

# Fluid Mechanics - MTF053

## Lecture 2

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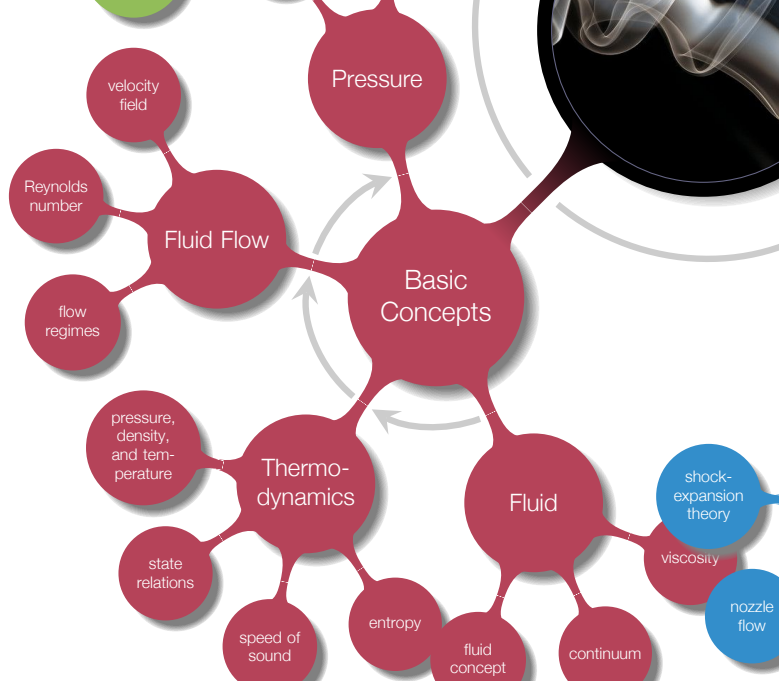
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## Chapter 1 - Introduction

# Overview

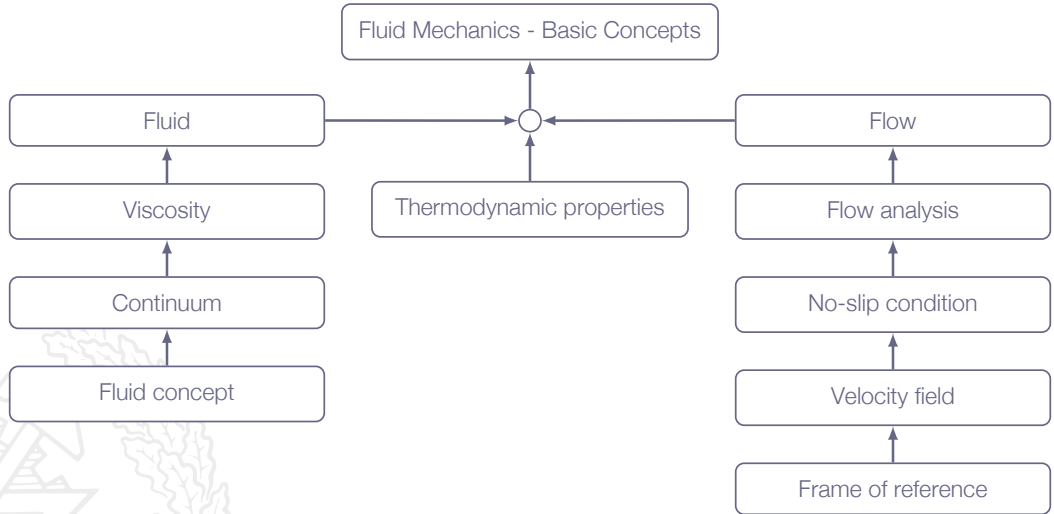


# Learning Outcomes

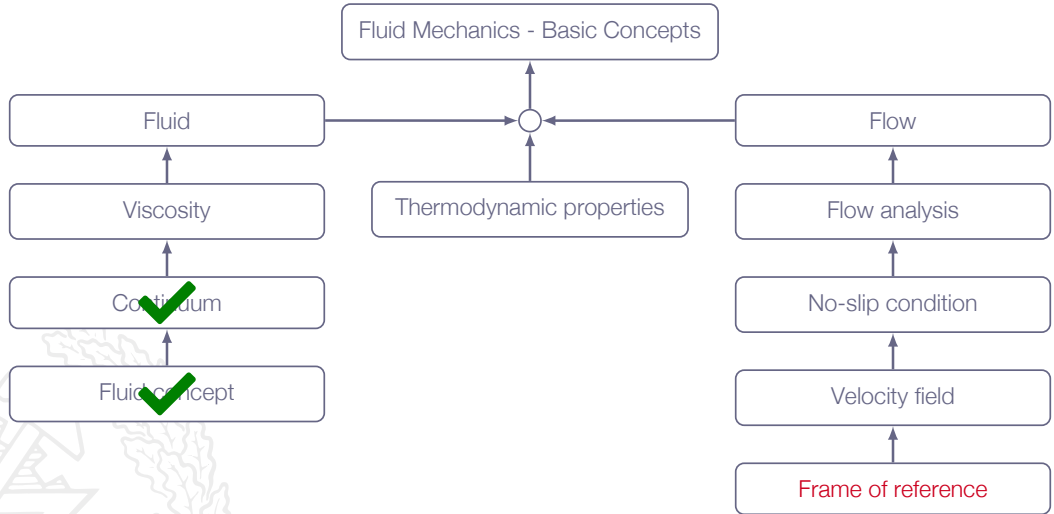
- 1 **Explain** the difference between a fluid and a solid in terms of forces and deformation
- 2 **Understand** and be able to explain the viscosity concept
- 3 **Define** the Reynolds number
- 5 **Explain** the difference between Lagrangian and Eulerian frame of reference and know when to use which approach
- 7 **Explain** the concepts: streamline, pathline and streakline
- 8 **Understand** and be able to **explain** the concept shear stress
- 9 **Explain** how to do a force balance for fluid element (forces and pressure gradients)
- 10 **Understand and explain** buoyancy and cavitation
- 16 **Understand** and **explain** the concept Newtonian fluid

