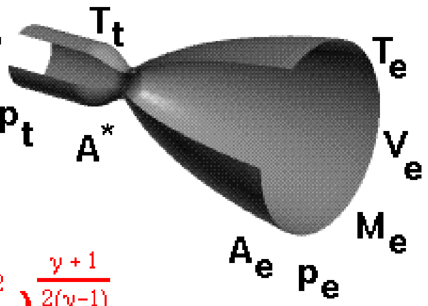
Why supersonic flow?

Rocket Thrust Summary

Known:

- p_t = Total Pressure
- T_t = Total Temperature
- p_o = Free Stream Pressure
- γ = Specific Heat Ratio
- R = Gas Constant
- A = Area



Mass Flow Rate: $\dot{m} = \dfrac{A^* p_t}{\sqrt{T_t}} \sqrt{\dfrac{\gamma}{R}} \left(\dfrac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$

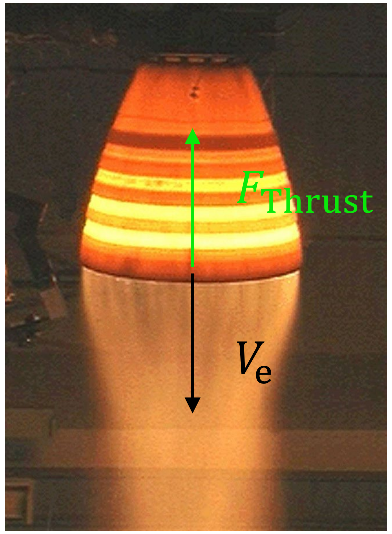
Exit Mach: $\dfrac{A_e}{A^*} = \left(\dfrac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}} \dfrac{\left(1+\frac{\gamma-1}{2} M_e^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{M_e}$

Exit Temperature: $\dfrac{T_e}{T_t} = \left(1 + \dfrac{\gamma-1}{2} M_e^2\right)^{-1}$

Exit Pressure: $\dfrac{p_e}{p_t} = \left(1 + \dfrac{\gamma-1}{2} M_e^2\right)^{-\frac{\gamma}{\gamma-1}}$

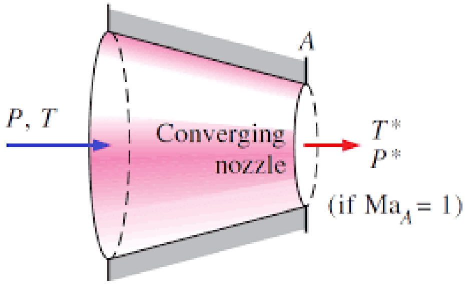
Exit Velocity: $V_e = M_e \sqrt{\gamma R T_e}$

Thrust: $F = \dot{m} V_e + (p_e - p_0) A_e$



Exercise 15.

- Sonic flow $(\mathrm{M} = 1)$
  - Sonic values: A^*, ρ^*, V^* ...
  - If $\mathrm{M} = 1$ achieved → no higher mass flow is possible!
    $\dot{m}_{\max} = \rho^* A^* V^*$ Eq. (9.46a)
    - ➢ The flow is choked



- Sub- and super-sonic flow
  - Pressure-waves moves with the speed of sound
  - Flow behaves differently in nozzles if sub- or supersonic

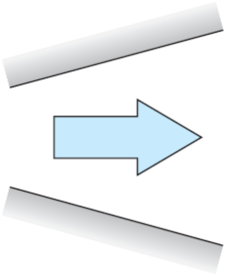
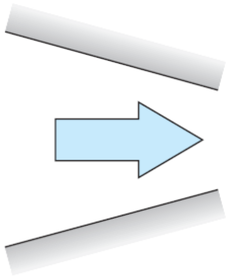
Duct geometry		Subsonic Ma < 1	Supersonic Ma > 1
	$dA > 0$	$dV < 0$ $dp > 0$ Subsonic diffuser	$dV > 0$ $dp < 0$ Supersonic nozzle
	$dA < 0$	$dV > 0$ $dp < 0$ Subsonic nozzle	$dV < 0$ $dp > 0$ Supersonic diffuser

