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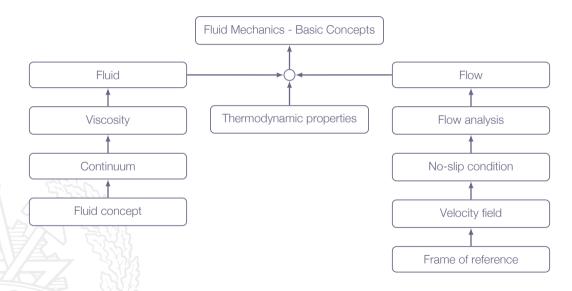
#### Overview



### Learning Outcomes

- 1 Explain the difference between a fluid and a solid in terms of forces and deformation
- 2 **Understand** and be able to explain the viscosity concept
- 3 **Define** the Reynolds number
- 5 **Explain** the difference between Lagrangian and Eulerian frame of reference and know when to use which approach
- 7 **Explain** the concepts: streamline, pathline and streakline
- 8 Understand and be able to explain the concept shear stress
- 9 **Explain** how to do a force balance for fluid element (forces and pressure gradients)
- 10 Understand and explain buoyancy and cavitation
- 16 Understand and explain the concept Newtonian fluid

in this lecture we will find out what a fluid flow is



#### Fluid Mechanics

"Fluid mechanics is the branch of physics concerned with the mechanics of fluids (liquids, gases, and plasmas) and the forces on them. It has applications in a wide range of disciplines, including mechanical, civil, chemical and biomedical engineering, geophysics, oceanography, meteorology, astrophysics, and biology."

Wikipedia

## Fluid Flows in Your Daily Life

"When you think about it, almost everything on this planet either is a fluid or moves within or near a fluid"

Frank M. White























# Governing Equations

#### Conservation of mass (continuity)

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

Conservation of linear momentum (Newton's 2:nd law)

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \rho g_x - \frac{\partial \rho}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

$$\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = \rho g_y - \frac{\partial \rho}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)$$

$$\rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = \rho g_z - \frac{\partial \rho}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

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Conservation of energy (1:st law of thermodynamics)

$$\left[\rho C_{v}\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right) = \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + k\left(\frac{\partial^{2} T}{\partial x^{2}} + \frac{\partial^{2} T}{\partial y^{2}} + \frac{\partial^{2} T}{\partial z^{2}}\right) + \Phi\right]$$

#### Fluid Flow Applications

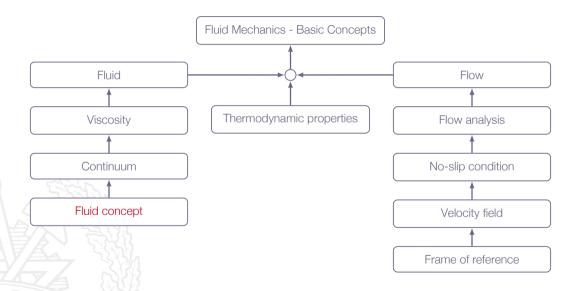
Analytical solutions limited to very specific simplified cases

Complex geometries and flows leads to the need for experiments and Computational Fluid Dynamics (CFD)

Chief obstacles to a general theory:

Geometry Viscosity Non-linearity Turbulence

Understanding the basic principles is a key factor for a correct analysis

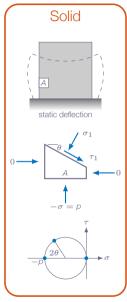


# The Concept of a Fluid

"In physics, a fluid is a substance that continually deforms (flows) under an applied shear stress, or external force. Fluids are a phase of matter and include liquids, gases and plasmas. They are substances with zero shear modulus, or, in simpler terms, substances which cannot resist any shear force applied to them."