*in this lecture we will find out what a fluid flow is*

# Roadmap - Introduction to Fluid Mechanics



# Roadmap - Introduction to Fluid Mechanics



# Frame of Reference



## Eulerian Frame of Reference

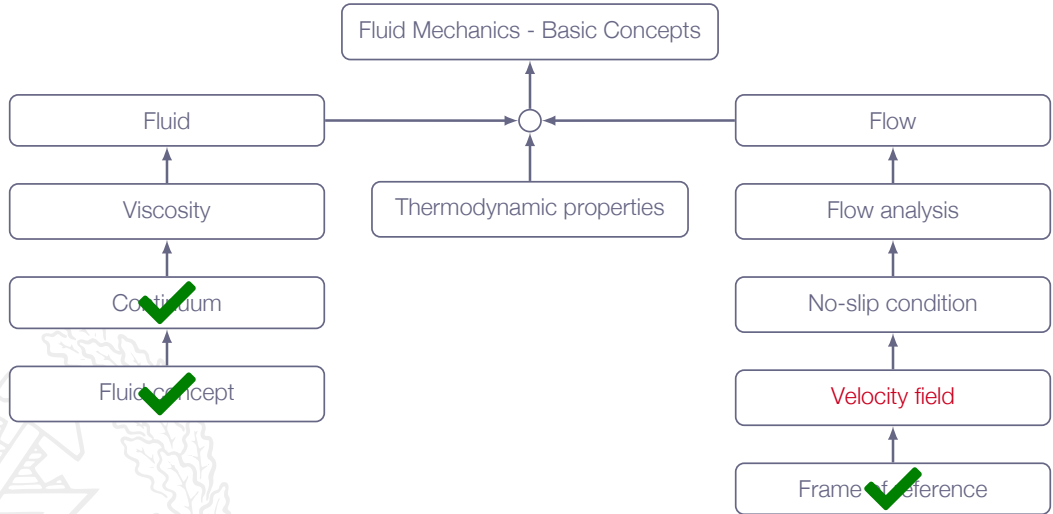
fluid properties as function of position and time



## Lagrangian Frame of Reference

follows a system in time and space

# Roadmap - Introduction to Fluid Mechanics





# Properties of the Velocity Field

The fluid velocity is a function of position and time

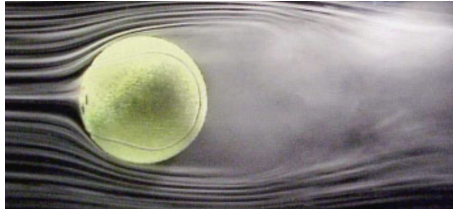
Three components  $u$ ,  $v$ , and  $w$  (one in each spatial direction)

$$\mathbf{V}(x, y, z, t) = u(x, y, z, t)\mathbf{e}_x + v(x, y, z, t)\mathbf{e}_y + w(x, y, z, t)\mathbf{e}_z$$



# Properties of the Velocity Field

Acceleration:

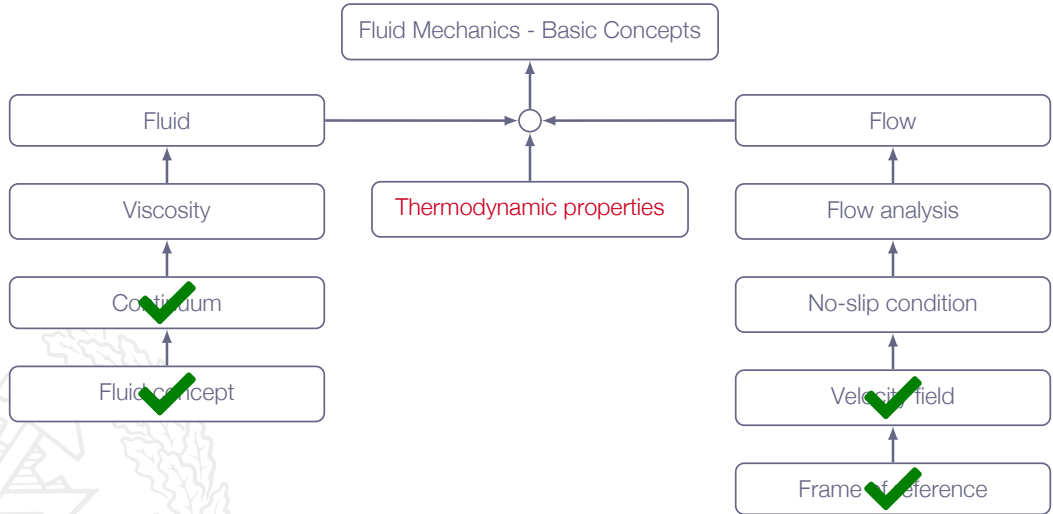


$$\mathbf{V}(x, y, z, t) = u(x, y, z, t)\mathbf{e}_x + v(x, y, z, t)\mathbf{e}_y + w(x, y, z, t)\mathbf{e}_z$$

