Exercise 7.

- Three differential equations that together describes the fluid motion
  - **Continuity equation**
    - ➢ Conservation of mass
  - **Navier-Stokes (NS),** or momentum
    - ➢ Conservation of linear momentum (force balance)
  - **Energy equation**
    - ➢ Conservation of energy
- The three equations are solved numerically, referred to as CFD

- Note that conservation of angular momentum vanishes as the CV → 0
  - The leaver → 0, ($\tilde{r} \to 0$)

Continuity equation

- Cartesian coordinates (4.4):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial z}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

- Cylindrical coordinates (4.9), or appendix D

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho w_z) = 0$$

Change in fluid volume = flow out - flow in

Navier-Stokes (momentum)

- Correlates to the **RTT** with **linear momentum** for a CV
  - Difference is that we use **differential equations** for an **infinitesimal volume**
- **Newtonian fluid** with **constant viscosity** and **density** we have (4.38):

Total derivative
(Material derivative)

$$\rho \frac{\mathrm{d}u}{dt} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \rho g_x$$

$$\rho \frac{\mathrm{d}v}{dt} = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + \rho g_y$$

$$\rho \frac{\mathrm{d}w}{dt} = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) + \rho g_w$$

density x acceleration = pressure forces + viscous (friction) forces + gravity forces

For a cylindrical coordinate system, see Appendix D!

Energy equation

- Most **general** form (4.51)

$$\rho \frac{\mathrm{d}\hat{u}}{\mathrm{d}t} + p(\nabla \cdot \tilde{v}) = \nabla \cdot (k \nabla T) + \Phi$$

Change of internal energy + pressure work = heat conductivity + viscous (friction) losses

- This equation is usually simplified with various assumptions
  - The most common is constant: ρ, μ and k:

$$\rho C_{\mathrm{v}} \frac{\mathrm{d}T}{\mathrm{d}t} = (k \nabla^2 T) + \Phi \qquad (4.53)$$

Laplace-operator:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

For a cylindrical coordinate system, see Appendix D!

Use the correct equation

- If the task is to determine a profile/function, e.g. $u(y)$), use diff. eqn.
  - Velocity profile → Navier-Stokes (momentum)
  - Temperature profile/distribution → Energy equation
  - The continuity equation must always hold! It is common that one needs to combine continuity equation with the other eqns.

- If the task is to determine a value (scalar), the integral form of the conservation equations (RTT) for a control volume is simpler to use.

How to solve the differential equation:

Chose the correct differential equation (and form)

Simplify the differential equation with assumptions (justify this!)

Integrate the expression

Apply boundary conditions

The rest is *just* algebra (skyffla siffror) ☺