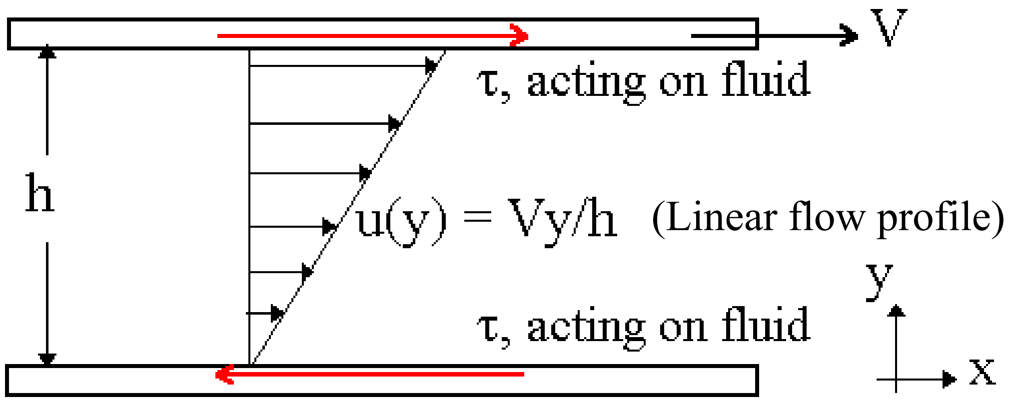
Chapter 1. (E01)

- Shear stress τ in a fluid (motion in *x* only)

$$\tau = \mu \frac{du}{dy} \qquad (1.23)$$

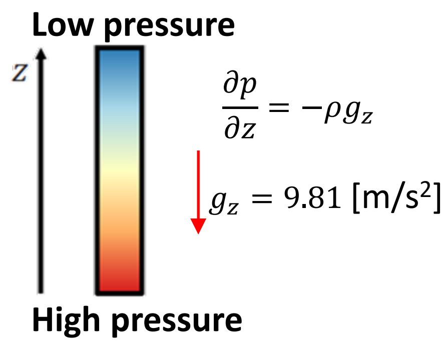
Fluid Viscosity (μ) — Velocity gradient in *y*-direction ($\frac{du}{dy}$)



- Total shear force on a surface:
  - $F = \int \tau \, \mathrm{d}A = \tau A$ (force = pressure x area)
- Re-number: $\mathrm{Re} = \frac{\rho U L}{\mu} = \frac{UL}{\nu} = \frac{\text{Inertia}}{\text{Viscous}}$ (1.24)
  - Dynamic viscosity: μ [kg/(m s)]
  - Kinematic viscosity: $\nu = \frac{\mu}{\rho}$ [m^2/s]

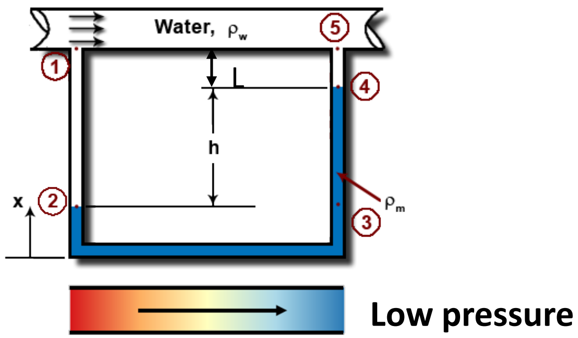
Chapter 2. (E02)

- Hydrostatic = fluid at rest
  - No viscous shear stress ($\tilde{f}_{\text{visc}} = 0$), no acceleration ($\tilde{a} = 0$) and gravity in *z* ($\tilde{g} = -g_z$)
  - $\sum \tilde{f} = 0 = \tilde{f}_{\text{press}} + \tilde{f}_{\text{grav}} \rightarrow \frac{\partial p}{\partial z} = -\rho g_z$ Eq. (2.11)
  - Integration gives: $\Delta p = \int_1^2 -\rho g_z \, \mathrm{dz} \rightarrow \Delta p = -\rho g_z \Delta z$ Eq. (2.14)
  - Pressure increase downward!