$$\mathbf{a} = \frac{d\mathbf{V}}{dt} = \frac{\partial \mathbf{V}}{\partial t} + \left( \frac{\partial \mathbf{V}}{\partial x} \right) \left( \frac{\partial x}{\partial t} \right) + \left( \frac{\partial \mathbf{V}}{\partial y} \right) \left( \frac{\partial y}{\partial t} \right) + \left( \frac{\partial \mathbf{V}}{\partial z} \right) \left( \frac{\partial z}{\partial t} \right)$$

$$\frac{d\mathbf{V}}{dt} = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + v \frac{\partial \mathbf{V}}{\partial y} + w \frac{\partial \mathbf{V}}{\partial z}$$

# Roadmap - Introduction to Fluid Mechanics



# Thermodynamic Properties

Thermodynamic properties describe the state of a system, i.e., a collection of matter of fixed identity which interacts with its surroundings

In this course, the system will be a small fluid element, and all properties will be assumed to be continuum properties of the flow field



# Thermodynamic Properties

Pressure:  $p$  (Pa)

Density:  $\rho$  ( $\text{kg}/\text{m}^3$ )

Temperature:  $T$  (K)

**most common properties**



# Thermodynamic Properties

Pressure:  $p$  (Pa)

Density:  $\rho$  (kg/m<sup>3</sup>)

Temperature:  $T$  (K)

**most common properties**

Internal energy:  $\hat{u}$  (J/kg)

Enthalpy:  $h = \hat{u} + p/\rho$  (J/kg)

Entropy:  $s$  (J/(kg K))

Specific heats:  $C_p$  and  $C_v$  (J/(kg K))

**work, heat, and energy balances**

# Thermodynamic Properties

Pressure:  $p$  (Pa)

Density:  $\rho$  (kg/m<sup>3</sup>)

Temperature:  $T$  (K)

**most common properties**

Internal energy:  $\hat{u}$  (J/kg)

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Entropy:  $s$  (J/(kg K))

Specific heats:  $C_p$  and  $C_v$  (J/(kg K))

**work, heat, and energy balances**

Viscosity:  $\mu$  (kg/(m s))

Thermal conductivity:  $k$  (W/(m K))

**friction and heat conduction**

# Thermodynamic Properties

For a single-phase substance, two basic properties are sufficient to get the values of all others

$$\rho = \rho(p, T), h = h(p, T), \mu = \mu(p, T)$$

In the following it will be assumed that all thermodynamic properties exists as point functions in a flowing fluid ( $\rho = \rho(x, y, z, t)$ )

large enough number of molecules  $\Rightarrow$  **continuum**

any changes in thermodynamic conditions are faster than the flow time scale  $\Rightarrow$  **equilibrium**



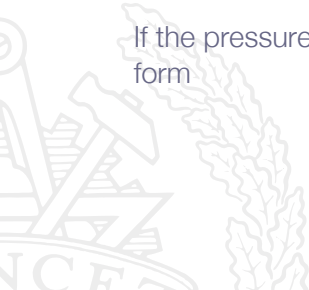
# Primary Thermodynamic Properties

## Pressure

The compression stress at a point in a static fluid

A fluid flow is often driven by pressure gradients

If the pressure drops below the vapor pressure in a liquid, vapor bubbles will form



# Primary Thermodynamic Properties

Temperature

Related to internal energy

Large temperature differences  $\Rightarrow$  heat transfer may be important



# Primary Thermodynamic Properties

## Density

Mass per unit volume

Nearly constant in liquids (incompressible) - for water, the density increases about one percent for a pressure increase by a factor of 220

Not constant for gases

$$\rho = \frac{p}{RT}$$

# Potential and Kinetic Energies

The total stored energy per unit mass:

$$e = \hat{u} + \frac{1}{2}V^2 + gz$$

the internal energy is a function of temperature

the potential and kinetic energies are kinematic quantities

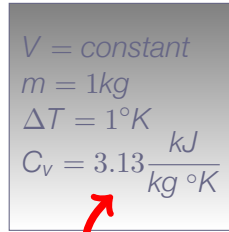
# State Relations for Gases

The perfect gas law:

$$p = \rho RT$$

where  $R$  is the gas constant

$$R = C_p - C_v$$

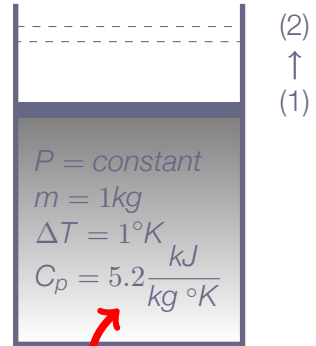


A diagram of a gas cylinder with a piston fixed in place, indicated by a dashed line at the top. The cylinder is shaded grey and contains the following text:

$$\begin{aligned} V &= \text{constant} \\ m &= 1\text{kg} \\ \Delta T &= 1^\circ\text{K} \\ C_v &= 3.13 \frac{\text{kJ}}{\text{kg } ^\circ\text{K}} \end{aligned}$$

A red arrow points from the bottom of the cylinder to the value 3.13 kJ.

