

MTF053 - Fluid Mechanics

2025-10-31 08.30 – 13.30

Approved aids:

- The formula sheet handed out with the exam
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- *Formelblad Matematik 5*
- Any calculator with cleared memory

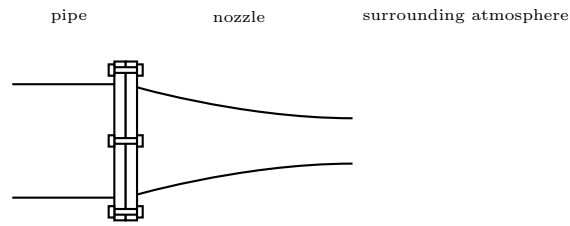
Exam Outline:

- In total 6 problems each worth 10p

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

PROBLEM 1 - WATER (10 P.)



Water @ 20°C is accelerated through a nozzle. The nozzle is attached to a pipe by four bolts. The pipe diameter is the same as the inlet diameter of the nozzle (10.0 cm) and the nozzle-outlet diameter is 4.0 cm. At the exit of the nozzle, the pressure is 100 kPa.

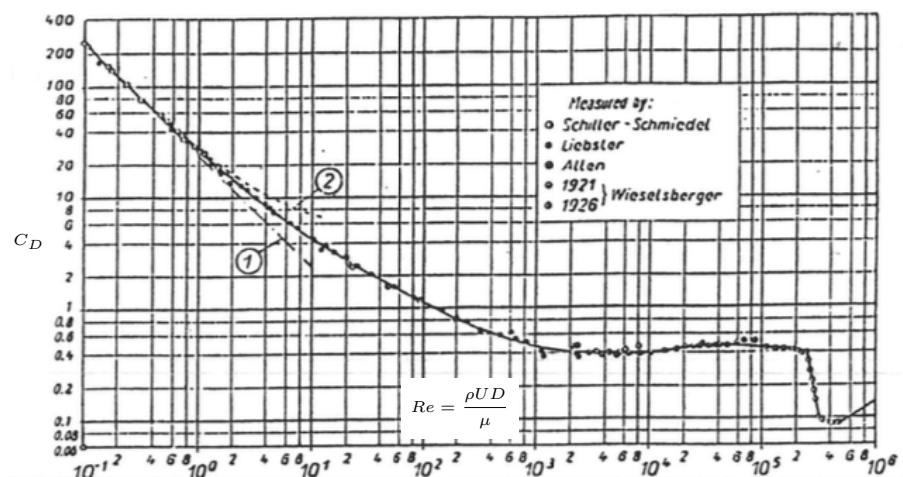
- (a) Calculate the maximum flow rate in m^3/s if the force in each of the four bolts is limited to 450.0 N (8p)

Theory questions related to the topic:

- (b) Derive the continuity equation on integral form for a fixed control volume using Reynolds transport theorem (2p)

PROBLEM 2 - ICING (10 P.)

A hockey puck glides over ice at a velocity of $V = 10 \text{ m/s}$. The hockey puck has a diameter of 7.0 cm and a thickness of 2.5 cm. A 0.1 mm water film is formed between the ice and the puck (you can assume that the temperature of the water in the water film is 0°C and that the temperature of the air is 10°C).



- (a) What is the relation between drag related to the contact with the water film and form drag of the puck as it moves along the ice? (8p)

Theory questions related to the topic:

- (b) What does it mean that a fluid is Newtonian? (1p)
- (c) The variation of fluid viscosity with temperature is different for liquids and gases. What is the difference and why is it so? (1p)

PROBLEM 3 - COOLING WATER PIPE (10 P.)

A 320 m long pipe with the inner diameter of 400 mm is used for delivery of cooling water. The inlet of the pipe is located 4.5 m above the outlet and the pressure at the inlet and outlet is 150 kPa and 120 kPa, respectively. Under these conditions, the pipe delivers cooling water at the flow rate $1100 \text{ m}^3/\text{h}$. The temperature of the cooling water can be assumed to be 10°C .

- (a) There is a wish to increase the flow rate by 40% and to do so the idea is to increase the inlet pressure by replacing the upstream pump. How much must the inlet pressure be increased? (8p)

Theory questions related to the topic:

- (b) What do we mean when we say that a pipe flow is *fully developed*? (1p)
- (c) Why does the Moody chart not give reliable values in the Reynolds number range $2000 < Re < 4000$? (1p)

PROBLEM 4 - BOUNDARY LAYER (10 P.)

