MTF053 - Fluid Mechanics

2026-01-08 08.30 – 13.30

Approved aids:

- *Fluid Mechanics MTF053 - Formulas, Tables & Graphs*
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- *Formelblad Matematik 5*
- Any calculator with cleared memory

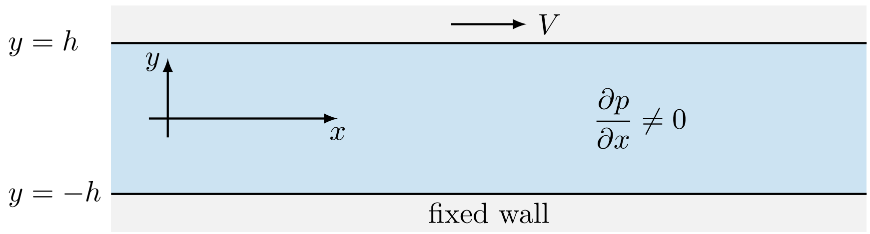
Exam Outline:

- In total 6 problems each worth 10p

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

PROBLEM 1 - FLOW BETWEEN PARALLEL PLATES (10 P.)



The fluid flow between two parallel plates is driven by a moving boundary (the upper plate is moving at a constant velocity V in the positive x-direction) and a non-zero pressure gradient. The fluid is incompressible and the flow can be assumed to be steady and fully developed. There is no flow in the z-direction and gravity can be neglected.

Find expressions for:

(a) The flow velocity distribution between the parallel plates (4.0 p.)

(b) A relation between the velocity of the upper plate (V) and the pressure gradient ($\partial p/\partial x$) such that the shear-stress at the lower wall is zero (2.0 p.)

(c) An expression for the area-averaged velocity between the parallel plates (2.0 p.)

Theory questions related to the topic:

(d) The x-component of the Navier-Sokes equations reads:

$$\rho g_x - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = \rho \frac{Du}{Dt}$$

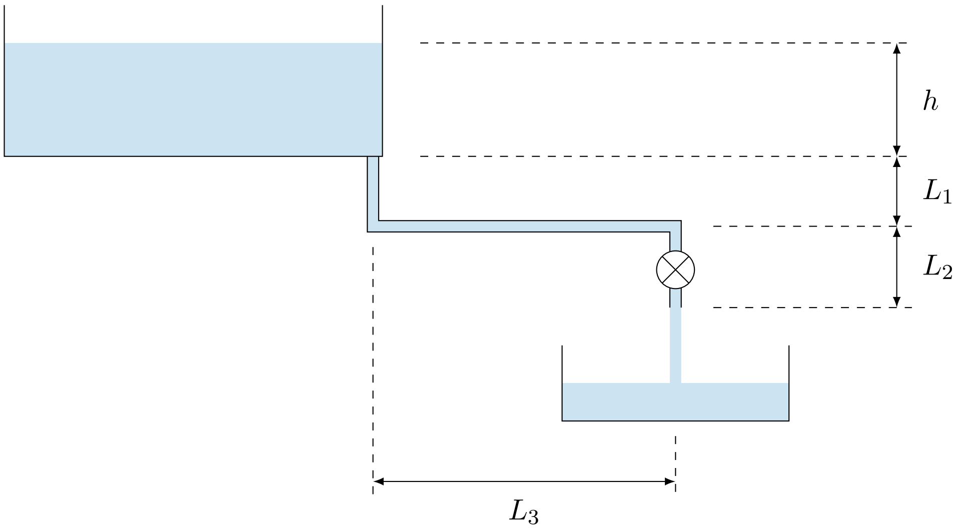
where Du/Dt is the substantial derivative

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

What is the physical meaning of each of the terms in the equation? (1.0 p.)

(e) What boundary condition is often used for velocity at a solid wall and what is the physical meaning of the boundary condition? (1.0 p.)

Problem 2 - Water Tank (10 p.)



Water @ 20°C is drained from a tank through a 0.1 m diameter (D) pipe attached to the bottom of the tank (see figure above). The pipe has two 90° bends, each with a loss coefficient of $K = 0.23$ and a valve with a loss coefficient of $K = 2.0$. The tank can be assumed to be very large which implies that the water level (h) can be approximated to be constant. The pipe lengths $L_1 - L_3$ are given in the table below.

D	0.1 m
h	3.0 m
L_1	0.5 m
L_2	0.5 m
L_3	5.0 m

(a) How long time will it take to drain 1.0 m^3 of water from the tank given that the pipe is made of a material with $\varepsilon = 0.2\ mm$ (7.0 p.)

The total loss related to the local loss coefficients can be calculated as

$$\Delta h_{local} = \frac{V^2}{2g}\sum_{i=1}^{n} K_i$$

where K_i is a local loss coefficient and V is the average pipe velocity

Theory questions related to the topic:

(b) What do we mean when we say that a pipe flow is *fully developed*? (1.0 p.)

(c) For laminar pipe flows, show that the friction factor is (2.0 p.)

$$f = \frac{64}{Re_D}$$

Problem 3 - Terminal Velocity (10 p.)

A group of students who recently took a course in fluid mechanics get into an argument about the details provided in a YouTube clip. In the clip, the terminal velocity of a parachutist is discussed and some of the students finds the provided information to be unrealistic and therefore they decide to try to make an estimate of the terminal velocity (the maximum velocity that a falling object can reach). They assume that the **weight of the parachutist including equipment is 80 kg**. Furthermore, they assume that the **drag of the parachutist is negligible compared with the drag of the parachute**. One of the students finds, after some googling, data from a **water tunnel experiment** where the drag of a model scale parachute is measured at different velocities. In the description of the parachute experiment it says that the tested parachute is a **1:10 scale model**. Data from the water tunnel experiment is provided below.