3.13kJ



A diagram of a gas cylinder with a piston that can move, indicated by a dashed line at the top and an upward arrow. The cylinder is shaded grey and contains the following text:

$$\begin{aligned} P &= \text{constant} \\ m &= 1\text{kg} \\ \Delta T &= 1^\circ\text{K} \\ C_p &= 5.2 \frac{\text{kJ}}{\text{kg } ^\circ\text{K}} \end{aligned}$$

A red arrow points from the bottom of the cylinder to the value 5.2 kJ.

5.2kJ

# State Relations for Gases

The ideal gas law requires:  $\hat{u} = \hat{u}(T)$  and thus

specific heat (constant volume):

$$C_v = \left( \frac{\partial \hat{u}}{\partial T} \right)_\rho = \frac{d\hat{u}}{dT} = C_v(T)$$



# State Relations for Gases

specific heat (constant pressure):

$$h = \hat{u} + \frac{p}{\rho} = \hat{u} + RT = h(T)$$

$$C_p = \left( \frac{\partial h}{\partial T} \right)_p = \frac{dh}{dT} = C_p(T)$$

ratio of specific heats:

$$\gamma = \frac{C_p}{C_v} \geq 1$$



# State Relations for Gases

$$C_v = \frac{R}{\gamma - 1}$$

$$C_p = \frac{\gamma R}{\gamma - 1}$$





# Speed of Sound

Speed of sound plays an important role when compressible effects are important (Chapter 9)

$$a^2 = \left( \frac{\partial p}{\partial \rho} \right)_s$$

$$\tau_s = \frac{1}{\rho} \left( \frac{\partial \rho}{\partial p} \right)_s \Rightarrow a = \sqrt{\frac{1}{\rho \tau_s}}$$

where  $\tau_s$  is the fluid **compressibility**

for an ideal gas:

$$a = \sqrt{\gamma R T}$$

# Vapor Pressure

*"the pressure at which a liquid boils and is in equilibrium with its own vapor"*

Vapor pressure for water:

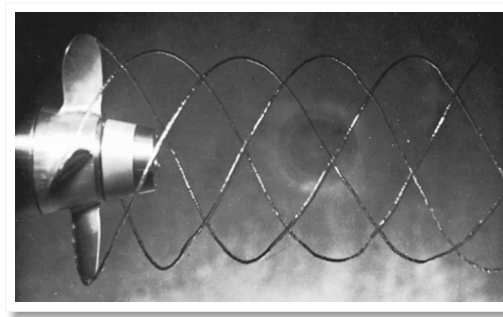
$T[^\circ\text{C}]$	vapor pressure [Pa]
20	2340
100	101300



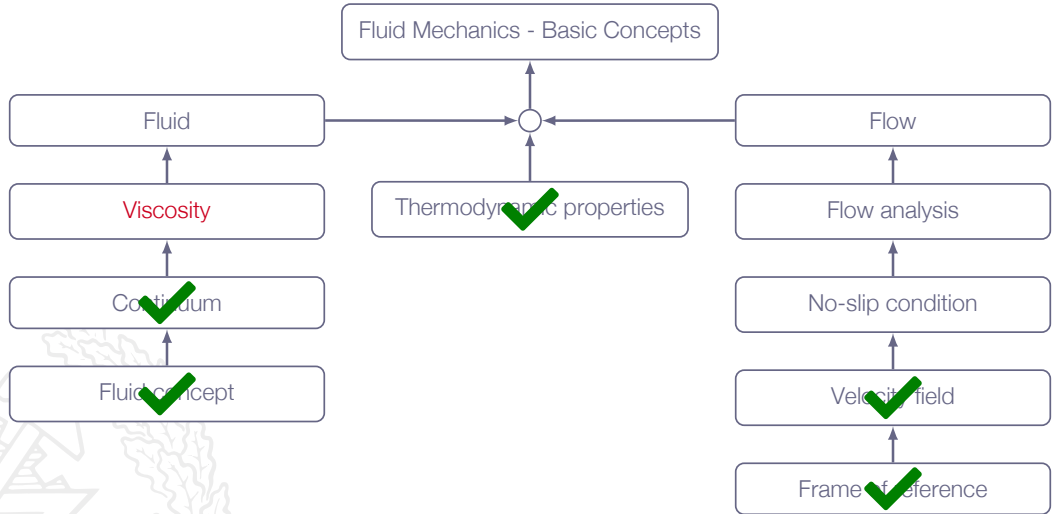
# Vapor Pressure

If the pressure in a liquid gets lower than the vapor pressure, vapor bubbles will appear in the liquid

If the pressure drops below the vapor pressure due to the flow itself we get cavitation



# Roadmap - Introduction to Fluid Mechanics



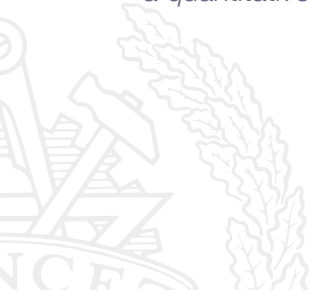


Viscosity

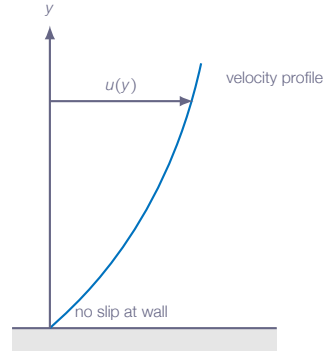
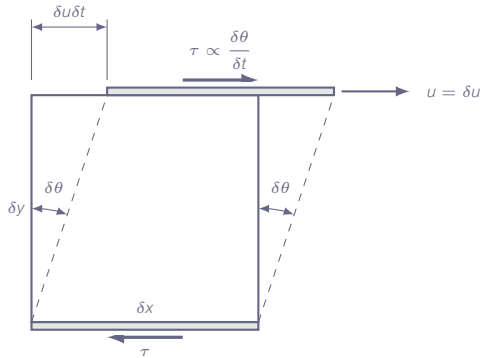
# Viscosity

