Exercise 6.

- The continuity equation:
  - **Integral form**, from RTT for a system:

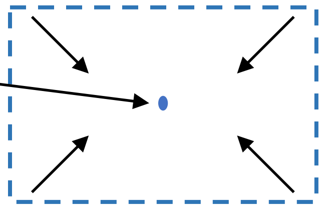
$$\frac{\mathrm{d}}{\mathrm{d}t}(m) = 0 = \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{\mathrm{CV}} \rho \, \mathrm{d}V\right) + \int_{\mathrm{CS}} \rho(\tilde{v}_r \circ \tilde{n})\mathrm{d}A \qquad \text{Eq. (3.20)}$$

    - ➢ Valid for a finite CV

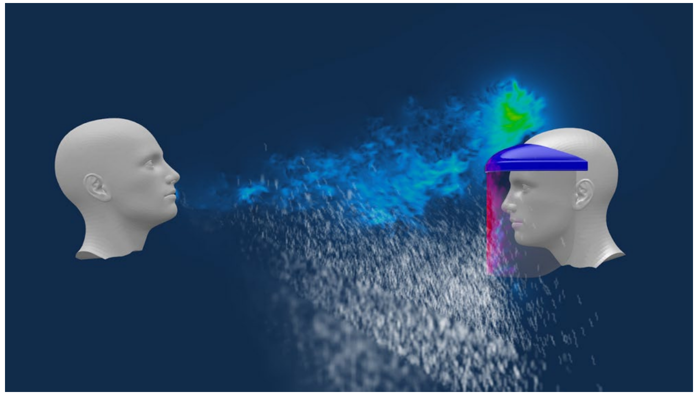
  - **Differential form**, the CV → 0. Eqn. only in one point (*x,y,z,t*)

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial z}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \qquad \text{Eq. (4.4)}$$

$$\Rightarrow \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \tilde{v}) = 0 \qquad \text{Eq. (4.6)}$$



- The basics of Computational Fluid Dynamics (CFD)
  - By solving the diff. equations in this chapter the whole flow field can be described.



Exercise 6.

- Total/material-derivative:
  - The acceleration for a small fluid element, with the velocity field $\tilde{v}(x, y, z, t)$

$$\tilde{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \frac{\mathrm{d}\tilde{v}}{\mathrm{d}t}, \qquad \left\{\frac{\mathrm{d}}{\mathrm{d}t} = \text{Total derivative with respect to time}\right\}$$

$$\tilde{a} = \frac{\mathrm{d}\tilde{v}}{\mathrm{d}t} = \{\text{chain rule}\} = \frac{\partial \tilde{v}}{\partial t} + \frac{\partial \tilde{v}}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial \tilde{v}}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial \tilde{v}}{\partial z}\frac{\partial z}{\partial t}$$

$$\tilde{a} = \frac{\mathrm{d}\tilde{v}}{\mathrm{d}t} = \frac{\partial \tilde{v}}{\partial t} + \frac{\partial \tilde{v}}{\partial x} u + \frac{\partial \tilde{v}}{\partial y} v + \frac{\partial \tilde{v}}{\partial z} w \qquad \text{Eq. (4.2)}$$

  - Introducing Nabla: $\nabla = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{bmatrix}^T \Rightarrow$

$$\tilde{a} = \frac{\mathrm{d}\tilde{v}}{\mathrm{d}t} = \frac{\partial \tilde{v}}{\partial t} + \left(\frac{\partial \tilde{v}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} + \frac{\partial \tilde{v}}{\partial z}\right)\tilde{v} \Rightarrow$$

$$\tilde{a} = \frac{\partial \tilde{v}}{\partial t} + (\tilde{v} \cdot \nabla)\tilde{v}$$

Assumptions (examples)

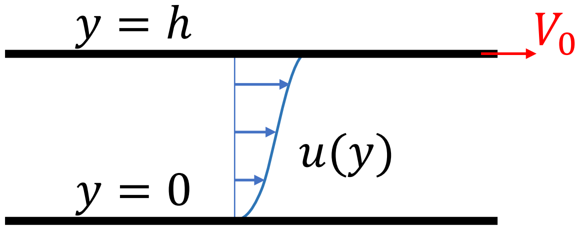
- Steady-state: $\frac{\partial}{\partial t} = 0$
- Incompressible: $\rho = \text{const.}$, $\frac{\mathrm{d}\rho}{\mathrm{d}} = 0$, $\frac{\partial \rho}{\partial} = 0$
  - Product rule on cont. Eq: $\frac{\partial(\rho u_i)}{\partial x_i} = 0 \Rightarrow \rho \frac{\partial u_i}{\partial x_i} + u_i \frac{\partial \rho}{\partial x_i} = 0 \Rightarrow \rho \frac{\partial u_i}{\partial x_i} = 0 \Rightarrow \frac{\partial u_i}{\partial x_i} = 0$
- 2D-flow e.g in xy: $w = 0$, $\frac{\partial}{\partial z} = 0$
- Only flow in x-dir: $w = v = 0$
- Fully developed flow in x: $\frac{\mathrm{d}}{\mathrm{d}x} = 0$, $\frac{\partial}{\partial x} = 0$
- Axisymmetric flow: $\frac{\partial}{\partial \theta} = 0$
- Only flow in z-dir (cyl-coord): $v_\theta = v_r = 0$

Boundary conditions

- No-slip → fluid velocity at the wall = wall velocity (Dirichlet)

$$u(0) = 0$$

$$u(h) = V_0$$



- Slip condition → velocity gradient = zero (Neumann)

$$\frac{\partial u(h)}{\partial y} = 0$$