U $[m/s]$	F_D $[N]$
2	600
4	1840
6	4000
8	6800
10	8400

(a) Find a value for the terminal velocity of the parachutist based on the information given above (8.0 p.)
you will most likely have to iterate to solve the problem. Suggested solution strategy:

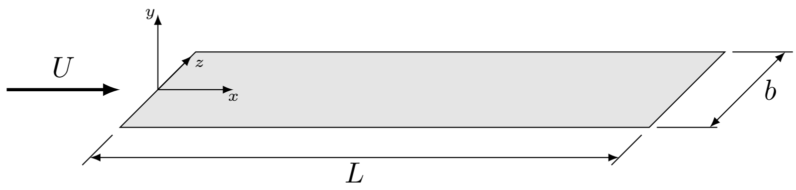
1. guess a value of the terminal velocity
2. use the information given above and your knowledge about similarity to evaluate if the velocity that you have guessed is correct. If not, make a new guess
3. keep trying until you find a velocity that is close enough (use your own judgement)

Theory questions related to the topic:

(b) Explain the concepts *geometric similarity*, *kinematic similarity*, and *dynamic similarity* (1.0 p.)

(c) Why do dimpled golf balls have lower pressure drag than golf balls with smooth surfaces? (1.0 p.)

Problem 4 - Flat Plate Flow (10 p.)



A flat plate is placed in a wind tunnel as part of an experimental setup. The boundary layer that builds up over the flat plate surface leads to a drag force that must be handled by the fastening devices. The flat plate will also be used for a similar experiment in a water tunnel, where the drag force will be greater. Flow and fluid properties are given in the table below (*use the same temperature and velocity for the water tunnel experiment and the wind tunnel experiment*)

Transition does not have to be accounted for, i.e. if you find that the boundary layer is turbulent, you can assume that it is turbulent all the way from the leading edge

L	2.0 m
b	2.0 m
U	3.0 m/s
T	20°C
p	100 kPa

(a) Calculate the total drag force for the two cases (wind tunnel and water tunnel) (5.0 p.)

(b) As part of the experiment, the flow velocity is measured at the location (x=2.0 m, y=0.5 mm, z=0). Calculate the average flow velocity at this location for the two cases (wind tunnel and water tunnel) (3.0 p.)

Theory questions related to the topic:

(c) For laminar flow over a flat plate, the velocity profile is self-similar - what does that mean? (1.0 p.)

(d) In what way is the transition location effected by (assume other properties to be constant) (1.0 p.)

i. increased freestream velocity U for a given $Re_{x,tr}$

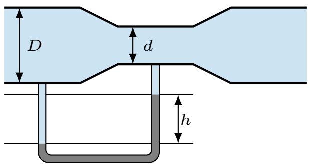
ii. surface roughness ε

iii. freestream turbulence

iv. positive pressure gradient

PROBLEM 5 - VENTURIMETER (10 P.)

A Venturimeter (a pipe with a contraction and a manometer) setup according to the illustration below is to be used to measure the flow velocity in a pipe. The fluid flowing through the pipe is air at $20.0°C$. The diameter of the pipe is $D = 20.0\ cm$ and in the contraction the tube diameter is reduced to $d = 10.0\ cm$.



(a) Find an expression for calculation of the flow rate Q $[m^3/s]$ in the pipe as a function of the manometer reading h, densities of the involved fluids, and pipe diameters. (5.0 p.)

(b) Calculate the flow rate Q and the change in average flow velocity when going from the larger pipe diameter (D) to the smaller pipe diameter (d) if mercury is used as manometer fluid and the manometer reading h is 6.0 mm. (3.0 p.)

Theory questions related to the topic:

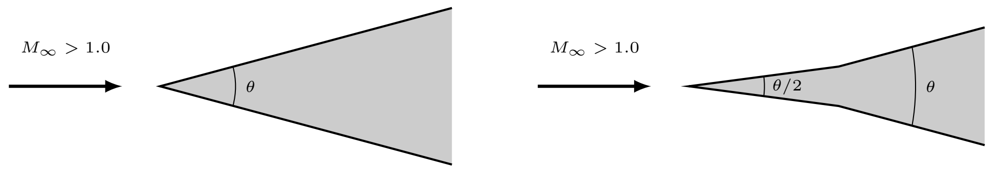
(c) Explain the physical meaning of each of the terms in Reynolds transport theorem (1.0 p.)

$$\frac{d}{dt}\left(B_{syst}\right) = \frac{d}{dt}\left(\int_{cv} \beta\rho d\mathcal{V}\right) + \int_{cs} \beta\rho\left(\mathbf{V}_r \cdot \mathbf{n}\right) dA$$

(d) What assumptions are made in the derivation of the Bernoulli equation? (1.0 p.)

Problem 6 - Supersonic Flow over Wedge (10 p.)

A vehicle for supersonic transport is to be constructed and a group of engineers investigate alternative designs of the nose cone. The left figure below (a) shows a nose cone where the flow deflection $\theta/2$ is done in one step and in the right figure (b), the same flow deflection is made but in two steps, each deflecting the flow an angle of $\theta/4$.



(a) (b)

(a) What is the minimum freestream Mach number that will lead to attached oblique shocks at the leading edge of the wedge in the two cases (a and b) if the wedge angle θ is 30°? (3.0 p.)