*"relates the local stresses in a moving fluid to the strain rate of the fluid element"*

*"a quantitative measure of the fluid's resistance to flow"*



# Viscosity



$$\tau \propto \frac{\delta \theta}{\delta t}, \tan \delta \theta = \frac{\delta u \delta t}{\delta y}$$

# Viscosity

for infinitesimal changes:

$$\frac{d\theta}{dt} = \frac{du}{dy}$$

from before we know that  $\tau \propto \frac{\delta\theta}{\delta t}$  and thus  $\tau \propto \frac{d\theta}{dt}$

For Newtonian fluids:

$$\tau = \mu \frac{d\theta}{dt} = \mu \frac{du}{dy}$$

where  $\mu$  is the fluid viscosity



# Viscosity

Liquids have high viscosity that decreases with temperature  
intermolecular forces decreases with temperature

Gases have low viscosity that increases with temperature  
increased temperature means increased molecular movement



# Viscosity

Fluid	$\mu$ ( $\text{kg m}^{-1} \text{s}^{-1}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$\nu$ ( $\text{m}^2 \text{s}^{-1}$ )
Hydrogen	8.8 E-06	8.400 E-02	1.05 E-04
Air	1.8 E-05	1.200 E+00	1.51 E-05
Gasoline	2.9 E-04	6.800 E+02	4.22 E-07
Water	1.0 E-03	9.980 E+02	1.01 E-06
Mercury	1.5 E-03	1.358 E+04	1.16 E-07
SAE-30 Oil	2.9 E-02	8.910 E+02	3.25 E-04
Glycerin	1.5 E+00	1.264 E+03	1.18 E-03

**Note!** there are two different viscosities in the table (dynamic viscosity  $\mu$  and kinematic viscosity  $\nu = \mu/\rho$ )

# Viscosity

Inviscid flows: flows where viscous forces are negligible

Viscous flows: flows where viscous forces are important



# Reynolds number

$$Re = \frac{\rho V L}{\mu}$$

Non-dimensional number that relates viscous forces to inertial forces

Very important parameter in fluid mechanics

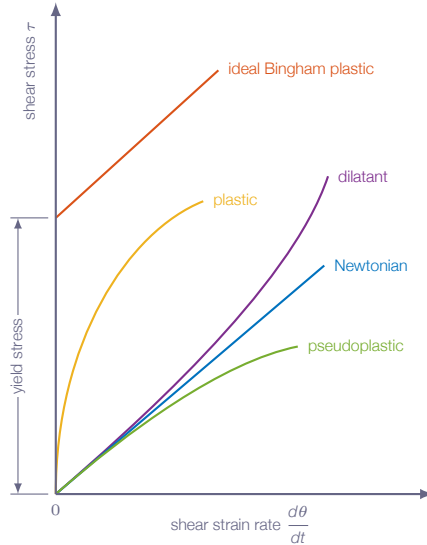
$V$  and  $L$  are characteristic velocity and length scales of the flow

# Reynolds number

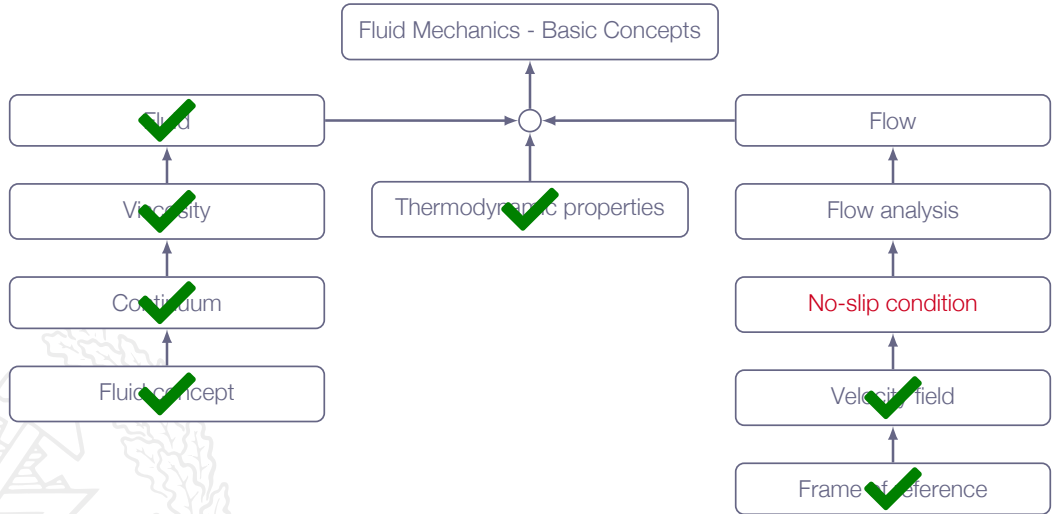
Reynolds number	flow description
low	viscous, creeping motion (inertial forces negligible)
moderate	laminar flow
high	turbulent flow