Wikipedia

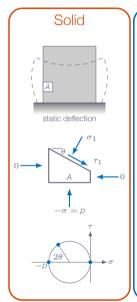
# The Concept of a Fluid

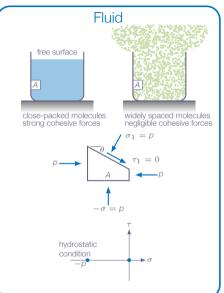


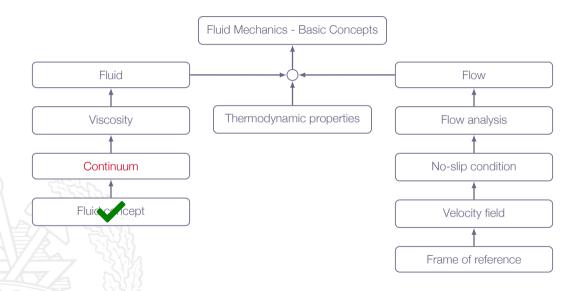
### The Concept of a Fluid

"A solid can resist a shear stress by a static deflection; a fluid cannot"







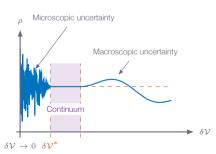


#### The Fluid as a Continuum

- Fluid **density** is essentially a **point function**
- Fluid properties can be thought of as varying continually in space
- Volume large enough such that the **number of molecules** within the volume is **constant**
- Volume small enough **not** to **introduce macroscopic fluctuations**

$$\rho = \lim_{\delta \mathcal{V} \to \delta \mathcal{V}^*} \frac{\delta m}{\delta \mathcal{V}}$$

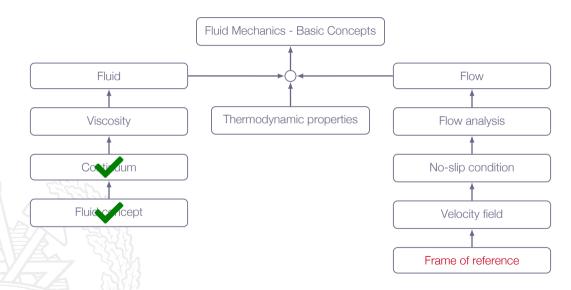
standard air:  $\delta \mathcal{V}^* \approx 10^{-9} \text{mm}^3 \Rightarrow \sim 3 \times 10^7 \text{ molecules}$ 



#### The Fluid as a Continuum

Flow properties varies smoothly ⇒ Differential calculus can be used

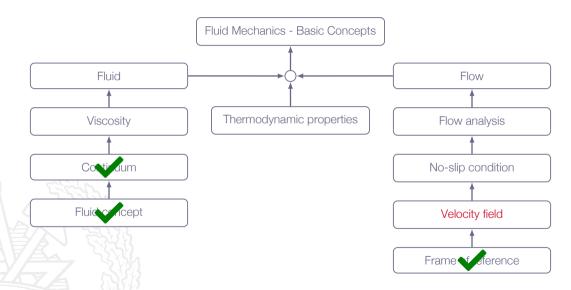




#### Frame of Reference



Eulerian Frame of Reference fluid properties as function of position and time Lagrangian Frame of Reference follows a system in time and space



# Properties of the Velocity Field

The fluid velocity is a function of position and time

Three components u, v, and w (one in each spatial direction)

$$\mathbf{V}(x,y,z,t) = \mathbf{u}(x,y,z,t)\mathbf{e}_x + \mathbf{v}(x,y,z,t)\mathbf{e}_y + \mathbf{w}(x,y,z,t)\mathbf{e}_z$$

# Properties of the Velocity Field

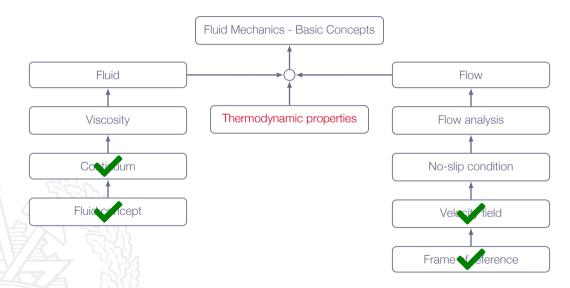


Acceleration:

$$\mathbf{V}(x,y,z,t) = \mathbf{u}(x,y,z,t)\mathbf{e}_x + \mathbf{v}(x,y,z,t)\mathbf{e}_y + \mathbf{w}(x,y,z,t)\mathbf{e}_z$$