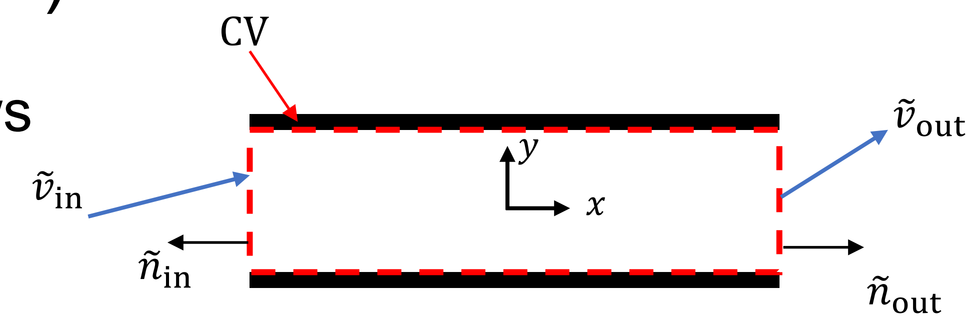
- Manometer:
  - Go (1) → (5) through the manometer (hydrostatic)
  - Measure friction (viscous) losses between (1) and (5)

$$p_1 - p_5 = -\rho_{\text{w}} g(z_1 - z_2) - \rho_{\text{m}} g(z_2 - z_4) - \rho_{\text{w}} g(z_4 - z_5)$$
$$\Delta \text{p} = -\rho_{\text{w}} g(L + h) - \rho_{\text{m}} g(-h) - \rho_{\text{w}} g(-L) \Rightarrow$$
$$\Delta \text{p} = (\rho_{\text{m}} - \rho_{\text{w}}) g h = \Delta p_{\text{visc}}$$



- Archimedes: $F_{\text{B}} = \rho_{\text{fluid}} \, g \, V$ Eq. (2.35)

Chapter 3. (E03-E05)



- Reynolds Transport Theorem, CV conservation laws
- Conservation of **mass**: $B = m$, $\beta = \frac{\mathrm{d}m}{\mathrm{d}m} = 1$
  $\frac{\mathrm{d}}{\mathrm{d}t}(m) = 0 = \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{\text{CV}} \rho \, \mathrm{d}V\right) + \int_{\text{CS}} \rho(\tilde{v}_r \circ \tilde{n})\mathrm{d}A$ Eq. (3.20)
- Conservation of **linear momentum**: $B = (m\tilde{v})_{\text{syst}}$, $\beta = \frac{\mathrm{d}(m\tilde{v})_{\text{syst}}}{\mathrm{d}m} = \tilde{v}_{\text{syst}}$
  $\frac{\mathrm{d}}{\mathrm{d}t}(m\tilde{v})_{\text{syst}} = \sum \tilde{F} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{\text{CV}} \tilde{v}\rho \, \mathrm{d}V\right) + \int_{\text{CS}} \tilde{v}\rho(\tilde{v}_r \circ \tilde{n})\mathrm{d}A$ Eq. (3.35)
- Conservation of **angular momentum**: $B_{\text{syst}} = \widetilde{H}_0 = \int_{\text{syst}}(\tilde{r} \times \tilde{v})\mathrm{d}m$, $\beta_{\text{syst}} = \frac{\mathrm{d}\widetilde{H}_0}{\mathrm{d}m} = \tilde{r} \times \tilde{v}$
  $\left.\frac{\mathrm{d}\widetilde{H}_0}{\mathrm{d}t}\right|_{\text{syst}} = \sum \widetilde{M}_0 = \frac{\mathrm{d}}{\mathrm{d}t}\left[\int_{\text{CV}}(\tilde{r} \times \tilde{v})\rho \, \mathrm{d}V\right] + \int_{\text{CS}}(\tilde{r} \times \tilde{v})\rho(\tilde{v}_r \circ \tilde{n})\mathrm{d}A$ Eq. (3.56)
- Conservation of **energy:** $B_{\text{syst}} = E$, $\beta_{\text{syst}} = \frac{\mathrm{d}E}{\mathrm{d}m} = e$
  $\dot{Q} - \dot{W}_{\text{s}} - \dot{W}_{\text{v}} = \frac{\partial}{\partial t}\left(\int_{\text{CV}}\left(\hat{u} + \frac{1}{2}v^2 + gz\right)\rho \, \mathrm{d}V\right) + \int_{\text{CS}}\left(h + \frac{1}{2}v^2 + gz\right)\rho(\tilde{v}_r \circ \tilde{n})\mathrm{d}A$ Eq. (3.67)