# Non-Newtonian Fluids



# Roadmap - Introduction to Fluid Mechanics



# No Slip/No Temperature Jump

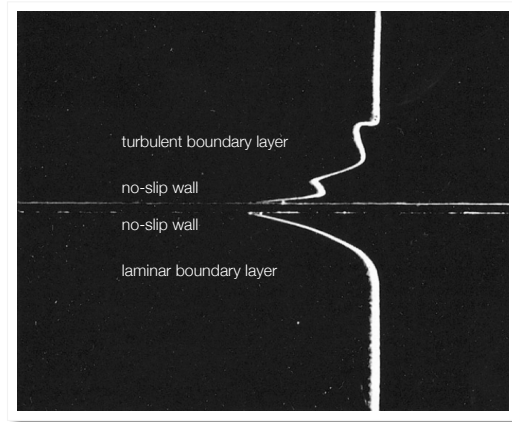
*"When a fluid flow is bounded by a solid surface, molecular interactions cause the fluid in contact with the surface to seek momentum and energy equilibrium with that surface"*



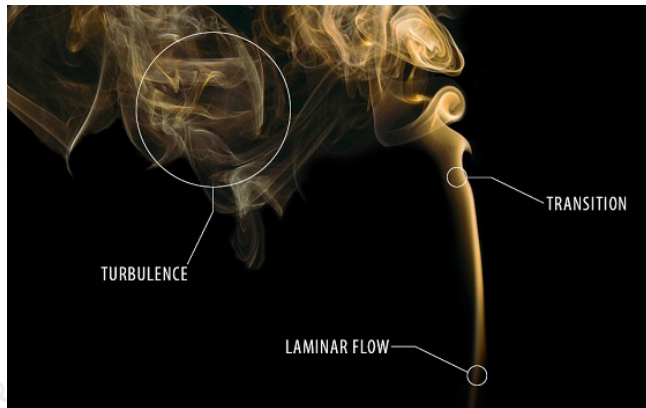


# No Slip/No Temperature Jump

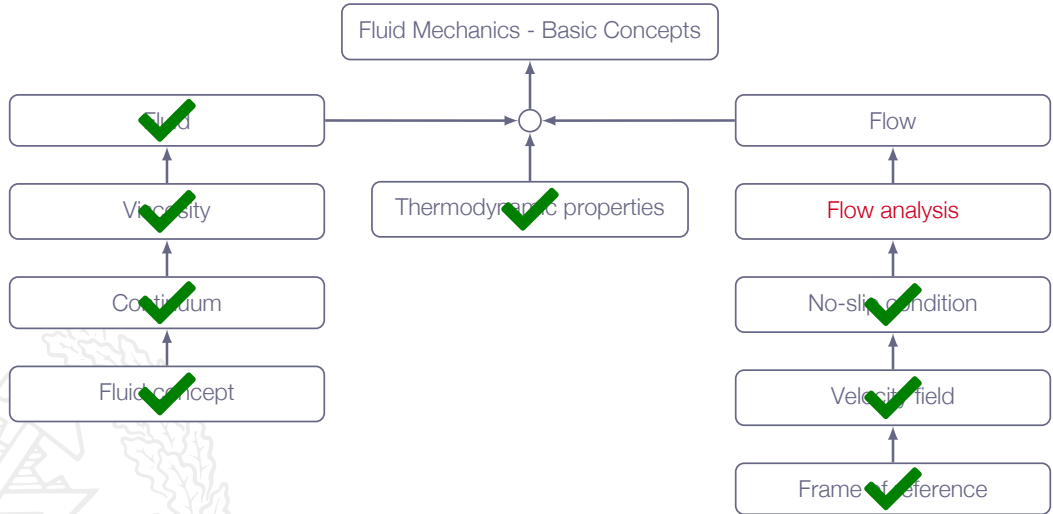
At a solid wall, the fluid will have the velocity and temperature of the wall



# Laminar/Turbulent Flow



# Roadmap - Introduction to Fluid Mechanics

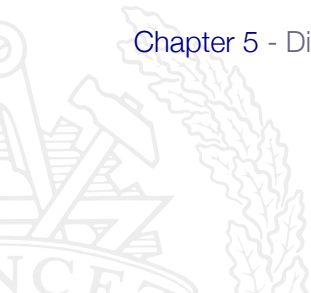


# Flow Analysis

Chapter 3 - Control-volume (integral) approach

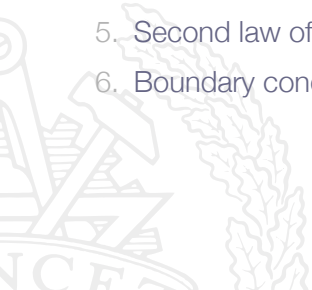
Chapter 4 - Infinitesimal system (differential) approach

Chapter 5 - Dimensional analysis approach



# Flow Analysis

1. Conservation of mass (continuity)
2. Conservation of momentum (Newton's second law)
3. Conservation of energy (first law of thermodynamics)
4. State relation (for example the ideal gas law)
5. Second law of thermodynamics
6. Boundary conditions



# Flow Visualization

## Streamline

a line that is tangent to the velocity vector everywhere at an instant in time

## Pathline

the actual path traversed by a fluid particle

## Streakline

the locus of particles that have earlier passed through a prescribed point

## Timeline

a line formed by a set of particles at a given instant



# Flow Visualization

## Streamline

a line that is tangent to the velocity vector everywhere at an instant in time

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the actual path traversed by a fluid particle