$$\mathbf{a} = \frac{d\mathbf{V}}{dt} = \frac{\partial \mathbf{V}}{\partial t} + \left(\frac{\partial \mathbf{V}}{\partial x}\right) \left(\frac{\partial x}{\partial t}\right) + \left(\frac{\partial \mathbf{V}}{\partial y}\right) \left(\frac{\partial y}{\partial t}\right) + \left(\frac{\partial \mathbf{V}}{\partial z}\right) \left(\frac{\partial z}{\partial t}\right)$$

$$\frac{d\mathbf{V}}{dt} = \frac{\partial \mathbf{V}}{\partial t} + u \frac{\partial \mathbf{V}}{\partial x} + \mathbf{v} \frac{\partial \mathbf{V}}{\partial y} + w \frac{\partial \mathbf{V}}{\partial z}$$



## Thermodynamic Properties

Thermodynamic properties describe the state of a system, i.e., a collection of matter of fixed identity which interacts with its surroundings

In this course, the system will be a small fluid element, and all properties will be assumed to be continuum properties of the flow field

# Thermodynamic Properties

Pressure: p (Pa)

Density:  $\rho$  ( $kg/m^3$ )

Temperature: T (K)

most common properties

### Thermodynamic Properties

Pressure: p (Pa)

Density:  $\rho$  ( $kg/m^3$ )

Temperature: T (K)

most common properties

Internal energy:  $\hat{u}$  (J/kg)

Enthalpy:  $h = \hat{u} + p/\rho (J/kg)$ 

Entropy: s(J/(kg K))

Specific heats:  $C_D$  and  $C_V$  (J/(kg K))

work, heat, and energy balances

### Thermodynamic Properties

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Specific heats:  $C_D$  and  $C_V$  (J/(kg K))

work, heat, and energy balances

Viscosity:  $\mu (kg/(m s))$ 

Thermal conductivity: k (W/(m K))

friction and heat conduction

## Thermodynamic Properties

For a single-phase substance, two basic properties are sufficient to get the values of all others

$$\rho = \rho(p, T), h = h(p, T), \mu = \mu(p, T)$$

In the following it will be assumed that all thermodynamic properties exists as point functions in a flowing fluid ( $\rho = \rho(x, y, z, t)$ )

large enough number of molecules ⇒ continuum

any changes in thermodynamic conditions are faster than the flow time scale  $\Rightarrow$  **equilibrium** 

## Primary Thermodynamic Properties

#### Pressure

The compression stress at a point in a static fluid

A fluid flow is often driven by pressure gradients

If the pressure drops below the vapor pressure in a liquid, vapor bubbles will form

## Primary Thermodynamic Properties

#### Temperature

Related to internal energy

Large temperature differences ⇒ heat transfer may be important

## Primary Thermodynamic Properties

#### Density

Mass per unit volume

Nearly constant in liquids (incompressible) - for water, the density increases about one percent for a pressure increase by a factor of 220

Not constant for gases

$$\rho = \frac{p}{RT}$$

# Potential and Kinetic Energies

The total stored energy per unit mass:

$$e = \hat{u} + \frac{1}{2}V^2 + gz$$

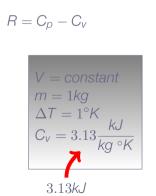
the internal energy is a function of temperature

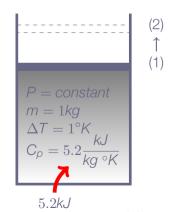
the potential and kinetic energies are kinematic quantities

The perfect gas law:

$$p = \rho RT$$

where R is the gas constant





The ideal gas law requires:  $\hat{u} = \hat{u}(T)$  and thus

specific heat (constant volume):

$$C_{V} = \left(\frac{\partial \hat{u}}{\partial T}\right)_{
ho} = \frac{d\hat{u}}{dT} = C_{V}(T)$$

specific heat (constant pressure):

$$h = \hat{u} + \frac{\rho}{\rho} = \hat{u} + RT = h(T)$$

$$C_{p} = \left(\frac{\partial h}{\partial T}\right)_{p} = \frac{dh}{dT} = C_{p}(T)$$

ratio of specific heats:

$$\gamma = \frac{C_p}{C_v} \ge 1$$

$$C_{v} = \frac{R}{\gamma - 1}$$

$$C_{p} = \frac{\gamma R}{\gamma - 1}$$

$$C_{p} = \frac{\gamma R}{\gamma - 1}$$



# Speed of Sound

Speed of sound plays an important role when compressible effects are important (Chapter 9)