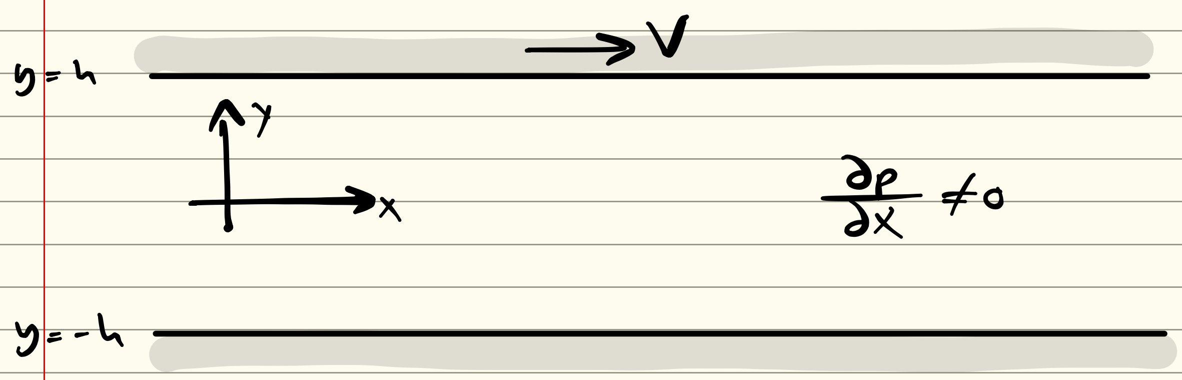
(b) Calculate the change in total pressure for the two cases if the freestream Mach number is 2.5 and the wedge angle θ is 30° (5.0 p.)

Theory questions related to the topic:

(a) Can the relations for total quantities for a normal shock be used for an oblique shock? Explain why/why not. (1.0 p.)

(b) The normal shock equation system has two solutions. How do we know which solution that is the correct one? (1.0 p.)

P1



ASSUMPTIONS:

- STEADY STATE $\Rightarrow \partial(\,)/\partial t = 0$
- FULLY DEVELOPED $\Rightarrow \partial(\,)/\partial x = 0$
- 2D-FLOW : $w = 0$, $\partial(\,)/\partial z = 0$
- CONFINED FLOW $\Rightarrow v = 0$
- NEGLECT GRAVITY $\Rightarrow (g_x, g_y, g_z) = (0,0,0)$

a) NAVIER STOKES:

$$\rho\left(\cancel{\frac{\partial u}{\partial t}} + u\cancel{\frac{\partial u}{\partial x}} + \cancel{v}\frac{\partial u}{\partial y} + \cancel{w}\cancel{\frac{\partial u}{\partial z}}\right) =$$

$$= -\frac{\partial p}{\partial x} + \cancel{\rho g_x} + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right)$$

$$\Rightarrow \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial p}{\partial x} = 0$$

INTEGRATE $\Rightarrow$

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial p}{\partial x}y + C_1$$

$$u(y) = \frac{1}{2\mu}\frac{\partial p}{\partial x}y^2 + C_1 y + C_2$$

BOUNDARY CONDITIONS :

$$u(-h) = 0$$

$$\frac{1}{2\mu}\frac{\partial p}{\partial x}(h^2) - C_1 h + C_2 = 0 \qquad (1)$$

$$u(h) = V$$

$$\frac{1}{2\mu}\frac{\partial p}{\partial x}(h^2) + C_1 h + C_2 = V \qquad (2)$$

$$(1) + (2) \Rightarrow$$

$$\frac{h^2}{\mu}\frac{\partial p}{\partial x} + 2C_2 = V$$

$$\Rightarrow C_2 = \frac{V}{2} - \frac{h^2}{2\mu}\frac{\partial p}{\partial x}$$

$$(1) - (2) \Rightarrow$$

$$-2C_1 h = -V \Rightarrow C_1 = \frac{V}{2h}$$

$$\boxed{u(y) = \frac{1}{2\mu}\frac{\partial p}{\partial x}(y^2 - h^2) + \frac{V}{2}\left(\frac{y}{h} + 1\right)}$$

b) $\tau_w(y = -h) = 0$:

$$\tau_w = \mu\frac{\partial u}{\partial y}$$

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial p}{\partial x} y + \frac{V}{2h}$$

$$\tau_w(y=-h) = 0 \Rightarrow$$

$$\left(-\frac{1}{\rho}\frac{\partial p}{\partial x}h + \frac{V}{2h}\right)\mu = 0$$

$$\Rightarrow \frac{V}{2h} = \frac{1}{\mu}\frac{\partial p}{\partial x}h \Rightarrow$$

$$\boxed{V = \frac{2h^2}{\mu}\frac{\partial p}{\partial x}}$$

c) AREA-AVERAGED VELOCITY:

$$\bar{u} = \frac{1}{A} w \int_{-h}^{h} u(y)\,dy$$

$$u(y) = \frac{1}{2\mu}\frac{\partial p}{\partial x}(y^2 - h^2) + \frac{V}{2}\left(\frac{y}{h} + 1\right)$$

