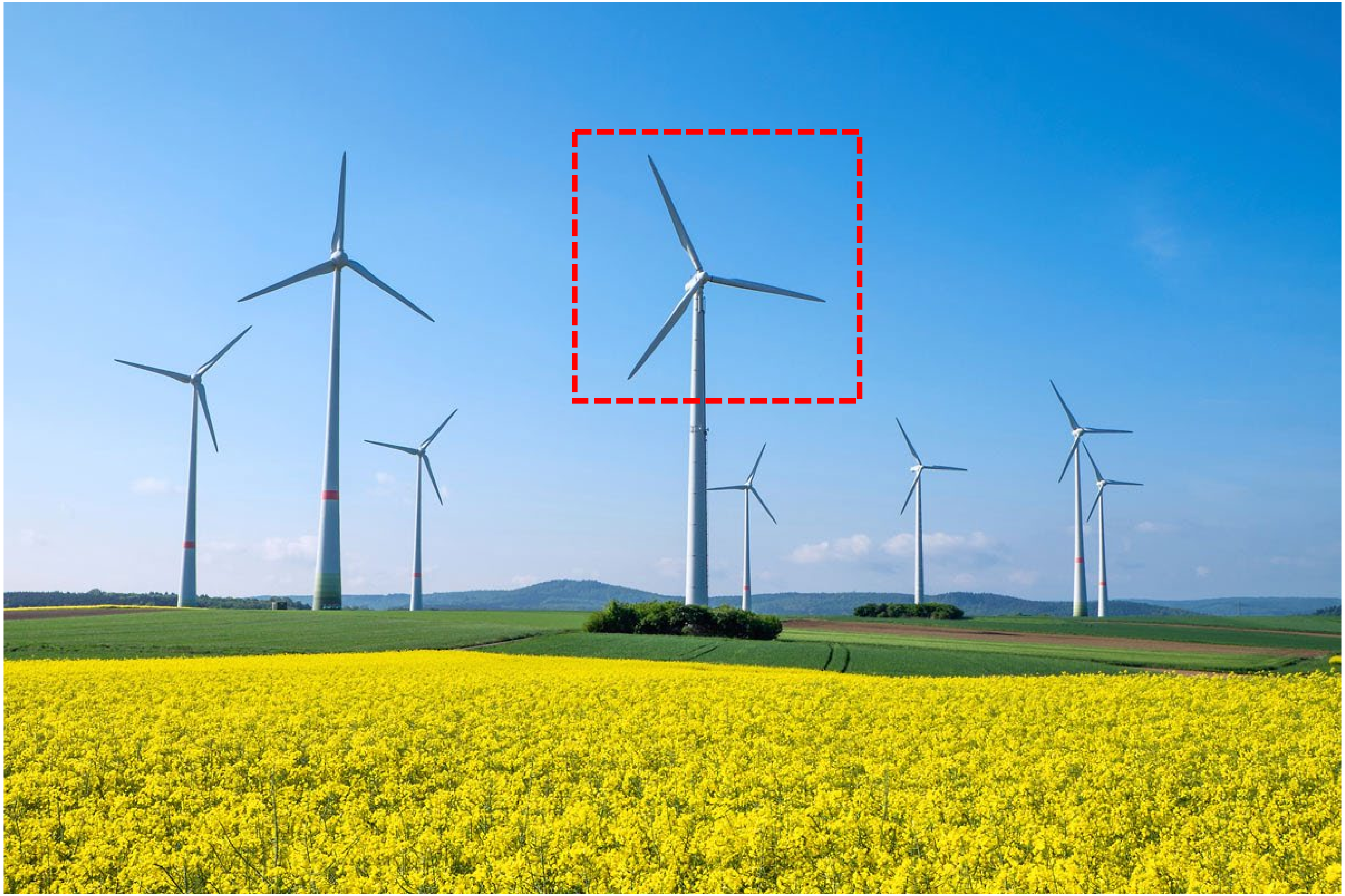
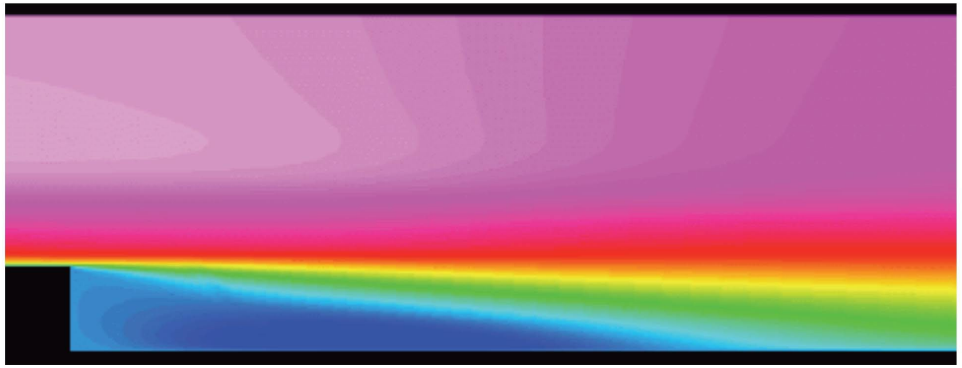