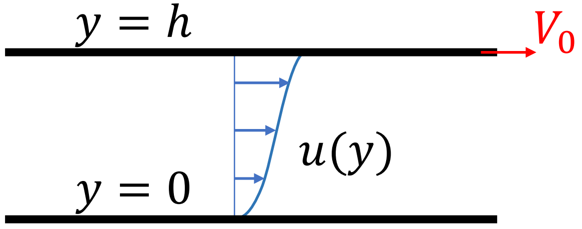
Chapter 4. (E06-E07)

- Differential form of conservation equations from Chapter 3
- Continuity: $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial z}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$ Eq. (4.4)
- Momentum (NS) in x-dir: $\rho \frac{\mathrm{d}u}{dt} = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \rho g_x$ Eq. (4.38)
- Energy: $\rho \frac{\mathrm{d}\hat{u}}{\mathrm{d}t} + p(\nabla \cdot \tilde{v}) = \nabla \cdot (k \nabla T) + \Phi$ Eq. (4.51)
- Boundary conditions:

No-slip

$u(h) = V_0$

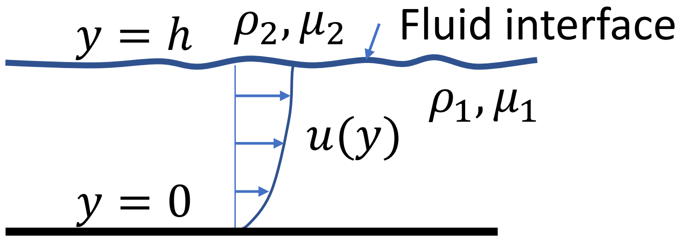
$u(0) = 0$



Slip (zero gradient) + **no-slip**

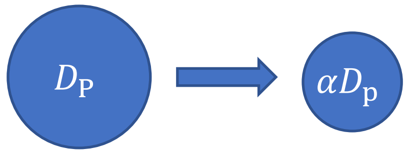
$\frac{\partial u(h)}{\partial y} = 0$

$u(0) = 0$

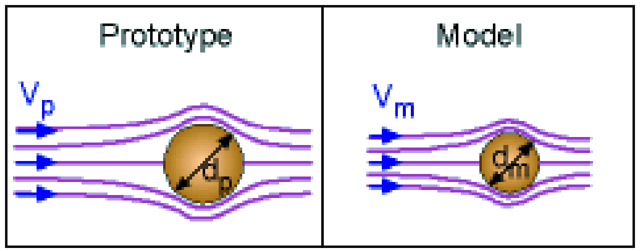


Chapter 5. (E08)

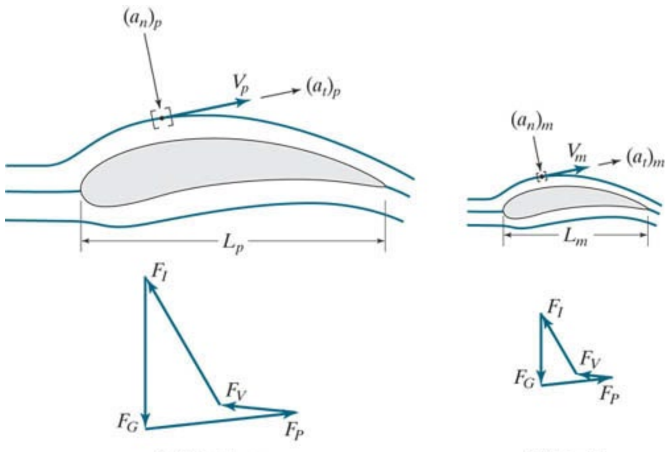
- **Geometric similarity**:
  - Dimensions scaled with one parameter
  - $\alpha L_{\text{Prototype}} = L_{\text{Model}}$