Fig. 9.5

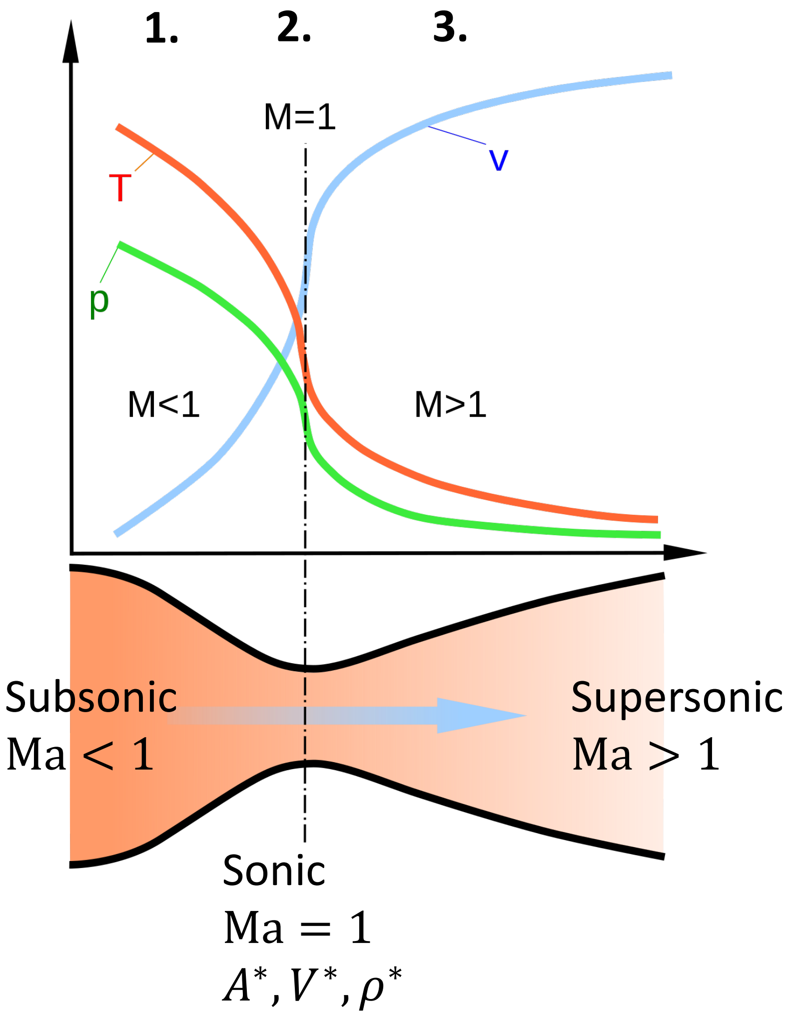
Exercise 15.

- Supersonic flow ($\mathrm{Ma} > 1$)
  - – Only possible from a reservoir if:

1. **Subsonic** flow is **accelerated** through **convergent nozzle**

2. **Sonic** flow is obtained at the **smallest cross-section** (throat)
   - ➢ $A^* = A_\mathrm{t} \rightarrow \mathrm{M} = 1$

3. **Supersonic** flow through **expanding** the flow via **divergent nozzle**



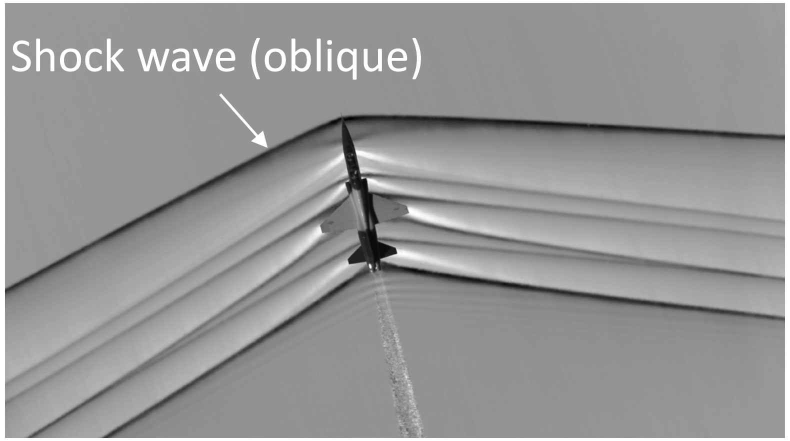
Exercise 15.

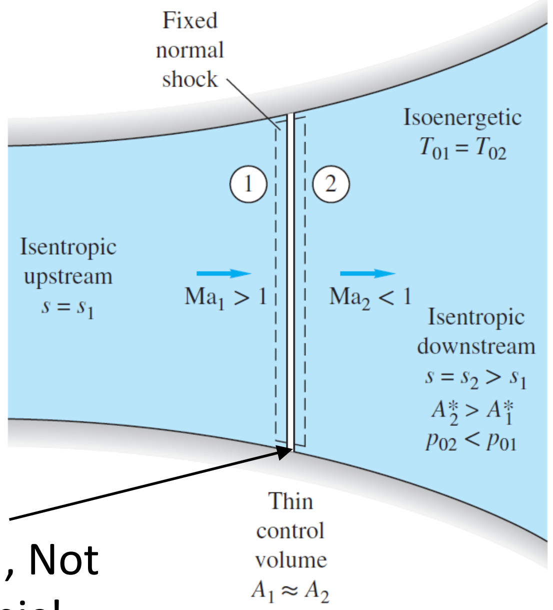
- **Normal shock**
  - Abrupt change in supersonic flow
  - If $\gamma > 1$ (e.g. air) the flow goes from super- to subsonic
  - The chock results in losses
    - ➢ Isentropic assumption not valid through chock
  - Need to use **normal shock relations** e.g.