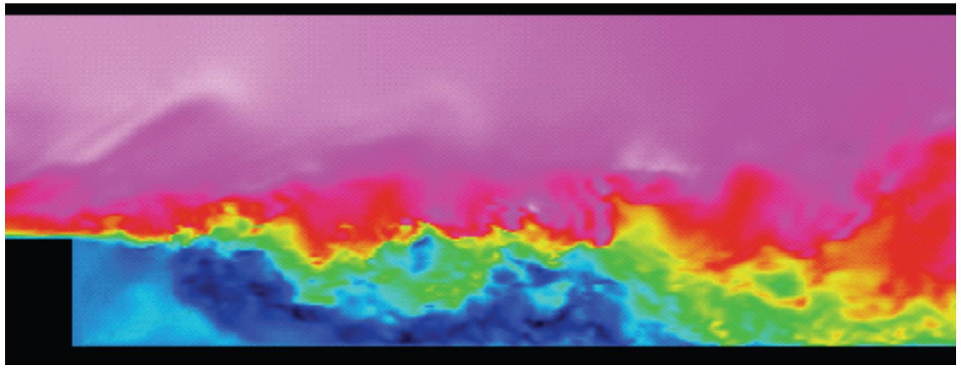
Steady and unsteady flow

Steady-state



Source: Rémy Fransen, 3rd INCA colloquium, ONERA, Toulouse (2011)

Unsteady



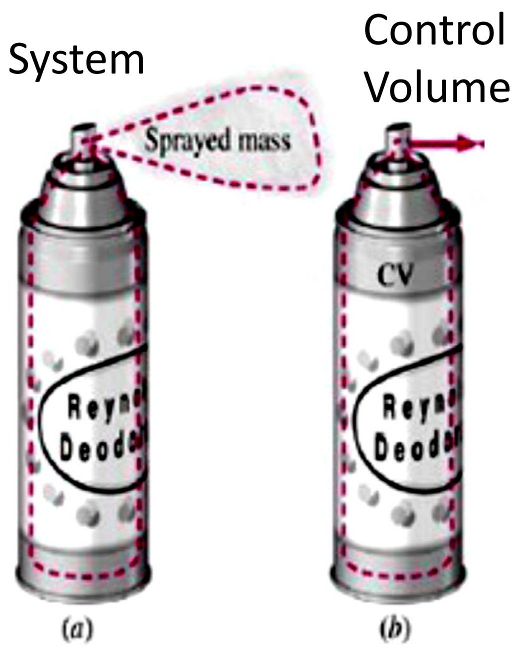
Source: Rémy Fransen, 3rd INCA colloquium, ONERA, Toulouse (2011)

Exercise 3.

- Reynolds Transport Theorem (RTT)
  - Integral relations for a control volume
  - One of the most fundamental theorems in fluid mechanics
  - From the theorem we get fundamental laws:
    - Mass conservation (continuity)
    - Linear momentum conservation (impulse)
    - Angular momentum conservation
    - Energy conservation
  - These expressions are the foundation of the Navier-Stokes equations, chapter 4.
- RTT allows us to analyse a system over a control volume

Exercise 3.