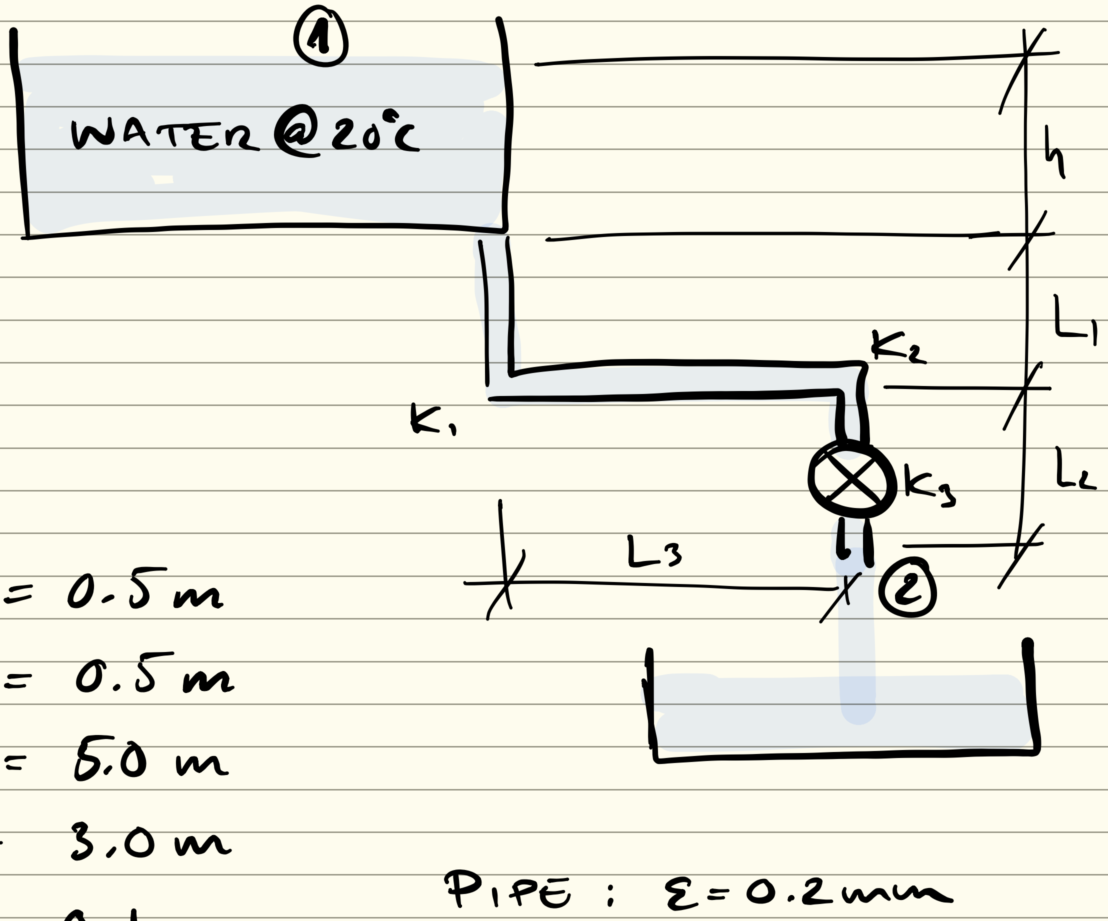
$$\Rightarrow \bar{u} = \frac{w}{2hw}\left[\frac{1}{2\mu}\frac{\partial p}{\partial x}\left(\frac{y^3}{3} - h^2 y\right) + \frac{V}{2}\left(\frac{y^2}{2h} + y\right)\right]_{-h}^{h}$$

$$= \frac{1}{2h}\left\{\frac{1}{2\mu}\frac{\partial p}{\partial x}\left(\frac{h^3}{3} - h^3\right) + \frac{V}{2}\left(\frac{h}{2} + h\right) + \right.$$

$$\left. - \left(\frac{1}{2\mu}\frac{\partial p}{\partial x}\left(\frac{-h^3}{3} + h^3\right) + \frac{V}{2}\left(\frac{h}{2} - h\right)\right)\right\}$$

$$\Rightarrow \boxed{\bar{u} = \frac{-h^2}{3\mu}\frac{\partial p}{\partial x} + \frac{V}{2}}$$

P2



$L_1 = 0.5\,m$

$L_2 = 0.5\,m$

$L_3 = 5.0\,m$

$h = 3.0\,m$

$D = 0.1\,m$

$K_1 = 0.23$

$K_2 = 0.23$

$K_3 = 2.0$

CALCULATE THE TIME THAT IT WOULD TAKE TO DRAIN $1.0\,m^3$ FROM THE TANK.

ASSUME STEADY-STATE, INCOMPRESSIBLE FLOW.

ENERGY EQUATION (3.73)
EVALUATED BETWEEN 1 AND 2

$$\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g} +$$

$$+ h_f + \cancel{h_t} + \cancel{h_p}$$

(NO PUMP OR TURBINE)

BIG TANK $\Rightarrow$ $V_1 \simeq 0$.

$P_1 = P_2 = P_{atm}$ $\Longrightarrow$

$$z_1 - z_2 = \frac{V^2}{2g} + h_f$$

$$\left.\begin{aligned} h_f &= \underbrace{f \frac{V^2}{2g} \frac{L}{D}}_{(6.10)} + h_{local} \\ h_{local} &= \frac{V^2}{2g} \sum_{i=1}^{3} k_i \end{aligned}\right\} h_f = \frac{V^2}{2g}\left(f \frac{L}{D} + \sum_{i=1}^{3} k_i\right)$$

$$\Rightarrow \boxed{\frac{V^2}{2g}\left(1 + f \frac{L}{D} + \sum_{i=1}^{3} k_i\right) = \Delta z_{1\to 2} \quad (1)}$$

SOLVE ITERATIVELY USING THE MOODY CHART.

$\frac{\varepsilon}{D} = 0.002$

$\nu = 10^{-6}\ m^2/s$ (WATER @ 20°C)

GUESS $V = 1\ m/s \Rightarrow 9.95 \times 10^4 \Rightarrow$

$\Rightarrow f = 0.0251$

$f = 0.0251$ in (1) $\Rightarrow V = 3.98\ m/s$

$V = 3.98\ m/s \Rightarrow Re = 3.96 \times 10^5 \Rightarrow f = 0.0239$

$f = 0.0239$ in (1) $\Rightarrow V = 4.0\ m/s$

(CLOSE ENOUGH..)