- **Kinematic similarity**:
  - Require geometric similarity
  - Flow field scaled with one coefficient, e.g. Re-number
  - Flow field the same between prototype and model
  - $\text{Re}_{\text{Prototype}} = \text{Re}_{\text{Model}}$



- **Dynamic similarity:**
  - Require both geometric and kinematic similarity
  - Force vectors are scaled
  - $C_{\text{D,Prototype}} = C_{\text{D,Model}}$



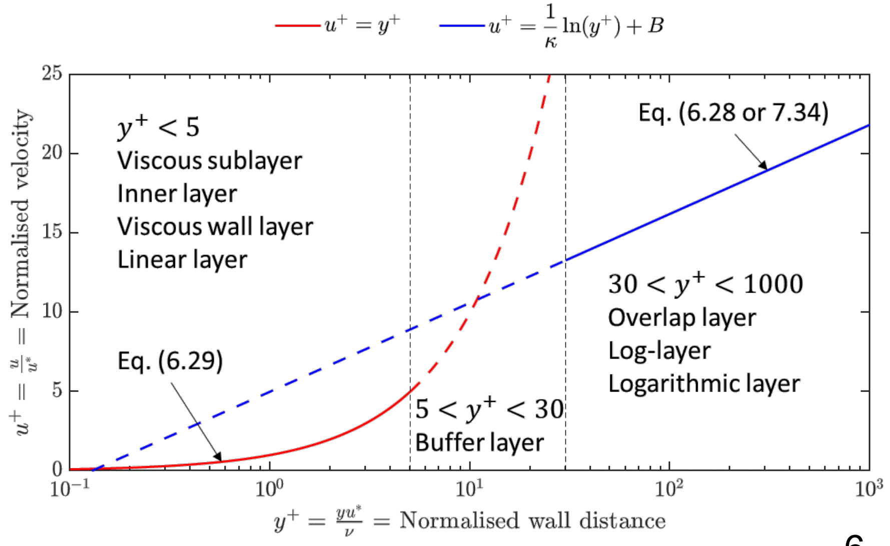
(*a*) Prototype (*b*) Model

Chapter 6. (E09-E10)

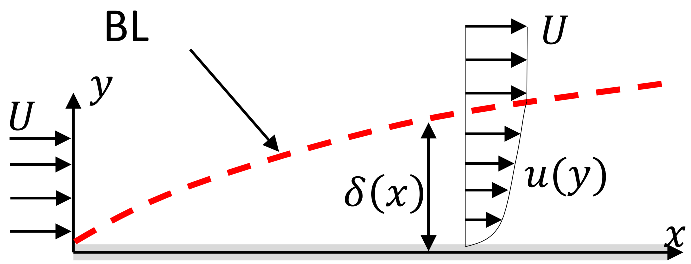
- Pressure drop in pipes:
  - Major losses, friction losses (viscous) = $\Delta p_\mathrm{f} = \rho g h_\mathrm{f}$
  - Minor losses, one-time losses in the flow path (engångsförluster) = $\Delta p_\mathrm{m} = \rho g h_\mathrm{m}$
- Bernoulli's Equation with head losses:
  - Follow the streamline from 1 → 2

    $p_1 + \frac{\rho V_1^2}{2} + \rho g z_1 = p_2 + \frac{\rho V_2^2}{2} + \rho g z_2 + \Delta p_\mathrm{f} + \Delta p_\mathrm{m}$ Eq. (3.73) (no pump or turbine)
- Wall Shear stress in the fluid (near the wall)