Air flows over a flat surface. At a certain distance, x , downstream of the leading edge of the flat surface, the freestream velocity (the velocity outside of the boundary layer built up over the surface) is 15 m/s and at the same axial location, x , inside of the boundary layer at $y^+ = 5$, the shear stress is $\tau = 0.39 \text{ N/m}^2$.

For non-dimensional wall-normal coordinates in the range $30 < y^+ < 1000$, the non-dimensional fluid velocity is given by

$$u^+ = 2.44 \ln(y^+) + 4.9$$

- (a) What is the wall shear stress τ_w at this location (2p)
- (b) At the upper limit for the log-law given above (at the wall-normal coordinate where relation is no longer valid), what is the ratio of local velocity and freestream velocity? (5p)
- (c) Can we say that we have reached the edge of the boundary layer at $y^+ = 1000$? (your answer has to be justified by calculated values) (1p)

Theory questions related to the topic:

- (d) For laminar flow over a flat plate, the velocity profile is self-similar - what does that mean? (1p)
- (e) Name two alternative ways to measure the boundary layer thickness other than δ . How can these measures be interpreted physically? (1p)

PROBLEM 5 - SMOKESTACK (10 P.)

A paper mill smokestack is to be replaced. The new smokestack is 20 meters high and has a cylindrical cross section with an outer diameter of 2.5 m. During a construction meeting it is decided to do model-scale measurements to verify the construction analyses. The parameters that the team wishes to verify are the bending moment at the base of the smokestack during wind gusts and the oscillating frequency during turbulent vortex shedding conditions. The construction team contacts a wind tunnel lab for consultation. When they send their specifications to the lab (wind gust speed: 30 m/s and height of model scale smokestack: 2.0 m), they get an immediate response that it will not be possible to do a test like that in the wind tunnel. Instead a water tunnel experiment is suggested.

- (a) Why do you think that the lab team say that a wind tunnel test is impossible? (justify your answer with calculations) (2p)
- (b) In the water tunnel test, a bending moment at the base of the smoke stack of 50 kNm is measured for wind gust conditions, what is the corresponding bending moment for the full scale smokestack? (3p)
- (c) Turbulent vortex shedding is observed at a flow velocity of 12.0 m/s in the water tunnel and the corresponding frequency is measured to be 10.56 Hz. What will frequency be for the full scale smokestack? *hint: the Strouhal number is the same for the model scale test and the full scale smokestack* (3p)

Theory questions related to the topic:

- (d) If you are going to do an experimental investigation of a problem including a number of important physical variables, why is it beneficial to divide the variables into non-dimensional groups? (1p)
- (e) The drag coefficient, C_D , can be divided into two components, which two components? What phenomena are associated with each of the two components? (1p)

PROBLEM 6 - CD NOZZLE FLOW (10 P.)

A convergent-divergent nozzle is attached to a large tank where the pressure and temperature is 2.0 MPa and 27°C, respectively. The nozzle is designed for a pressure ratio $p_o/p_e = 10$. The exit area of the nozzle is $A_e = 10 \text{ cm}^2$.

- (a) Calculate the massflow through the nozzle if the pressure downstream of the nozzle exit is 100 kPa (4p)
- (b) Calculate the exit velocity for the same condition (3 p)
- (c) The operation conditions for a convergent-divergent nozzle can be categorized as:
 - subcritical
 - critical
 - choked
 - internal shock
 - shock at exit
 - overexpanded
 - supercritical
 - underexpanded
 - isentropic internal flow
 - non-isentropic internal flow

which of these give a correct description of the flow described above? (can be more than one) your answer must be justified (1p)

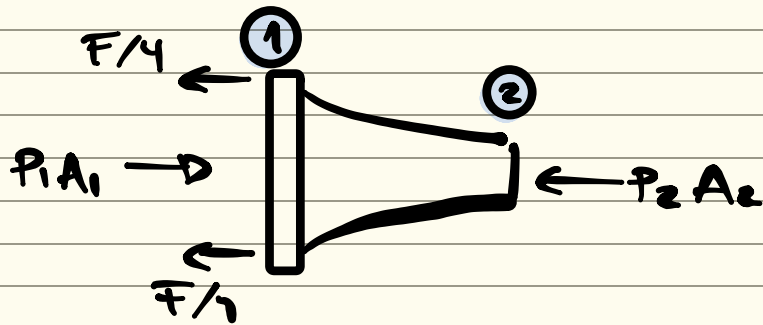
Theory questions related to the topic:

- (d) Which of the properties h_o , T_o , a_o , p_o , and ρ_o are constants in a flow if the flow is adiabatic and isentropic, respectively? (1p)
- (e) Show, using the area-velocity relation below, how the velocity changes in a flow through a divergent or convergent duct for initially subsonic flow or initially supersonic flow (1p)

$$\frac{dA}{A} = (M^2 - 1) \frac{dV}{V}$$

P_1

WATER @ 20°C IS ACCELERATED
THROUGH A NOZZLE



THE FORCE IN EACH OF THE FOUR BOLTS
IS LIMITED TO 450 N - FIND Q_{max} ..

$$D_1 = 10 \text{ cm} \quad , \quad D_2 = 4 \text{ cm} \quad , \quad P_2 = 100 \text{ kPa}$$

ASSUMPTIONS :

STEADY STATE

INCOMPRESSIBLE

(3.22) CONSERVATION OF MASS
(ONE INLET & ONE OUTLET)

STEADY - STATE + INCOMPRESSIBLE $\Rightarrow$

$$A_1 V_1 = A_2 V_2$$

(1)

(3.54)

$$P_1 + \frac{1}{2} \rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g z_2$$

$$z_1 = z_2 \Rightarrow P_1 + \frac{1}{2} \rho V_1^2 = P_2 + \frac{1}{2} \rho V_2^2$$

$$\Rightarrow \boxed{p_1 = p_2 + \frac{1}{2} \rho (V_2^2 - V_1^2)} \quad (2)$$

(3.70)

$$\sum F_x = \dot{m}_2 V_2 - \dot{m}_1 V_1$$

$$4F + p_1 A_1 - p_2 A_2 = \dot{m} (V_2 - V_1)$$

$$\dot{m} = \rho Q$$

$$V_1 = \frac{Q}{A_1}$$

$$V_2 = \frac{Q}{A_2}$$

$$\Rightarrow -4F + p_1 A_1 - p_2 A_2 = \rho Q \left(\frac{1}{A_2} - \frac{1}{A_1} \right)$$

(2) $\Rightarrow$

$$\begin{aligned} -4F + p_2 A_1 + \frac{1}{2} \rho (V_2^2 - V_1^2) A_1 - p_2 A_2 &= \\ &= \rho Q \left(\frac{1}{A_2} - \frac{1}{A_1} \right) \end{aligned}$$

$$\begin{aligned} -4F + p_2 (A_1 - A_2) + \frac{1}{2} \rho Q^2 \left(\frac{1}{A_2^2} - \frac{1}{A_1^2} \right) A_1 &= \\ &= \rho Q^2 \left(\frac{1}{A_2} - \frac{1}{A_1} \right) \end{aligned}$$

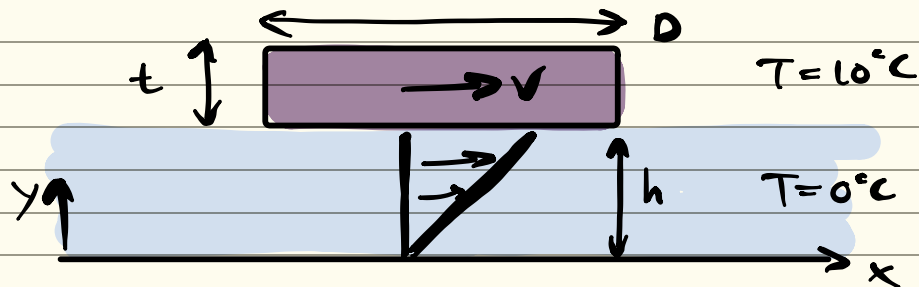
$$\Rightarrow 4F - p_2 (A_1 - A_2) =$$

$$= \rho Q^2 \left(\frac{A_1}{2} \left(\frac{1}{A_2^2} - \frac{1}{A_1^2} \right) - \left(\frac{1}{A_2} - \frac{1}{A_1} \right) \right)$$

$$F = 450 \text{ N}, \quad p_2 = 100 \text{ kPa}, \quad \rho = 998 \text{ kg/m}^3$$

$$\Rightarrow \boxed{Q = 0.025 \text{ m}^3/\text{s}}$$

P₂



$$t = 0.025 \text{ m}$$

WHAT IS THE RELATION

$$D = 0.07 \text{ m}$$

BETWEEN THE WATER -

$$h = 0.0001 \text{ m}$$

FILM DRAG AND THE

$$V = 10 \text{ m/s}$$

FORM DRAG?