$V = 4.0\ m/s \Rightarrow Q = VA = V\frac{\pi D^2}{4} = 3.15 \cdot 10^{-2}\ m^3/s$

$t = \frac{Vol}{Q} = \frac{1.0}{Q} = 32\ s$

P3

FIND THE TERMINAL VELOCITY OF A PARACHUTIST

WEIGHT : 80 kg

PROVIDED EXPERIMENTAL DATA
(WATER TUNNEL, 1:10 SCALE MODEL)

U m/s	F_D N
2	600
4	1840
6	4000
8	6800
10	8400

ASSUMPTIONS : NEGLECT DRAG OF PARACHUTIST
TEMPERATURE: 20°C (BOTH AIR AND WATER)

$$20°C \Rightarrow \nu_{air} = 15 \cdot 10^{-6} \ m^2/s$$
$$\nu_{water} = 10^{-6} \ m^2/s$$

TERMINAL VELOCITY $\Rightarrow$ NO ACCELERATION
$\Rightarrow$ FORCE BALANCE : $F_D = mg$

DRAG FORCE (7.666)

$$\boxed{F_D = \frac{1}{2} C_D A \rho_{air} U^2 = mg \quad (1)}$$

$$Re = \frac{UD}{\nu_{air}}$$

FOR THE MODEL:

$$Re_m = \frac{U_m D_m}{\nu_{water}} = \frac{U_m D}{\nu_{water} 10} \quad (1:10 \text{ scale})$$

$$\boxed{Re = Re_m \Rightarrow U_m = \frac{10U}{15} \quad (2)}$$

SOLUTION:

1. GUESS U
2. CALCULATE DRAG FORCE USING THE PROVIDED DATA
3. CALCULATE MASS
4. MAKE NEW GUESS AND REPEAT (IF NEEDED)

#1. FIRST GUESS: $U = 10$ m/s

(2) $\Rightarrow U_m = 6.6$ m/s

INTERPOLATION $\Rightarrow F_m = 4980$ N

(7.66 b)

$$F_m = \frac{1}{2} C_{D_m} \left(\frac{D}{10}\right)^2 \rho_{water} U_m^2$$

$$F_D = \frac{1}{2} C_D D^2 \rho_{air} U^2$$

$C_{D_m} = C_D$ (SINCE $Re_m = Re$)

$$\Rightarrow \boxed{F_D = \frac{\rho_{air} F_m U^2 \cdot 100}{\rho_{water} U_m^2} \quad (3)}$$

$\Rightarrow$ $F_D = 1334$ N

$F_D = mg \Rightarrow m = 136$ kg > 80 kg.

#2. NEW GUESS : $U = 5.0$ m/s

(2) $\Rightarrow$ $U_m = 3.33$ m/s

INTERPOLATION $\Rightarrow$ $F_m = 1437$ N

(3) $\Rightarrow$ $F_D = 385$ N

$F_D = mg \Rightarrow m = 39$ kg < 80 kg

#3. NEW GUESS : $U = 7.5$ m/s

(2) $\Rightarrow$ $U_m = 5.0$ m/s

INTERPOLATION $\Rightarrow$ $F_m = 2947$ N

(3) $\Rightarrow$ $F_D = 789$ N

$F_D = mg \Rightarrow \underline{\underline{m = 80\text{ kg}}}$

$\Rightarrow$ TERMINAL VELOCITY : $\underline{7.5 \text{ m/s}}$

P4

FLAT PLATE INSTALLED IN WINDTUNNEL AND WATERTUNNEL.

$L = 2.0\,m$ (length)

$b = 2.0\,m$ (width)

$U = 3.0$ (freestream velocity)

$T = 20°C$

$p = 100\,kPa$

AIR:

USE THE IDEAL GAS LAW AND SUTHERLAND'S LAW TO CALCULATE DENSITY AND VISCOSITY:

(TABLE A.2)

$$\rho = \frac{p}{RT} \quad (\text{WHERE } R = 287\ J/kgK)$$

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{3/2} \left(\frac{T_0 + S}{T + S}\right)$$

WHERE:

$T_0 = 273\,K$

$\mu_0 = 1.71 \cdot 10^{-5}\ kg/ms$

$S = 110.4\,K$

$$\Rightarrow \rho_{air} = 1.189\ kg/m^3$$

$$\mu_{air} = 1.807 \cdot 10^{-5}\ kg/(ms)$$

$$\nu_{air} = \mu_{air}/\rho_{air} = 1.520 \cdot 10^{-5}\ m^2/s$$

WATER:

USE DATA FOR 1 atm.

$\rho_{water} = 998\ kg/m^3$

$\mu_{water} = 1.003 \cdot 10^{-3}\ kg/(ms)$

$\nu_{water} = 1.005 \cdot 10^{-6}\ m^2/s$

a) CALCULATE TOTAL DRAG FOR THE TWO CASES.