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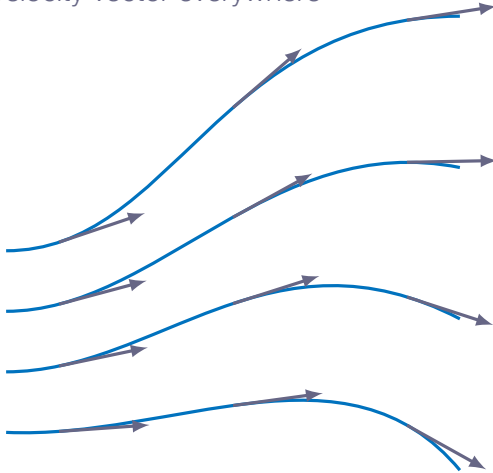
## Timeline

a line formed by a set of particles at a given instant

**Note!** In a steady-state flow, streamlines, pathlines and streaklines are identical

# Streamline

Tangent to flow velocity vector everywhere

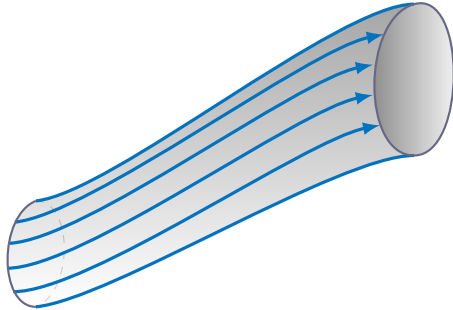




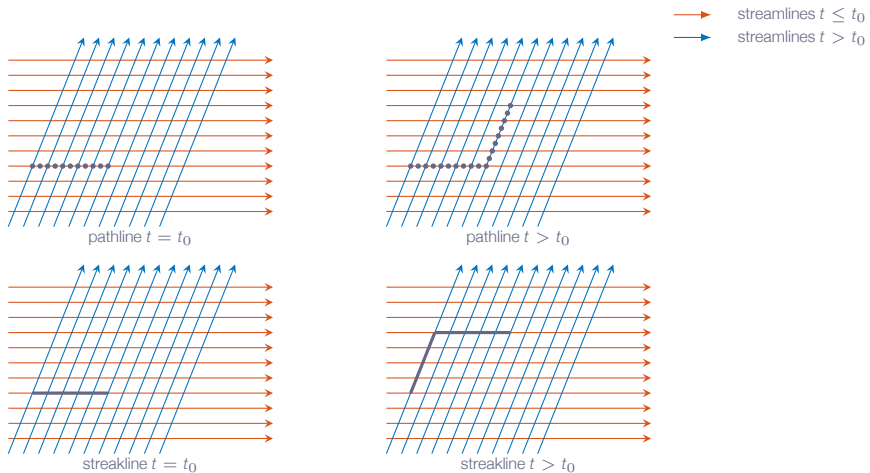
# Streamtube

"Constructed" from individual streamlines

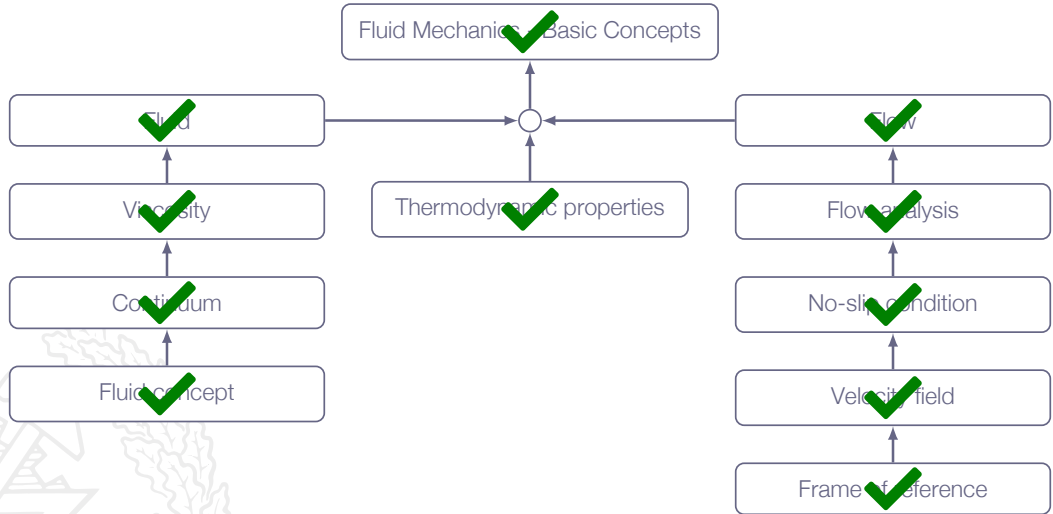
No flow across streamtube "walls" (by definition)



