

Exercise 4.

- Conservation of **Linear Momentum** with RTT Eq. (3.16)

– Setting $B_{\text{syst}} = (m\tilde{v})_{\text{syst}}$, $\beta_{\text{syst}} = \left. \frac{dm\tilde{v}}{dm} \right|_{\text{syst}} = \tilde{v}_{\text{syst}}$, gives:

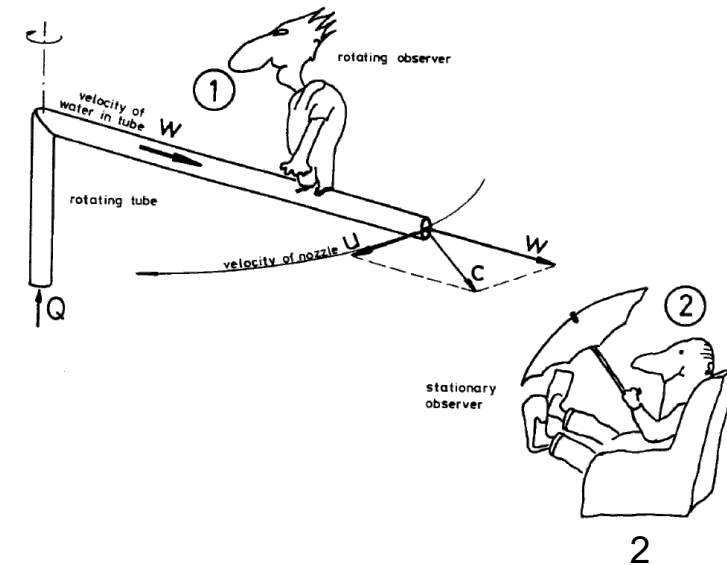
$$\frac{d}{dt}(m\tilde{v})_{\text{syst}} = \sum \tilde{F} = \frac{d}{dt} \left(\int_{\text{CV}} \tilde{v} \rho \, dV \right) + \int_{\text{CS}} \tilde{v} \rho (\tilde{v}_r \circ \tilde{n}) \, dA \quad (\text{Eq. 3.35})$$

– Note the LHS remains if external forces acts on the system:

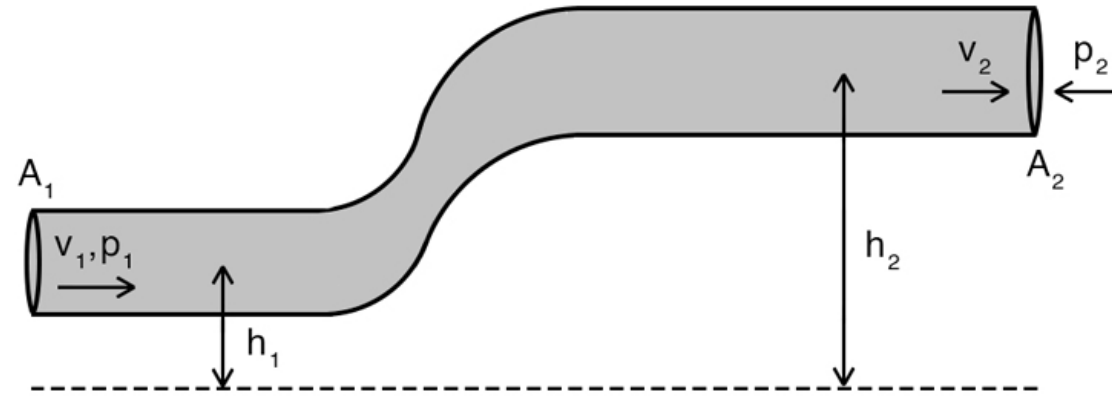
$$\frac{d}{dt}(m\tilde{v})_{\text{syst}} = m \frac{d\tilde{v}_{\text{syst}}}{dt} + \tilde{v}_{\text{syst}} \frac{dm}{dt} = m \tilde{a}_{\text{syst}} = \sum F_{\text{syst}}$$

- The relative velocity \tilde{v}_r is used in (3.35), for moving CV

– $\tilde{v}_r = \tilde{v} - \tilde{v}_s$, where \tilde{v}_s is the velocity of the CV



Bernoulli's Equation



$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

A_1, A_2 : Cross-sectional areas at points 1 and 2

p_1, p_2 : Pressures

v_1, v_2 : Velocities

h_1, h_2 : Elevations

Exercise 4.

- Bernoulli's equation

- Combine the **continuity** and **linear momentum** equations along a **streamline**
- Assume steady-state ($\frac{d}{dt} = 0$), friction less (no viscous effects) and incompressible ($\frac{d\rho}{d} = 0$) flow, gives:

$$p_1 + \frac{\rho v_1^2}{2} + \rho g z_1 = p_2 + \frac{\rho v_2^2}{2} + \rho g z_2 \quad \text{Eq. (3.54)}$$

- Can be used to convert between different types of pressure in the system.
- Is a type of energy conservation.
- Can be extended to include energy losses/addition between 1 → 2 as:

$$p_1 + \frac{\rho v_1^2}{2} + \rho g z_1 = p_2 + \frac{\rho v_2^2}{2} + \rho g z_2 + \Delta p_{\text{friction}} - \Delta p_{\text{pump}} + \Delta p_{\text{turbine}} \quad \text{Eq. (3.73)}$$