Exercise 5.

- Conservation of **Angular Momentum** with RTT Eq. (3.16)
  - Setting $B_{\text{syst}} = \widetilde{H}_0 = \int_{\text{syst}} (\tilde{r} \times \tilde{v}) \mathrm{d}m,$ $\beta_{\text{syst}} = \frac{\mathrm{d}\widetilde{H}_0}{\mathrm{d}m} = \tilde{r} \times \tilde{v},$ gives:

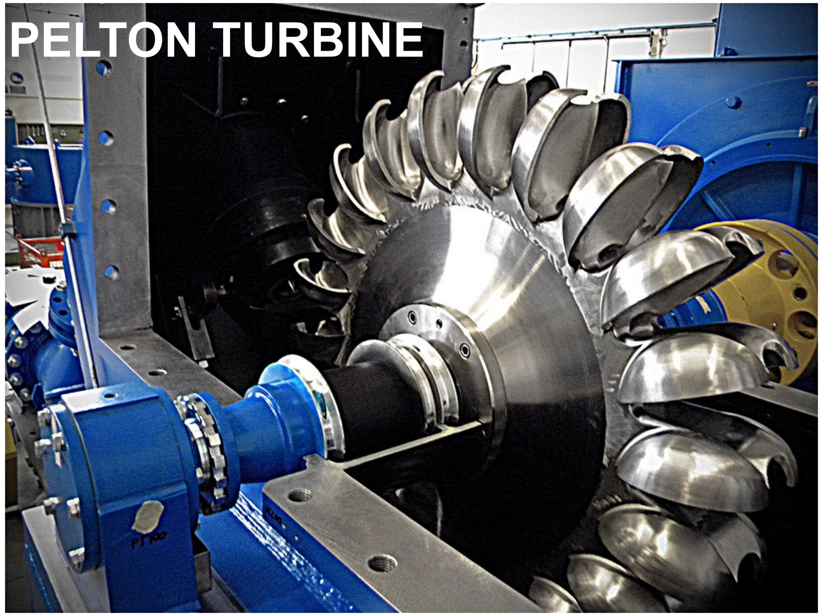
$$\left.\frac{\mathrm{d}\widetilde{H}_0}{\mathrm{d}t}\right|_{\text{syst}} = \frac{\mathrm{d}}{\mathrm{d}t}\left[\int_{\text{CV}} (\tilde{r} \times \tilde{v}) \rho \, \mathrm{d}V\right] + \int_{\text{CS}} (\tilde{r} \times \tilde{v}) \rho (\tilde{v}_r \circ \tilde{n}) \mathrm{d}A \qquad \text{Eq. (3.56)}$$

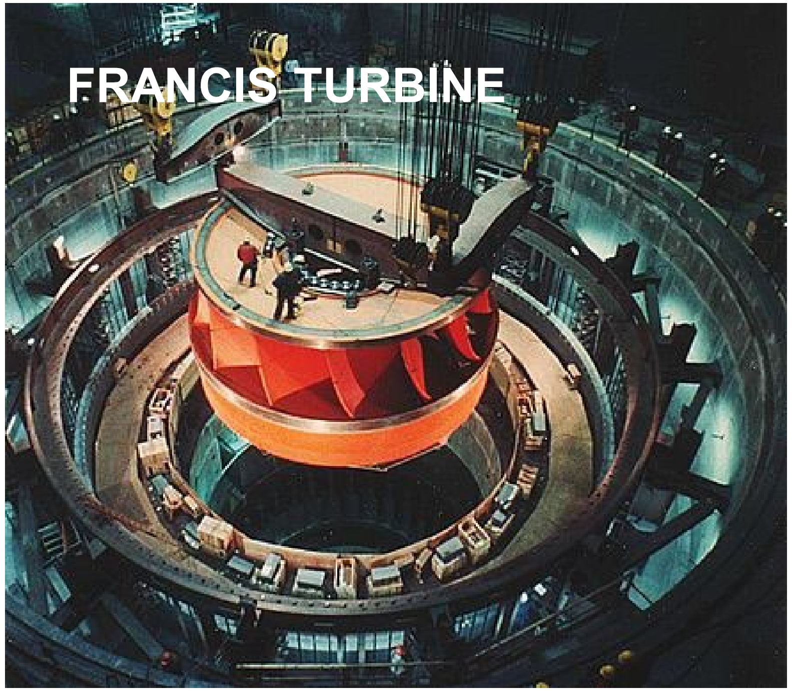
  - Non-deformable and fix CV gives:

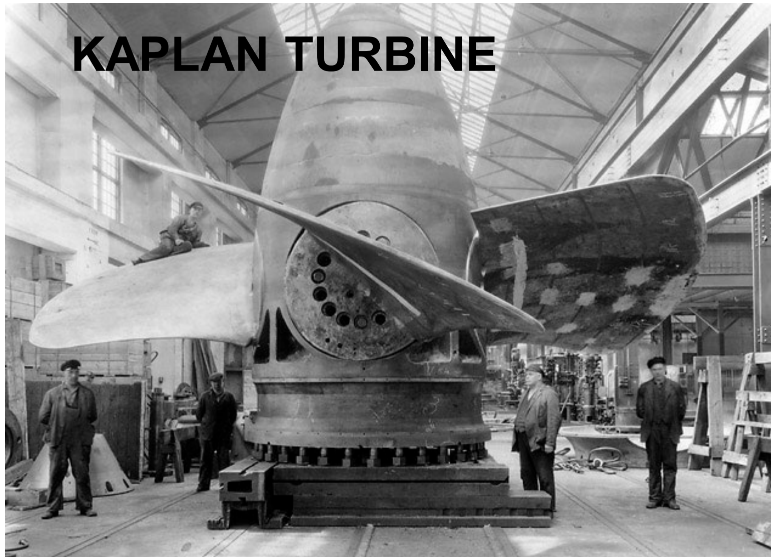
$$\sum \widetilde{M}_0 = \frac{\partial}{\partial t}\left[\int_{\text{CV}} (\tilde{r} \times \tilde{v}) \rho \, \mathrm{d}V\right] + \int_{\text{CS}} (\tilde{r} \times \tilde{v}) \rho (\tilde{v} \circ \tilde{n}) \mathrm{d}A \qquad \text{Eq. (3.59)}$$

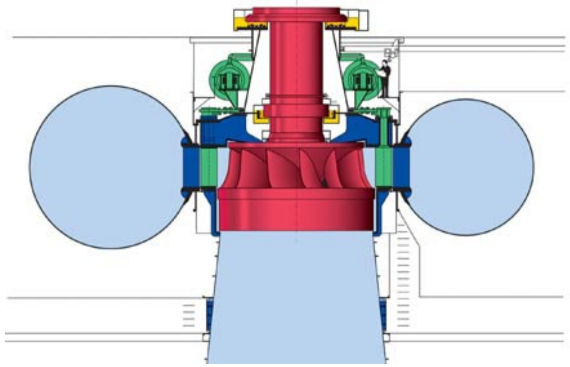
  - Note the LHS remains if external momentum (force x distance) acts on the system:

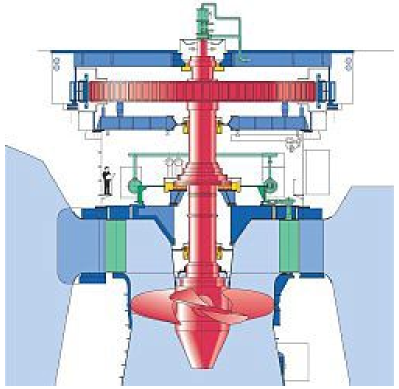
$$\left.\frac{\mathrm{d}\widetilde{H}_0}{\mathrm{d}t}\right|_{\text{syst}} = \sum \widetilde{M}_0 = \sum (\tilde{r} \times \tilde{v})_0$$











Euler turbine equation:

$$M_0 = \dot{m}[(v_\theta\, r)_{\text{out}} - (v_\theta\, r)_{\text{in}}]$$

$$P = \omega M_0$$

-Used in the initial design phase of all new turbines

Exercise 5.

- Conservation of **Energy** with RTT Eq. (3.16)
  - Setting $B_{\text{syst}} = E,\ \ \beta_{\text{syst}} = \frac{\mathrm{d}E}{\mathrm{d}m} = e,$ which gives:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{\mathrm{CV}} e\rho\,\mathrm{d}V\right) + \int_{\mathrm{CS}} e\rho(\tilde{v}_r \circ \tilde{n})\mathrm{d}A \qquad \text{Eq. (3.61)}$$

  - Total energy (LHS) can be divided as: $\dot{E}_{\text{syst}} = \dot{Q} - \dot{W}_{\text{shaft}} - \dot{W}_{\text{visc}} - \dot{W}_{\text{press}}$
    - The work done by pressure as: $\dot{W}_{\text{press}} = \int_{\mathrm{CS}} p\ (\tilde{v} \circ \tilde{n})\mathrm{d}A$ Eq. (3.63)
  - The energy per unit mass as: $e = \hat{u} + 1/2\, v^2 + gz$ Eq. (3.62)
  - Adding the definition of enthalpy (sum of internal and pressure energy): $h = \hat{u} + \frac{p}{\rho}$
  - Non deformable and fix CV → $\frac{\mathrm{d}}{\mathrm{d}t} \Rightarrow \frac{\partial}{\partial t}$

Exercise 5.

- Inserting all of this in the energy equation (3.61), move the pressure work to the RHS, and include the enthalpy h:

$$\dot{Q} - \dot{W}_{\mathrm{s}} - \dot{W}_{\mathrm{v}} = \frac{\partial}{\partial t}\left(\int_{\mathrm{CV}}\left(\widehat{u} + \frac{1}{2}v^2 + gz\right)\rho\,\mathrm{d}V\right) + \int_{\mathrm{CS}}\left(h + \frac{1}{2}v^2 + gz\right)\rho(\tilde{v}_r \circ \tilde{n})\mathrm{d}A \qquad \text{Eq. (3.67)}$$

  – Here:

    $\dot{Q}$= Heat transfer

    $\dot{W}_s$= Shaft work

    $\dot{W}_{\mathrm{v}}$= Viscous (friction) work, often negligible

    $\hat{u}$ = Internal energy

    $v/2$ = Kinetic energy

    gz = Potential energy

    h = Enthalpy

  – Heat, Q, added or work, W, done by the system is positive!