$$a^{2} = \left(\frac{\partial \rho}{\partial \rho}\right)_{s}$$

$$\tau_{s} = \frac{1}{\rho} \left(\frac{\partial \rho}{\partial \rho}\right)_{s} \Rightarrow a = \sqrt{\frac{1}{\rho \tau_{s}}}$$

where  $\tau_s$  is the fluid **compressibility** 

for an ideal gas:

$$a = \sqrt{\gamma RT}$$

# Vapor Pressure

"the pressure at which a liquid boils and is in equilibrium with its own vapor"

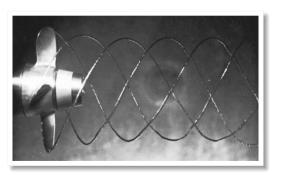
Vapor pressure for water:

$T[^{\circ}C]$	vapor pressure [Pa]
20	2340
100	101300

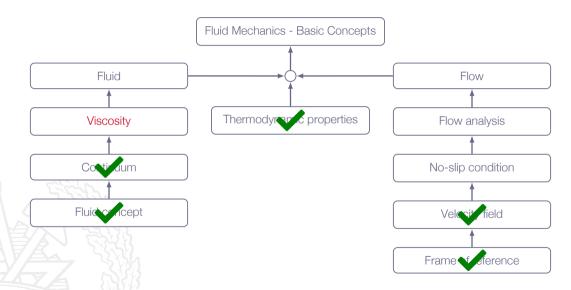
## Vapor Pressure

If the pressure in a liquid gets lower than the vapor pressure, vapor bubbles will appear in the liquid

If the pressure drops below the vapor pressure due to the flow itself we get cavitation



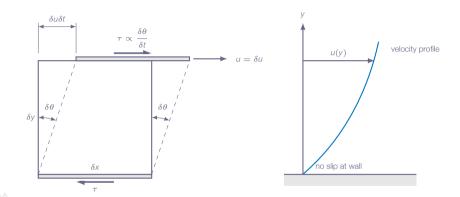
### Roadmap - Introduction to Fluid Mechanics





"relates the local stresses in a moving fluid to the strain rate of the fluid element"

"a quantitative measure of the fluid's resistance to flow"



$$au \propto rac{\delta heta}{\delta t}, \ an \delta heta = rac{\delta u \delta t}{\delta y}$$

for infinitesimal changes:

$$\frac{d\theta}{dt} = \frac{du}{dy}$$

from before we know that  $\tau \propto \frac{\delta \theta}{\delta t}$  and thus  $\tau \propto \frac{d\theta}{dt}$ 

For Newtonian fluids:

$$\tau = \mu \frac{d\theta}{dt} = \mu \frac{du}{dy}$$

where  $\mu$  is the fluid viscosity

Liquids have high viscosity that decreases with temperature intermolecular forces decreases with temperature

Gases have low viscosity that increases with temperature increased temperature means increased molecular movement

Fluid	$\mu \ (kg \ m^{-1} \ s^{-1})$	$\rho$ (kg m <sup>-3</sup> )	$\nu \ (m^2 \ s^{-1})$
Hydrogen	8.8 E-06	8.400 E-02	1.05 E-04
Air	1.8 E-05	1.200 E+00	1.51 E-05
Gasoline	2.9 E-04	6.800 E+02	4.22 E-07
Water	1.0 E-03	9.980 E+02	1.01 E-06
Mercury	1.5 E-03	1.358 E+04	1.16 E-07
SAE-30 Oil	2.9 E-02	8.910 E+02	3.25 E-04
Glycerin	1.5 E+00	1.264 E+03	1.18 E-03

**Note!** there are two different viscosities in the table (dynamic viscosity  $\mu$  and kinematic viscosity  $\nu = \mu/\rho$ )

Inviscid flows: flows where viscous forces are negligible

Viscous flows: flows where viscous forces are important

# Reynolds number

Re = 
$$\frac{\rho VL}{\mu}$$

Non-dimensional number that relates viscous forces to inertial forces

Very important parameter in fluid mechanics

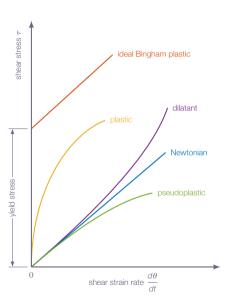
 $\ensuremath{V}$  and  $\ensuremath{L}$  are characteristic velocity and length scales of the flow