- **System:** Collection of materia with a fixed identity
  - Always the same atoms or fluid particles
  - A specific, and identifiable quantity of mass
- **Control volume (CV):** A fixed volume in space
  - Fluid flow through the CV
  - Geometrically constraint
  - Independent of mass



Exercise 3.

- RTT:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(B_{\mathrm{syst}}\right) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{\mathrm{CV}} \beta\rho \, \mathrm{d}V\right) + \int_{\mathrm{CS}} \beta\rho(\tilde{v}_r \circ \tilde{n})\mathrm{d}A$$

Eq. (3.16)

Change of *B* in the whole system	Change of *B* inside the CV	*B* “flowing” in and out of the CV

- By letting *B* equal mass, linear momentum, angular momentum, or energy, we get the four laws
- ***B*** is an **extensive** property and has units e.g. [kg], [kg m/s]…
- **β** is an **intensive** property, i.e. amount of ***B*** per unit mass, with units of [kg/kg], [kg m/s^2 /kg]…

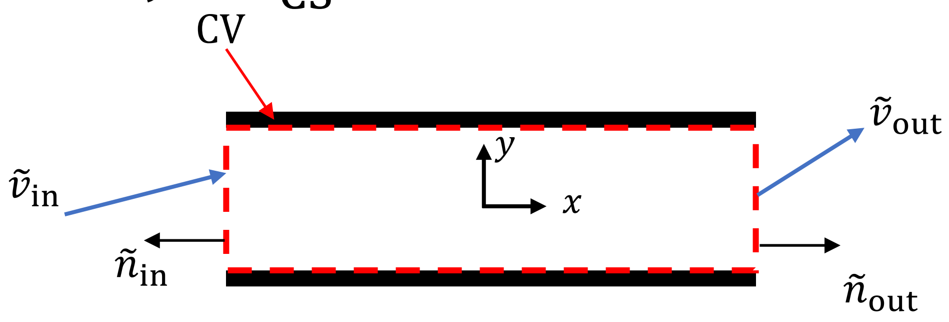
$$\beta = \frac{\mathrm{d}B}{\mathrm{d}m}$$

- For RTT to be true, the system and the CV must equal at the time *t*, syst=CV, $B_{\mathrm{syst}}(t) = B_{\mathrm{CV}}(t)$
  - At $t + \mathrm{d}t$: $B_{\mathrm{syst}}(t + \mathrm{d}t) = B_{\mathrm{CV}}(t + \mathrm{d}t) - (\text{inflow of } B)_{\mathrm{CS}} + (\text{outflow of } B)_{\mathrm{CS}}$
    - ➢ RTT can predict the change inside the CV at all times after *t*

Exercise 3.

- Setting *B* = mass → mass conservation and assume steady-state (3.20)

$$\frac{\mathrm{d}}{\mathrm{d}t}(m) = 0 = \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{\mathrm{CV}} \rho \,\mathrm{d}V\right) + \int_{\mathrm{CS}} \rho(\tilde{v}_r \circ \tilde{n})\mathrm{d}A \Rightarrow [\text{steady}] \Rightarrow \int_{\mathrm{CS}} \rho(\tilde{v} \circ \tilde{n})\mathrm{d}A = 0$$



$$\tilde{v}_{\mathrm{in}} = \begin{bmatrix} u_{\mathrm{in}} \\ v_{\mathrm{in}} \end{bmatrix}, \quad \tilde{n}_{\mathrm{in}} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \qquad \tilde{v}_{\mathrm{out}} = \begin{bmatrix} u_{\mathrm{out}} \\ v_{\mathrm{out}} \end{bmatrix}, \quad \tilde{n}_{\mathrm{out}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

- 1D mass conservation yield:

$$\sum_i (\rho_i A_i V_i)_{\mathrm{in}} = \sum_j \left(\rho_j A_j V_j\right)_{\mathrm{out}} \qquad (3.24)$$

$$\dot{m}_{\mathrm{in}} = \dot{m}_{\mathrm{out}} \qquad (3.27)$$