  $\tau_\mathrm{w} = \mu \frac{\partial \bar{u}}{\partial y} - \rho \overline{u'v'} = \tau_\mathrm{lam} + \tau_\mathrm{turb}$ Eq. (6.23)
  - $y^+ < 5$ (Sublayer): $u^+ = y^+$ Eq. (6.29)
  - $5 < y^+ < 30$ (Buffer): No exact model
  - $30 < y^+ < 1000$ (Log): $u^+ = \frac{1}{\kappa} \ln y^+ + B$ Eq. (6.28)



Chapter 7. (E11-E13)



- Flat plate boundary layer theory

- Blasius flat plate – **Laminar flow:**

$\frac{u(x,y)}{U} = f'(\eta)$ (**Blasius profile**) Eq. (7.21)

  – Use Blasius for laminar flow, if nothing else is stated!
  – Eq. (7.21) – (7.31), and Table 7.1

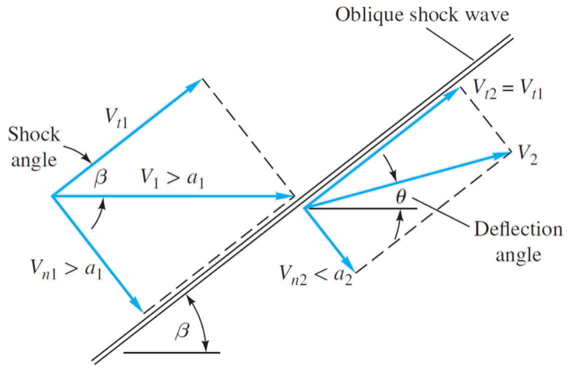
- Prandtl flat plate – **Turbulent flow:**

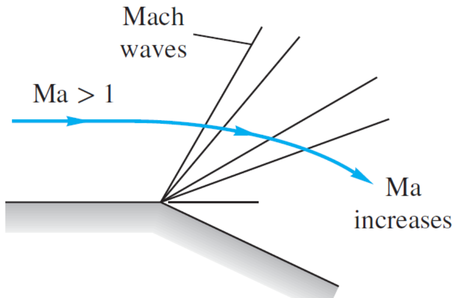
$\left(\frac{u}{U}\right)_{\text{Turb}} \approx \left(\frac{y}{\delta}\right)^{1/7}$ (**1/7-rule**) Eq. (7.39)

  – Eq. (7.39) – (7.49)

Chapter 9. (E14-E16)

- Compressible flow, density variations at $\mathrm{Ma} > 0.3$
- Find the right Eqns. in Ch. 9 depending on what type of flow we have
  - Perfect gas law, $p = \rho R T$
  - Continuity equation $\dot{m} = \mathrm{const.}$
  - Isentropic relations, e.g. Eqns. (9.26 – 9.28), A. B1
  - Area – Ma relations, e.g. Eq. (9.44) and (9.47), A. B1
  - Normal shock relations (see previous slide), A. B2
- Oblique shock
  - Use normal shock relations in normal direction to shock
  - Eqs. (9.82 – 9.86)
- Expansion fan (Prandtl Meyer)
  - Isentropic expansions
  - $\omega(\mathrm{Ma})$ with Eq. (9.99), or table B.5





(d)

What will be one the exam?

- Check exams and re-exams from 2021-2022 for structure
  - Older exams can be used for tasks
- From the last 14 exams on canvas (rough estimate):
  - Compressible flow (Chapter 9) – 100%
    - ➢ Niklas main area of research
  - Flat plate (Chapter 7) – 86%
  - Pipe flow (Chapter 6) – 71%
  - Reynolds transport theorem (Chapter 3) – 71%
  - Hydrostatic – 57%
  - Drag coefficient, or model/prototype scaling – 50%
- Niklas will not give you everything in the text
  - Need to use tables to find material data
  - Assumptions to make the equations manageable