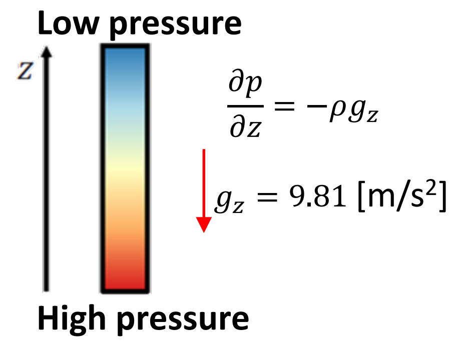
Exercise 2.

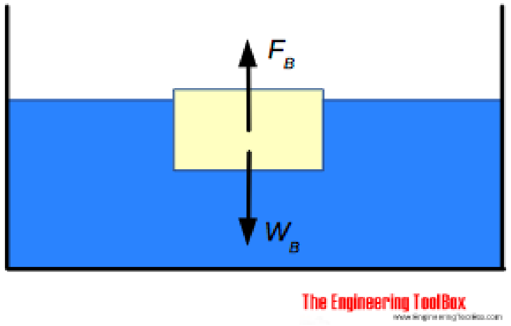
- Basic concepts:
  - **Incompressibility** = the density of the fluid is not dependent on pressure (or temperature)
    - ➢ **Liquids** are usually approximated as **incompressible** (i.e. const. density)
  - **Gases** are generally **compressible**
    - ➢ Equation of state for an ideal gas: $\rho = \frac{p}{RT}$
  - **Newtons 2a law** for a fluid (per unit volume)
    $\sum \tilde{f} = \rho \tilde{a} = \tilde{f}_{\text{press}} + \tilde{f}_{\text{grav}} + \tilde{f}_{\text{visc}} = -\tilde{\nabla} p + \rho \tilde{g} + \mu \tilde{\nabla}^2 \tilde{v}$ (2.8)
- Hydrostatic = fluid at rest
  - No viscous shear stress ($\tilde{f}_{\text{visc}} = 0$), no acceleration ($\tilde{a} = 0$) and gravity in *z* ($\tilde{g} = -g_z$)
  - $\sum \tilde{f} = 0 = \tilde{f}_{\text{press}} + \tilde{f}_{\text{grav}} \rightarrow \frac{\partial p}{\partial z} = -\rho g_z$ (2.11)
  - Integration gives: $\Delta p = \int_1^2 -\rho g_z \, \mathrm{d}z \rightarrow \Delta p = -\rho g_z \Delta z$ (2.14)
  - Pressure increase downward!



Exercise 2.

- Archimedes' principle:

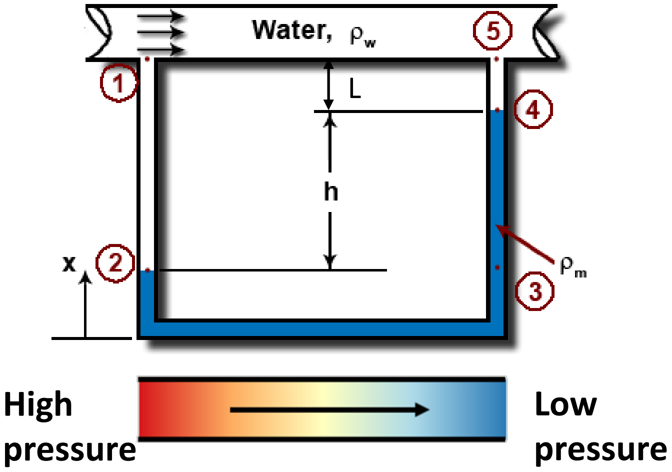
  ”Any object, totally or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object.”



$$F_{\mathrm{B}} = \rho_{\mathrm{fluid}}\, g\, V_{\mathrm{displaced\ fluid}}$$

- Manometer:
  - Go (1) → (5) through the manometer (hydrostatic)
  - Measure friction (viscous) losses between (1) and (5)

    $$p_1 - p_5 = -\rho_{\mathrm{w}} g(z_1 - z_2) - \rho_{\mathrm{m}} g(z_2 - z_4) - \rho_{\mathrm{w}} g(z_4 - z_5)$$
    $$\Delta \mathrm{p} = -\rho_{\mathrm{w}} g(L + h) - \rho_{\mathrm{m}} g(-h) - \rho_{\mathrm{w}} g(-L) \Rightarrow$$
    $$\Delta \mathrm{p} = (\rho_{\mathrm{m}} - \rho_{\mathrm{w}}) g h$$
    $$\mu \tilde{\nabla}^2 \tilde{v} = \Delta p = p_1 - p_5 = (\rho_m - \rho_w) g h$$
  - Note that $g = 9.81$, the minus sign is included in eqn.

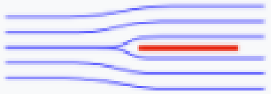
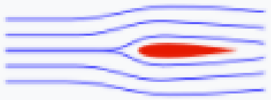
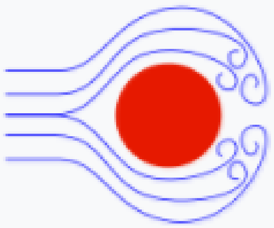
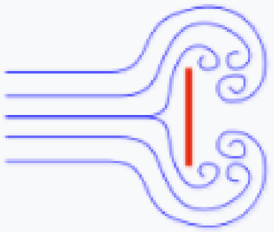


$$\tilde{\nabla} p = \mu \tilde{\nabla}^2 \tilde{v}$$

In x-direction: $\frac{\partial p}{\partial z} = \mu \tilde{\nabla}^2 u$

Two types of flow resistance:

- $\tilde{f}_{\text{press}}$= From drag
- $\tilde{f}_{\text{visc}}$ = Skin friction

Shape and flow	Form Drag	Skin friction
	0%	100%
	~10%	~90%
	~90%	~10%
	100%	0%