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left[\mathrm{Ma}_1^2 - 1\right] \quad \text{Eq. (9.55)}$$

$$\mathrm{Ma}_2^2 = \frac{(\gamma-1)\mathrm{Ma}_1^2+2}{2\gamma\,\mathrm{Ma}_1^2-(\gamma-1)} \quad \text{Eq. (9.57)}$$

  - Can also use relations from Eq. (9.58)
  - Table B.1 isentropic relations
  - Table B.2 normal shock relations





$s_2 > s_1$, Not isentropic!

Fig. 9.8

Exercise 15.

- Converged-Divergent nozzle
  - The flow behaviour depends on $p_0 - p_{\rm e}$
    - ➢ p_0 is the reservoir pressure
    - ➢ $p_{\rm e}$ is the exit/jet pressure
    - ➢ $p_{\rm b}$ is the back pressure ($p_{\rm b} = p_{\rm e}$ at A – C and H)

- Find the right Eqns. in Ch. 9 depending on what type of flow we have
  - Perfect gas law, $p = \rho RT$ Eq. (1.10)
  - Continuity equation $\dot{m} = \text{const.}$
  - Isentropic relations, e.g. Eqs. (9.26 – 9.28) or B.1
  - Area – Ma relations, e.g. Eq. (9.44) and (9.47) or B.1
  - Normal shock relations (see previous slide) or B.2