TOTAL DRAG:

$$D = 2\ \frac{1}{2} C_D \rho U^2 b L \qquad (1)$$

(TWO SIDES)

AIR

$$Re_L = \frac{\rho_{air} U L}{\mu_{air}} = 3.95 \times 10^5 < 5.0 \times 10^5$$

$\Rightarrow$ LAMINAR

$$(7.27): \quad C_D = \frac{1.328}{Re_L^{1/2}} = 2.11 \times 10^{-3}$$

$$(1) \Rightarrow \boxed{D = 9.05 \times 10^{-2}\ N}$$

WATER:

$$Re_L = \frac{\rho_{water} U L}{\mu_{water}} = 5.97 \times 10^6 > 5.0 \times 10^5$$

$\Rightarrow$ TURBULENT

$$(7.45): \quad C_D = \frac{0.031}{Re_L^{1/7}} = 3.34 \times 10^{-3}$$

$$(1) \Rightarrow \boxed{D = 120\,N}$$

b) CALCULATE THE VELOCITY AT ($x = 2.0$ m, $y = 0.5$ mm)

AIR:

LAMINAR $\Rightarrow$ BLASIUS (TABLE 7.1)

$$\eta = y\left(\frac{U}{\nu x}\right)^{1/2} = 0.157$$

INTERPOLATION $\Rightarrow \frac{u}{U} = 0.52$

$$\Rightarrow \boxed{u(x,y) = 0.156 \text{ m/s}}$$

WATER:

TURBULENT FLOW:

(7.44)

$$\tau_w = \frac{0.0135\, \mu^{1/7}\, \rho^{6/7}\, U^{13/7}}{x^{1/7}} = 13.05\ Pa$$

$$u^* = \sqrt{\frac{\tau_w}{\rho}} = 1.144 \times 10^{-1}\ m/s$$

(6.29)

$$y^+ = \frac{y u^*}{\nu} = 56.9 > 30$$

$\Rightarrow$ LOG REGION

(6.28)

$$\frac{u}{u^*} = \frac{1}{\varkappa} \ln(y^+) + B$$

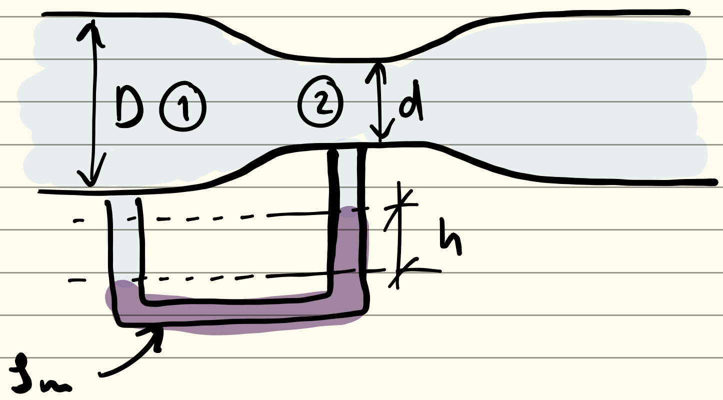
$(\varkappa = 0.41\ ,\ B = 5.0)$

$$\Rightarrow \boxed{u(x, y) = 1.7\ m/s}$$

P5

FIND AN EXPRESSION FOR THE FLOW RATE USING THE GIVEN VENTURI-METER SPECIFICATION.

ASSUME: STEDY STATE
INCOMPRESSIBLE



(3.22) CONTINUITY EQN $\Rightarrow$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$\rho_1 = \rho_2$ (INCOMPRESSIBLE)

$$\Rightarrow A_1 V_1 = A_2 V_2 = Q$$

$$V_1 \frac{\pi D^2}{4} = V_2 \frac{\pi d^2}{4} \Rightarrow V_2 = V_1 \left(\frac{D}{d}\right)^2 \quad (1)$$

(3.54) BERNOULLI BETWEN 1 AND 2

$z_1 = z_2 \Rightarrow$

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2 \quad (2)$$

(1) AND (2) $\Rightarrow$

$$V_1^2 = \frac{2(P_2 - P_1)}{\rho\left(\left(\frac{D}{d}\right)^4 - 1\right)} \quad (3)$$

$$(2.14) \Rightarrow P_2 - P_1 = h(\rho_m - \rho)g \quad (4)$$

(4) IN (3) $\Rightarrow$

$$V_1 = \sqrt{2gh\left(\frac{\rho_m}{\rho} - 1\right) \Big/ \left(\left(\frac{D}{d}\right)^4 - 1\right)} \quad (5)$$

$$\boxed{Q = V_1 A_1 = \frac{\pi D_1^2}{4}\sqrt{2gh\left(\frac{\rho_m}{\rho} - 1\right) \Big/ \left(\left(\frac{D}{d}\right)^4 - 1\right)} \quad (6)}$$

b)