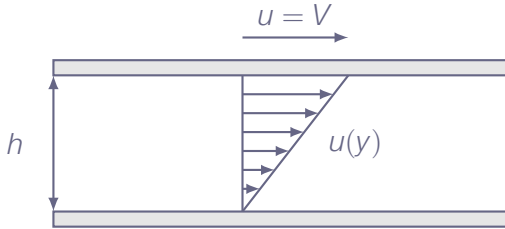
# Pathline vs Streakline



# Roadmap - Introduction to Fluid Mechanics



## Example - Flow Between Plates

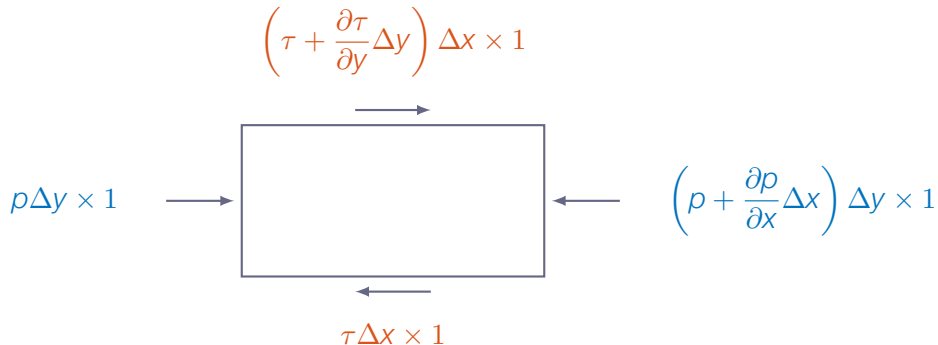


No acceleration

No pressure gradients

two-dimensional flow

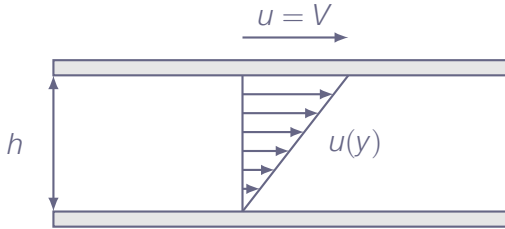
## Example - Flow Between Plates



$$\sum F_x = p \Delta y - \left(p + \frac{\partial p}{\partial x} \Delta x\right) \Delta y + \left(\tau + \frac{\partial \tau}{\partial y} \Delta y\right) \Delta x - \tau \Delta x = 0$$

$$\frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x} = 0 \Rightarrow \tau = \text{const}$$

## Example - Flow Between Plates



$$\frac{du}{dy} = \frac{\tau}{\mu} = \text{const}$$

$$u = a + by$$

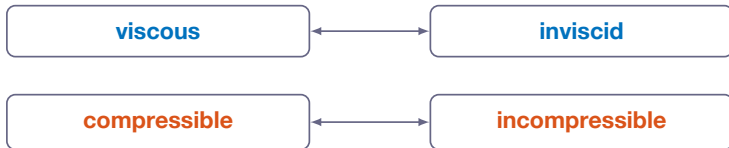
$$\begin{cases} y = 0 \Rightarrow u = 0 \\ y = h \Rightarrow u = V \end{cases}$$

$$u = \frac{V}{h}y$$

# Flow Categories

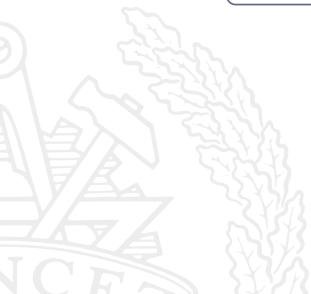
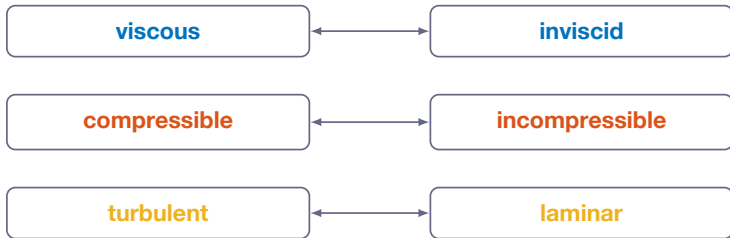


# Flow Categories

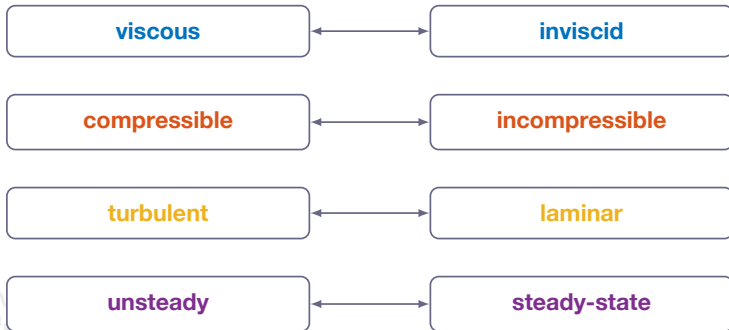




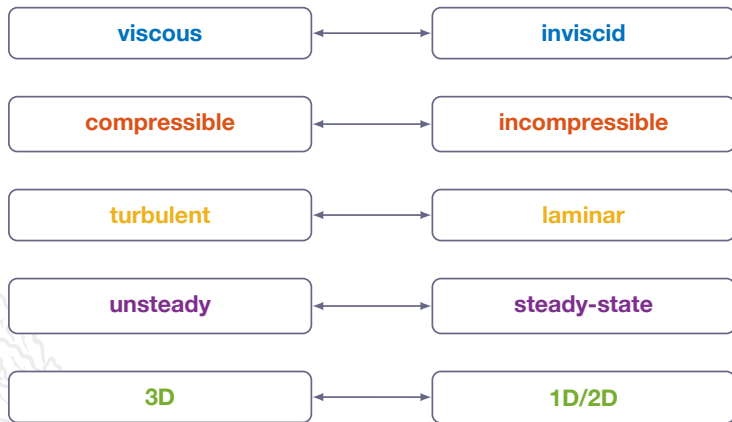
# Flow Categories



# Flow Categories



# Flow Categories



# Clouds