# Reynolds number

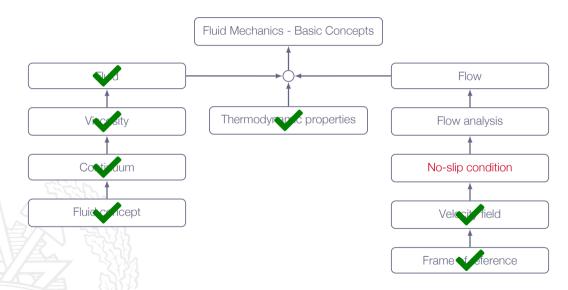
Reynolds number	flow description
low	viscous, creeping motion (inertial forces negligible)
moderate	laminar flow
high	turbulent flow

#### Non-Newtonian Fluids





### Roadmap - Introduction to Fluid Mechanics

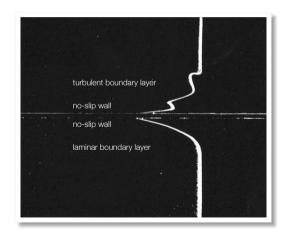


### No Slip/No Temperature Jump

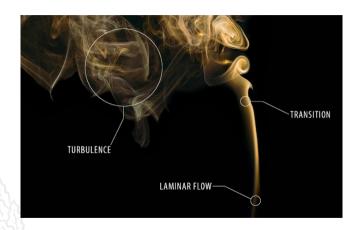
"When a fluid flow is bounded by a solid surface, molecular interactions cause the fluid in contact with the surface to seek momentum and energy equilibrium with that surface"

## No Slip/No Temperature Jump

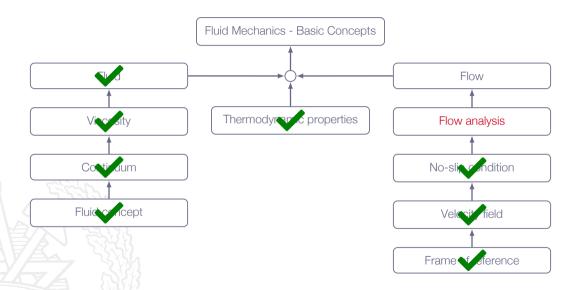
At a solid wall, the fluid will have the velocity and temperature of the wall



#### Laminar/Turbulent Flow



### Roadmap - Introduction to Fluid Mechanics



### Flow Analysis

Chapter 3 - Control-volume (integral) approach

Chapter 4 - Infinitesimal system (differential) approach

Chapter 5 - Dimensional analysis approach

# Flow Analysis

- 1. Conservation of mass (continuity)
- 2. Conservation of momentum (Newton's second law)
- 3. Conservation of energy (first law of thermodynamics)
- 4. State relation (for example the ideal gas law)
- 5. Second law of thermodynamics
- 6. Boundary conditions

#### Flow Visualization

#### Streamline

a line that is tangent to the velocity vector everywhere at an instant in time

#### **Pathline**

the actual path traversed by a fluid particle

#### Streakline

the locus of particles that have earlier passed through a prescribed point

#### **Timeline**

a line formed by a set of particles at a given instant

#### Flow Visualization

#### Streamline

a line that is tangent to the velocity vector everywhere at an instant in time

#### **Pathline**

the actual path traversed by a fluid particle

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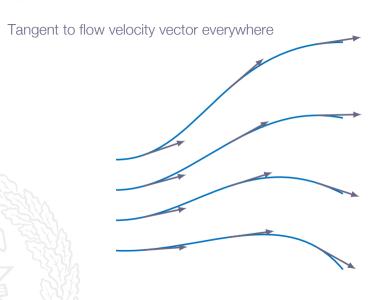
the locus of particles that have earlier passed through a prescribed point

#### **Timeline**

a line formed by a set of particles at a given instant

Note! In a steady-state flow, streamlines, pathlines and streaklines are identical

