*}{\partial y^{*2}}}_{\sim \frac{1}{\delta^{*2}}} \right)$$

the boundary layer is assumed to be very thin $\Rightarrow \delta^* \ll 1$ and thus

$$\frac{\partial^2 u^*}{\partial x^{*2}} \ll \frac{\partial^2 u^*}{\partial y^{*2}}$$

assuming the inertial forces to be of the same size as the friction forces in the boundary layer we get: $1/Re_L \sim \delta^{*2}$

Boundary Layer Equations

y-momentum:

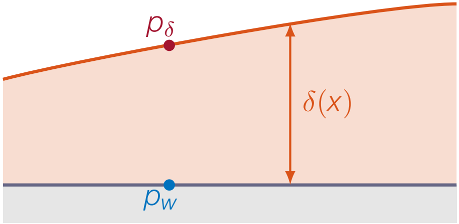
$$\underbrace{u^*\frac{\partial v^*}{\partial x^*}}_{\sim 1\times\delta^*=\delta^*} + \underbrace{v^*\frac{\partial v^*}{\partial y^*}}_{\sim\delta^*\frac{\delta^*}{\delta^*}=\delta^*} = -\frac{\partial p^*}{\partial y^*} + \underbrace{\frac{1}{Re_L}}_{\sim{\delta^*}^2}\left(\underbrace{\frac{\partial^2 v^*}{\partial {x^*}^2}}_{\sim\delta^*} + \underbrace{\frac{\partial^2 v^*}{\partial {y^*}^2}}_{\sim\frac{1}{\delta^*}}\right)$$

examining the equation we see that all terms are at most of size $\delta^* \Rightarrow \dfrac{\partial p^*}{\partial y^*} \sim \delta^*$

δ^* is small $\Rightarrow$ p is independent of y

Boundary Layer Equations

The pressure can be assumed to be constant in the vertical direction through the boundary layer and thus $p = p(x)$



$$|p_\delta^* - p_w^*| \approx \frac{\partial p^*}{\partial y^*}\delta^* \sim {\delta^*}^2$$

Boundary Layer Equations

With the knowledge gained, we now move back to the dimensional equations

laminar

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

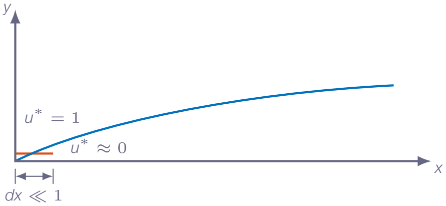
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$

turbulent

$$\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} = 0$$

$$\overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y} = -\frac{1}{\rho}\frac{d\overline{p}}{dx} + \nu\frac{\partial^2 \overline{u}}{\partial y^2} - \frac{\partial}{\partial y}\overline{u'v'}$$

Boundary Layer Equations



Limitations

1. The boundary layer equations **do not apply close to the start of the boundary layer** where $\dfrac{\partial u^*}{\partial x^*} \gg 1$
2. The equations are derived assuming a **thin boundary layer**

Boundary Layer Equations

The pressure derivative can be replaced with a velocity derivative

Outside of the boundary layer the flow is inviscid $\Rightarrow$ we can use the Bernoulli equation

$$p + \frac{1}{2}\rho U_\infty^2 = const \Rightarrow \frac{dp}{dx} + \rho U_\infty \frac{dU_\infty}{dx} = 0 \Rightarrow -\frac{1}{\rho}\frac{dp}{dx} = U_\infty \frac{dU_\infty}{dx}$$

Boundary Layer Equations

laminar boundary layer

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_\infty \frac{dU_\infty}{dx} + \nu \frac{\partial^2 u}{\partial y^2}$$

Two equations and two unknowns $\Rightarrow$ possible to solve ☺

Boundary Layer Equations

Note! the boundary layer equations can be used for curved surfaces if the boundary layer thickness δ is small compared to the curvature radius r