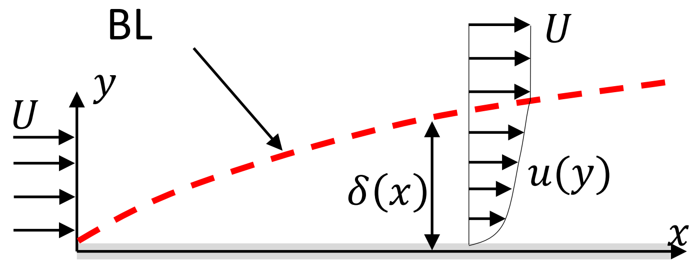
# Exercise 12.




- Boundary Layer (BL) theory

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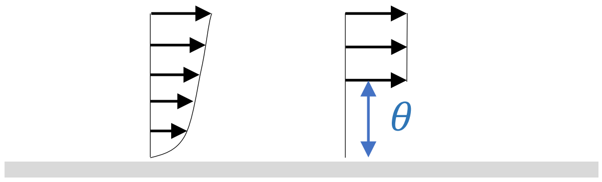
- Momentum thickness $\theta$
  - Same linear momentum
  - Linear flow profile

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) \mathrm{d}y \qquad \text{Eq. (7.3)}$$

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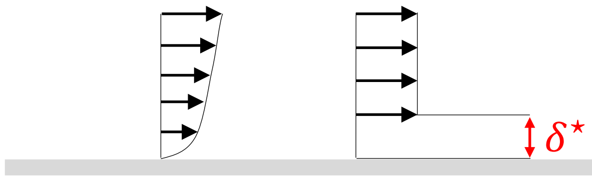
- Displacement thickness $\delta^\star$
  - Same mass flow rate
  - Linear flow profile

$$\delta^\star = \int_0^\delta \left(1 - \frac{u}{U}\right) \mathrm{d}y \qquad \text{Eq. (7.12)}$$

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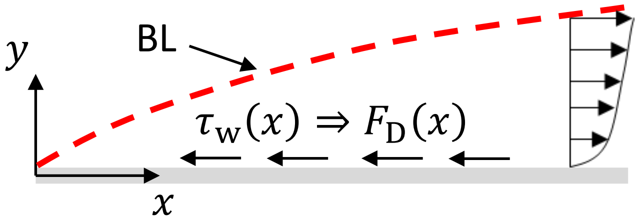
- Skin friction coefficient $c_\mathrm{f}$
  - Compare with friction coefficient for pipe flow

$$c_\mathrm{f} = \frac{2\tau_\mathrm{w}}{\rho U^2} \quad \text{Eq. (7.10),} \quad \text{with } \tau_\mathrm{w} = \rho U^2 \frac{\mathrm{d}\theta}{\mathrm{d}x} \qquad \text{Eq. (7.5)}$$



# Exercise 12.

- Karman flat plate – **Laminar flow (don't use this):**
  $\frac{u(x,y)}{U} \approx \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)$ (**parabolic**) Eq. (7.6)

- Blasius flat plate – **Laminar flow:**
  $\frac{u(x,y)}{U} = f'(\eta)$ (**Blasius profile**) Eq. (7.21)
  - Use Blasius for laminar flow, if nothing else is stated!
  - Eq. (7.21) – (7.31), and Table 7.1

- Prandtl flat plate – **Turbulent flow:**
  $\left(\frac{u}{U}\right)_{\text{Turb}} \approx \left(\frac{y}{\delta}\right)^{1/7}$ (**1/7-rule**) Eq. (7.39)
  - Eq. (7.39) – (7.49)

# Exercise 12.

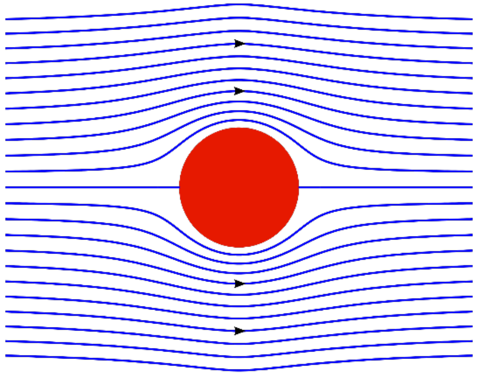
- **Laminar flow** (Blasius)

$$C_{\mathrm{D}} = \frac{F_{\mathrm{D}}}{\frac{\rho}{2}U^2 bL} = \frac{1.328}{\mathrm{Re}_L^{1/2}} \qquad \text{Eq. (7.27)}$$

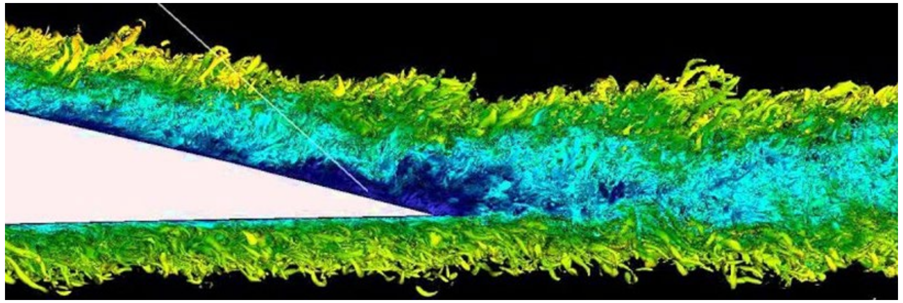
- Transition to turbulence $\mathrm{Re}_{\mathrm{trans}} = 5\mathrm{E}5 - 1\mathrm{E}6$ (if nothing else stated)
- **Turbulent flow** (Prandtl)

Smooth wall: $C_{\mathrm{D}} = \frac{0.031}{\mathrm{Re}_L^{1/7}}$ Eq. (7.45)

Rough wall: Use Fig. 7.6 and e.g (7.48) or (7.49)



*Potential flow = stream function (Ψ), solve Laplace eqn + Bernoulli (Ch. 4)*



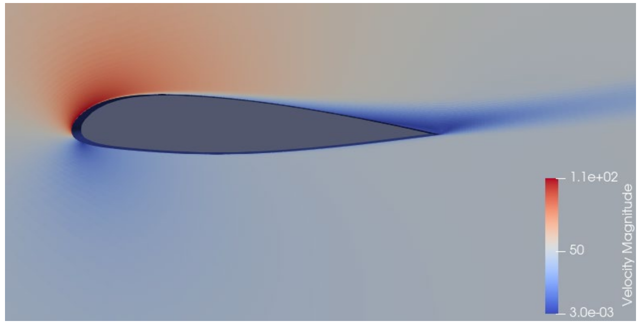
*DNS (**D**irect **N**umerical **S**imulation) = Navier-Stokes numerically (Ch. 4)*

Fast and cheap, but not so accurate

**Cost, computer resources**

Slow and expensive, accurately resolved flow field

*Potential flow + Von Karman (or similar boundary layer theory) → Streamlines can move away from the body, boundary layer thickness can be estimated, friction resistance can be estimated (Ch. 4 + Ch. 7)*




*RANS (**R**eynolds **A**veraged **N**avier-**S**tokes) = Solve the time-averaged Navier-Stokes + Boussinesq assumption*

$$\tau_{turb} = -\rho\overline{u'v'} = \mu_t\left(\frac{\partial\bar{u}}{\partial y} + \frac{\partial\bar{v}}{\partial x}\right)$$