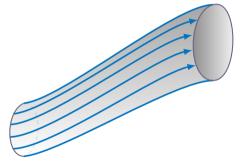
#### Streamline



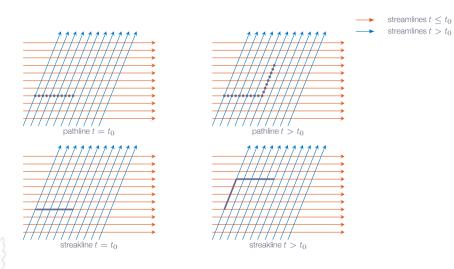
#### Streamtube

"Constructed" from individual streamlines

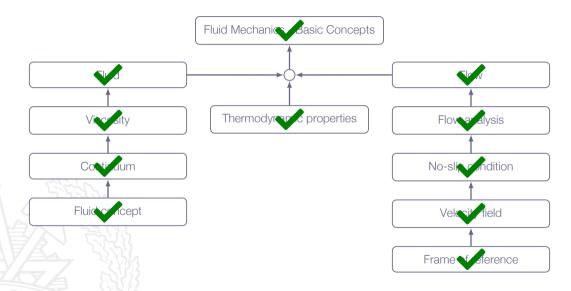
No flow across streamtube "walls" (by definition)



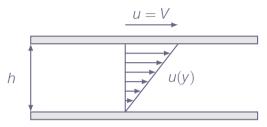
#### Pathline vs Streakline



### Roadmap - Introduction to Fluid Mechanics



## Example - Flow Between Plates



- No acceleration
- No pressure gradients
- two-dimensional flow

## Example - Flow Between Plates

$$\left(\tau + \frac{\partial \tau}{\partial y} \Delta y\right) \Delta x \times 1$$

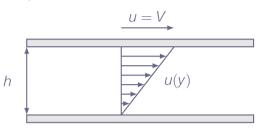
$$\rho \Delta y \times 1 \qquad \left(\rho + \frac{\partial \rho}{\partial x} \Delta x\right) \Delta y \times 1$$

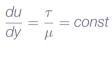
$$\tau \Delta x \times 1$$

$$\sum F_{x} = \rho \Delta y - \left(\rho + \frac{\partial \rho}{\partial x} \Delta x\right) \Delta y + \left(\tau + \frac{\partial \tau}{\partial y} \Delta y\right) \Delta x - \tau \Delta x = 0$$

$$\frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x} = 0 \Rightarrow \tau = const$$

## Example - Flow Between Plates





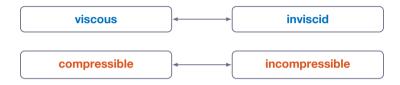
$$u = a + by$$

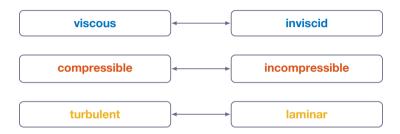
$$\begin{cases} y = 0 \Rightarrow u = 0 \\ y = h \Rightarrow u = V \end{cases}$$

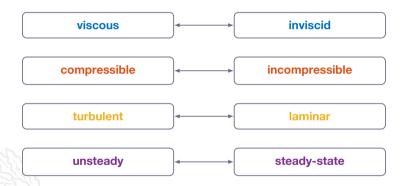
$$u = \frac{V}{h}y$$

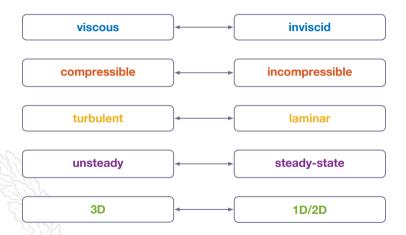












#### Clouds



YEAH, IT SEEMS LIKE A WASTE, THE UNIVERSE GETTING THE COMPLEX FLUID DYNAMICS RIGHT FOR EVERY MOMENTARY SUIRL OF CLOUD.

JUST A HUGE AMOUNT OF WORK.



MAYBE ATMOSPHERES JUST
HAVE SMOOTH LAMINAR FLOU
UNTIL SOMEONE LOOKS CLOSELY.
OR MAYBE THE UNIVERSE
JUST LIKES MAKING SUIRLY
CLOUDS, AND IS ANNOYED
THAT WE'RE WATCHING.