$h = 6\ mm$

$\rho_m = 13550\ kg/m^3$

$(6) \Rightarrow \boxed{Q = 0.3\ m^3/s}$

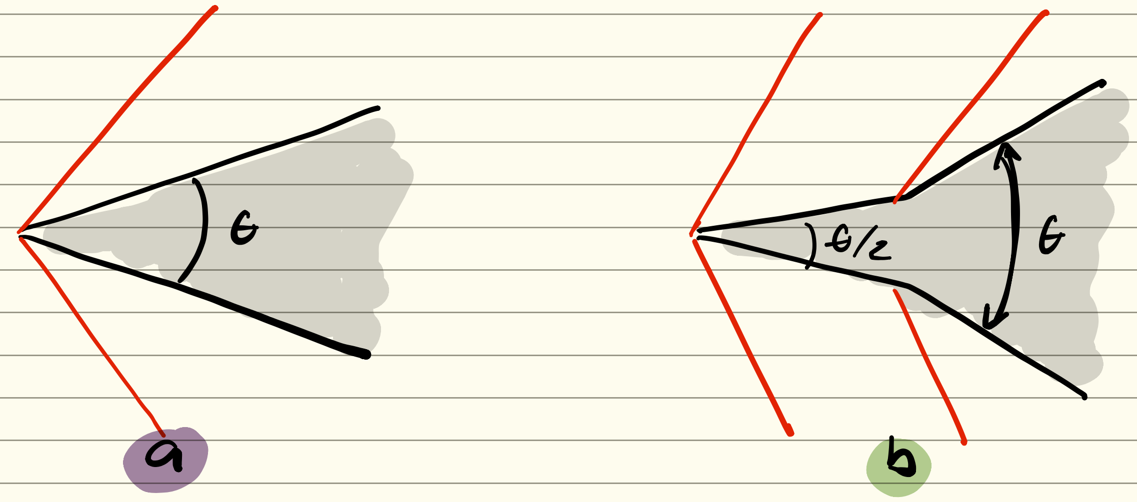
$(5) \Rightarrow V_1 = 9.4\ m/s$

$(1) \Rightarrow V_2 = 37.7\ m/s$

$$\boxed{V_2 - V_1 = 28.3\ m/s}$$

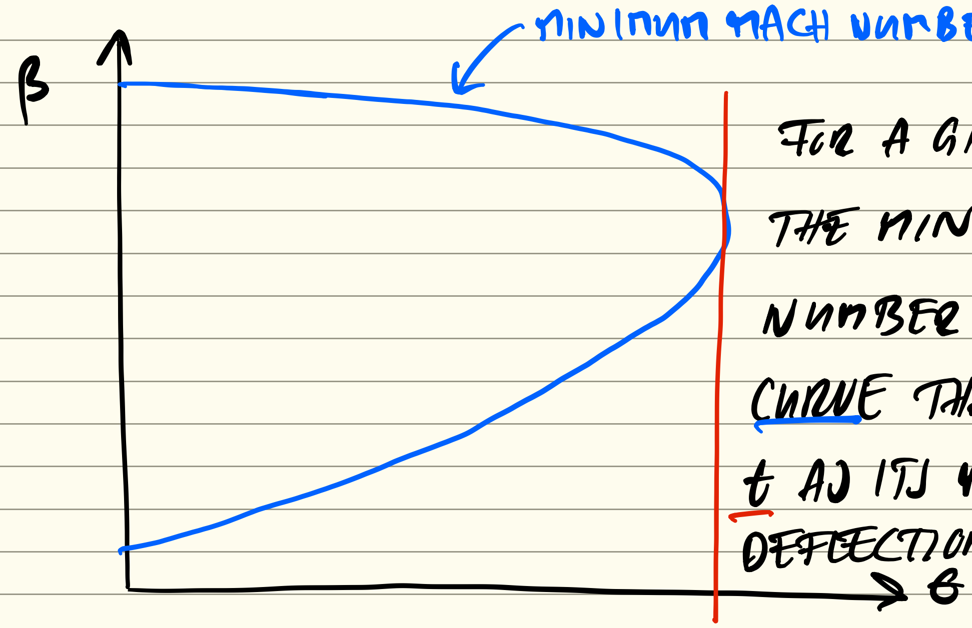
P6

EVALUATE THE OBLIQUE SHOCKS FORMED AT THE LEADING EDGE OF A NOSE-CONE WITH TWO DESIGN CONCEPTS



a) WHAT IS THE MINIMUM MACH NUMBER FOR WHICH AN ATTACHED OBLIQUE SHOCK WILL BE FORMED AT THE LEADING EDGE OF THE WEDGE IN THE TWO CASES?

THE MINIMUM MACH NUMBER CAN BE FOUND FROM THE θ-β-M-RELATION (FIGURE 9.23)



FOR A GIVEN θ, THE MINIMUM MACH NUMBER IS THE CURVE THAT HAS θ AS ITS MAXIMUM DEFLECTION

MINIMUM MACH NUMBER

WEDGE a:

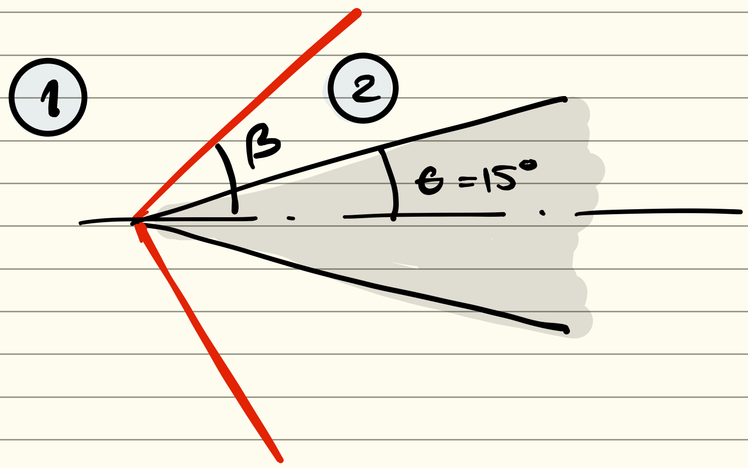
FLOW DEFLECTION : $\theta = 15°$
MINIMUM MACH NUMBER: 1.62

WEDGE b:

FLOW DEFLECTION : $\theta = 7.5°$
MINIMUM MACH NUMBER : 1.34

SO, SINCE CONCEPT b HAS A LEADING EDGE DEFLECTION THAT IS HALF THAT OF CONCEPT a, CONCEPT b WILL WORK FOR LOWER MACH NUMBERS THAN a

b) CALCULATE THE CHANGE IN TOTAL PRESSURE FOR THE TWO CASES IF THE FREESTREAM MACH NUMBER IS 2.5



$\theta - \beta - M$ RELATION (9.86)