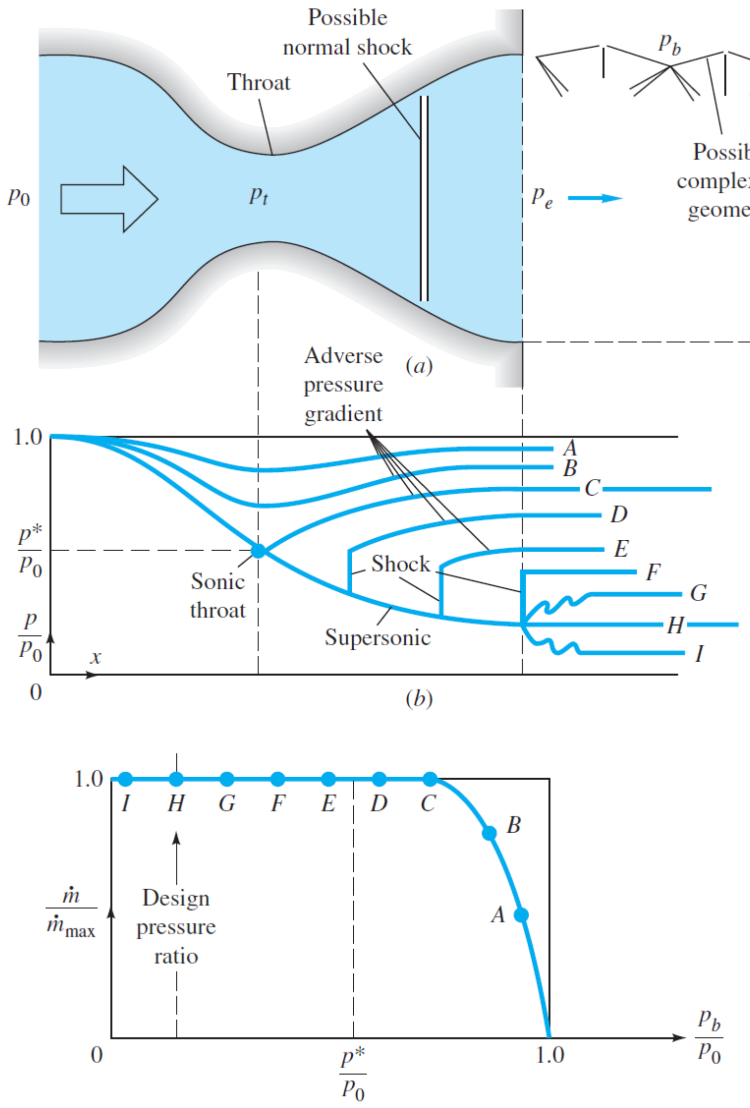
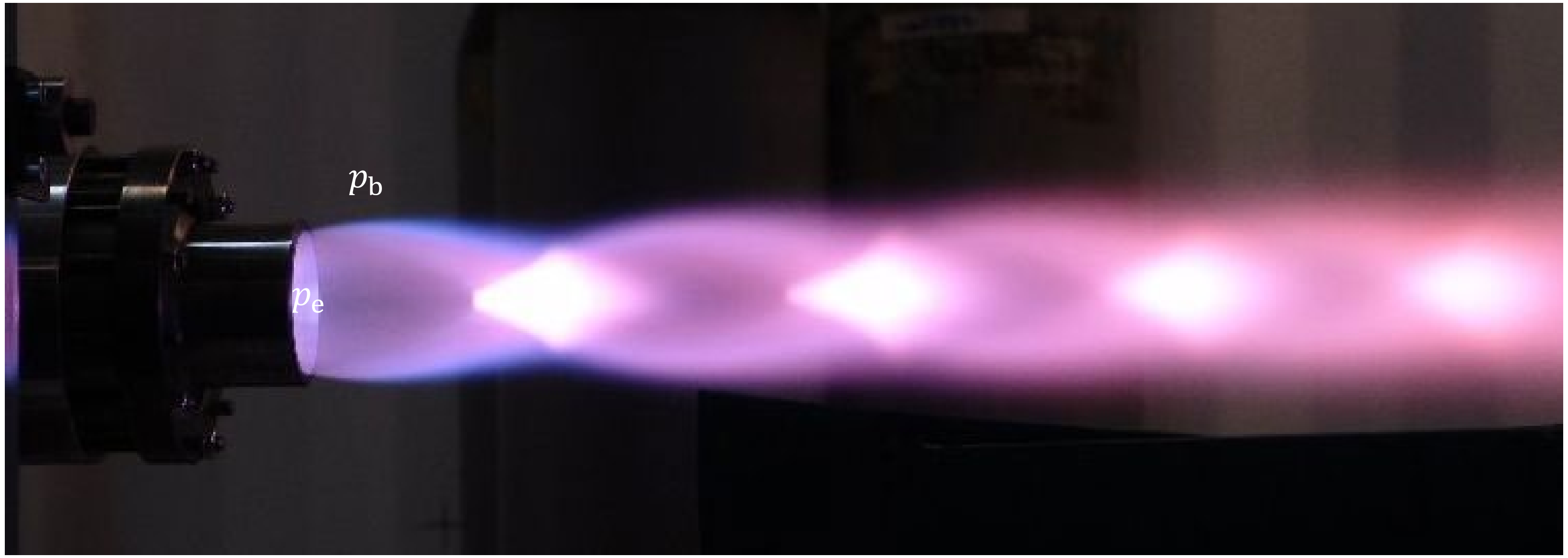
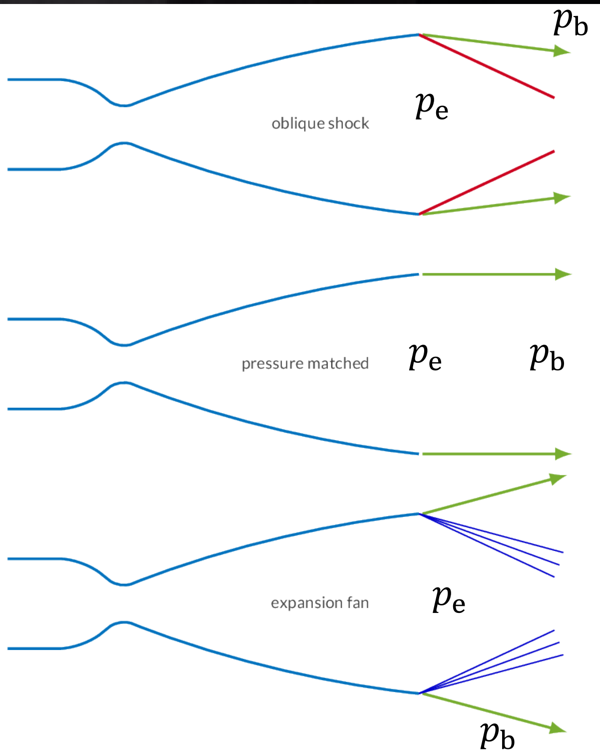


Fig. 9.12





Overexpanded nozzle:

$p_e < p_b$

Pressure matched nozzle:

$p_e = p_b$

Underexpanded nozzle:

$p_e > p_b$