$\theta = 15°, M = 2.5 \Rightarrow \boxed{\beta = 36.9°}$

(9.82)

$$M_{n_1} = M_1 \sin\beta$$

(9.57)

$$M_{n_2} = \frac{(\gamma-1)M_{n_1}^2 + 2}{2\gamma M_{n_1}^2 - (\gamma-1)}$$

(9.82)

$$M_2 = M_{n_2} / \sin(\beta - \theta)$$

$$\Rightarrow \boxed{M_2 = 1.87}$$

(9.55)

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_{n_1}^2 - 1)$$

$$\Rightarrow \boxed{\frac{p_2}{p_1} = 2.47}$$

TOTAL PRESSURE RATIO:

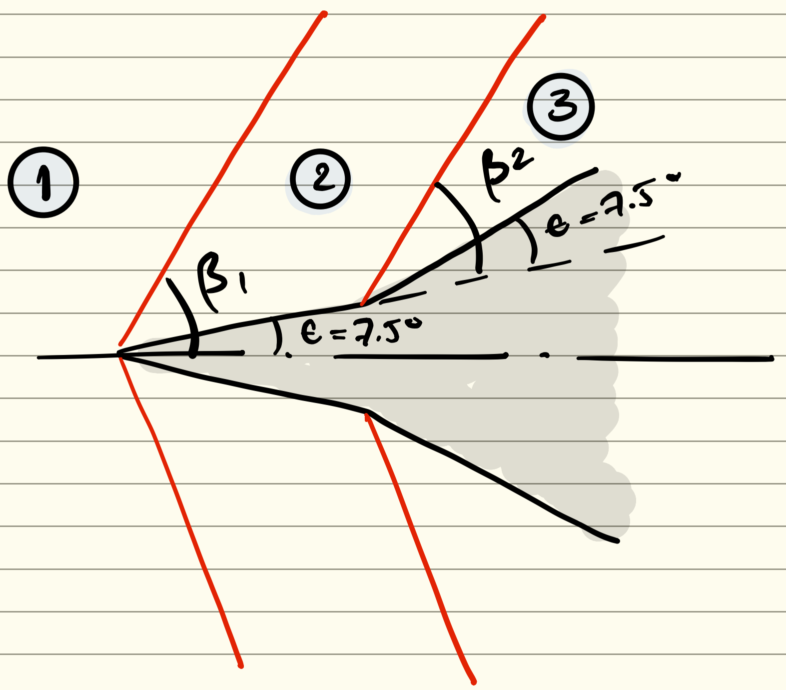
$$\frac{p_{02}}{p_{01}} = \frac{p_{02}}{p_2}\frac{p_2}{p_1}\frac{p_1}{p_{01}}$$

(9.28)

$$\frac{P_{02}}{P_2} = \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\gamma/(\gamma-1)}$$

$$\frac{P_{01}}{P_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\gamma/(\gamma-1)}$$

$$\Rightarrow \boxed{\frac{P_{02}}{P_{01}} = \frac{P_{02}}{P_2} \frac{P_2}{P_1} \frac{P_1}{P_{01}} = 0.93}$$



1 → 2

(9.86) $\epsilon - \beta - M$

$\epsilon = 7.5°$, $M_1 = 2.5$ $\Rightarrow \boxed{\beta_1 = 29.6°}$

(9.82)

$$M_{n1} = M_1 \sin \beta$$

(9.57)

$$M_{n_2}^2 = \frac{(\gamma-1) M_{n_1}^2 + 2}{2\gamma M_{n_1}^2 - (\gamma-1)}$$

(9.82)

$$M_2 = M_{n_2} / \sin(\beta_1 - \theta)$$

$$\Rightarrow \boxed{M_2 = 2.19}$$

(9.55)

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_{n_1}^2 - 1)$$

$$\Rightarrow \boxed{\frac{p_2}{p_1} = 1.61}$$

2 → 3

(9.86) $\theta - \beta - M$

$\theta = 7.5°$, $M_2 = 2.19 \Rightarrow \boxed{\beta_2 = 33.5°}$

(9.82)

$$M_{n_1} = M_2 \sin\beta_2$$

(9.57)

$$M_{n_2}^2 = \frac{(\gamma-1) M_{n_1}^2 + 2}{2\gamma M_{n_1}^2 - (\gamma-1)}$$

(9.82)

$$M_3 = M_{n2} / \sin(\beta_2 - \theta)$$

$$\Rightarrow \boxed{M_3 = 1.91}$$

(9.55)

$$\frac{p_3}{p_2} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right)$$

$$\Rightarrow \boxed{\frac{p_3}{p_2} = 1.54}$$

TOTAL PRESSURE RATIO:

$$\frac{p_{03}}{p_{01}} = \frac{p_{03}}{p_3}\,\frac{p_3}{p_2}\,\frac{p_2}{p_1}\,\frac{p_1}{p_{01}}$$

(9.28)

$$\frac{p_{03}}{p_3} = \left(1 + \frac{\gamma-1}{2} M_3^2\right)^{\gamma/(\gamma-1)}$$

$$\frac{p_{01}}{p_1} = \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\gamma/(\gamma-1)}$$

$$\boxed{\frac{p_{03}}{p_{01}} = \frac{p_{03}}{p_3}\,\frac{p_3}{p_2}\,\frac{p_2}{p_1}\,\frac{p_1}{p_{01}} = 0.98}$$

CASE a

$$\frac{P_{02}}{P_{01}} = 0.93$$

CASE b

$$\frac{P_{03}}{P_{01}} = 0.98$$

MAKING THE SAME TOTAL FLOW DEFLECTION WITH TWO SHOCKS LEADS TO LESS LOSSES THAN ONE SINGLE SHOCK (THE TOTAL PRESSURE IS LESS REDUCED IN CASE b, WHICH IS DIRECTLY RELATED TO FLOW LOSSES - ENTROPY INCREASE)