(9.38) X-DIRECTION

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \rho g_x - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

ASSUMPTIONS:

STEADY STATE $\Rightarrow \frac{\partial}{\partial t} () = 0$

NEGLECT GRAVITY: $g = 0$

ONLY FLOW IN X-DIRECTION ($v = w = 0$)

FULLY DEVELOPED $\Rightarrow \frac{\partial}{\partial x} () = 0$

INCOMPRESSIBLE: $\rho = \text{CONST.}$

NO PRESSURE GRADIENT $\Rightarrow \frac{\partial p}{\partial x} = 0$

$$\Rightarrow \frac{\partial^2 u}{\partial y^2} = 0$$

INTEGRATE:

$$\frac{\partial u}{\partial y} = C_1$$

$$u(y) = C_1 y + C_2$$

Boundary conditions: (No slip)

$$u(0) = 0$$

$$u(h) = V$$

$$\left. \begin{array}{l} u(0) = 0 \Rightarrow C_2 = 0 \\ u(h) = V \Rightarrow C_1 = \frac{V}{h} \end{array} \right\} \Rightarrow u(y) = \frac{V}{h} y$$

TABLE A1: $\rho_{\text{water}} = 1.79 \cdot 10^{-3} \text{ kg/(ms)}$

($T = 0^\circ\text{C}$)

$$\begin{aligned} F_{\text{water}} &= \rho_{\text{water}} \left. \frac{\partial u}{\partial y} \right|_{y=h} A = \\ &= \rho_{\text{water}} \frac{V}{h} \frac{\pi D^2}{4} = \boxed{0.69 \text{ N}} \end{aligned}$$

Form drag:

$$F_{\text{air}} = C_D \frac{1}{2} \rho_{\text{air}} V^2 A_p$$

$$Re_D = \frac{\rho_{\text{air}} V D}{\mu_{\text{air}}}$$

TABLE A2:

$$\rho_{\text{air}, 10^\circ\text{C}} = \rho_{\text{air}, 20^\circ\text{C}} \frac{293}{283} = 1.27 \text{ kg/m}^3$$

$$\frac{p_{\text{air}, 10^\circ\text{C}}}{p_0} = \left(\frac{T}{T_0} \right)^{5/2} \left(\frac{T_0 + S}{T + S} \right)$$

$$T_0 = 273\text{K}, S = 110.7\text{K}, p_0 = 1.71 \cdot 10^{-5} \text{ kg/(ms)}$$

$$\Rightarrow p_{\text{air}, 10^\circ\text{C}} = 1.759 \cdot 10^{-5} \text{ kg/(ms)}$$

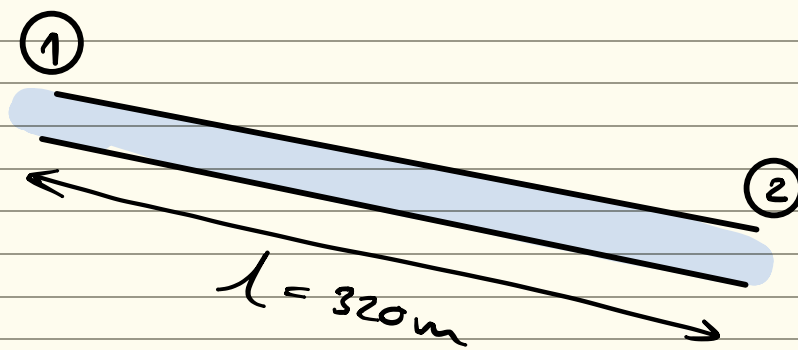
$$\Rightarrow Re_D = \frac{\rho_{\text{air}} V D}{\mu_{\text{air}}} \approx 4.7 \cdot 10^4$$

$$F_{\text{ic.}} \Rightarrow C_D \sim 0.5$$

$$\left. \begin{aligned} F_{\text{air}} &= C_D \frac{1}{2} \rho_{\text{air}} V^2 A_p \\ A_p &= t \cdot D \end{aligned} \right\} \boxed{F_{\text{air}} = 0.05 \text{ N}}$$

$$\Rightarrow \boxed{F_{\text{air}} / F_{\text{water}} = 0.077}$$

P₃



$$D = 0.4 \text{ m}$$

$$Q = 1100 \text{ m}^3/\text{h}$$

$$z_2 - z_1 = -9.5 \text{ m}$$

$$P_1 = 150 \text{ kPa}$$

$$P_2 = 120 \text{ kPa}$$

WATER @ 10°C

ASSUME:

- STEADY STATE
- INCOMPRESSIBLE
- Fully DEVELOPED

Q is to BE INCREASED 40%

CALCULATE THE NEW P₁

CASE 1: $Q = 1100 \text{ m}^3/\text{h}$

(3.75)

$$\left(\frac{P}{\rho g} + \frac{\alpha V^2}{2g} + z \right)_1 = \left(\frac{P}{\rho g} + \frac{\alpha V^2}{2g} + z \right)_2 + \cancel{h_{it}} - \cancel{h_{fp}} + h_f$$

Fully - DEVELOPED , STEADY - STATE $\Rightarrow$

$$\frac{\alpha_1 V_1^2}{2g} = \frac{\alpha_2 V_2^2}{2g}$$

$$\alpha_1 = \alpha_2$$

$$V_1 = V_2 \quad (QA = \text{CONST})$$

$\Rightarrow$

$$P_1 + \rho g z_1 = P_2 + \rho g z_2 + \Delta P_f$$

$$(z_1 = 0)$$

$$T = 10^{\circ}\text{C} \Rightarrow \rho = 1000 \text{ kg/m}^3$$

$$\Rightarrow \Delta p_f = 0.79 \cdot 10^5 \text{ Pa}$$

$$(6.10) \quad h_f = f \frac{L}{D} \frac{V^2}{2g} \Rightarrow$$

$$\Delta p_f = f \frac{L}{D} \rho \frac{V^2}{2}$$

$$V = \frac{Q}{A} = \frac{Q}{\pi D^2}$$

$$\Rightarrow \Delta p_f = f \frac{L}{D} \rho \frac{8Q^2}{\pi^2 D^5} = f \frac{8 \rho L Q^2}{\pi^2 D^5}$$

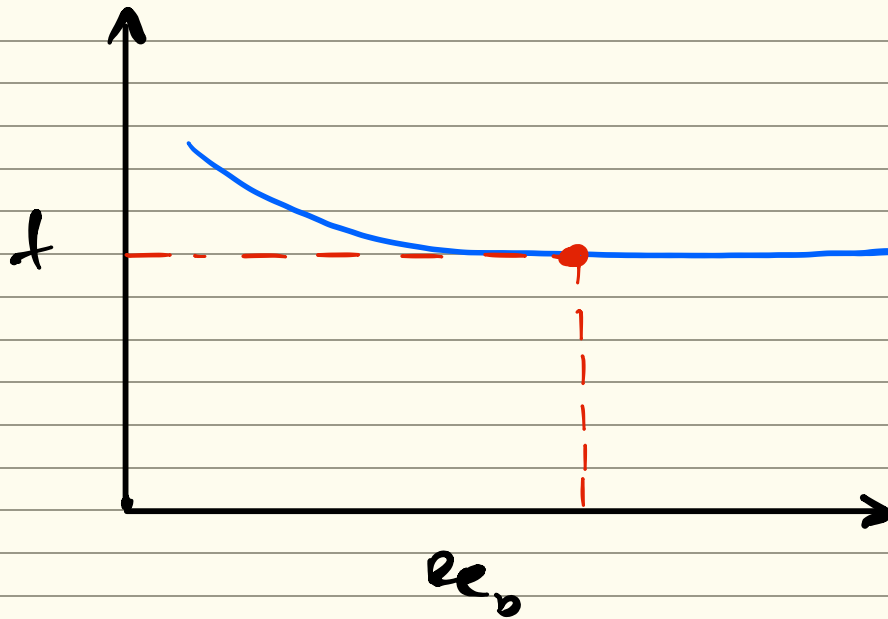
$$\Rightarrow f = 0.0319$$

$$Re_D = \frac{\rho V D}{\mu}$$

$$T = 10^{\circ}\text{C} \Rightarrow \mu = 1.81 \cdot 10^{-6} \text{ Pa}\cdot\text{s}$$

$$\Rightarrow Re_D = 7.5 \cdot 10^5$$

Moody chart:



$$Re_D = 7.5 \cdot 10^5 \text{ \& } f = 0.0314 \Rightarrow$$

Final result $\Rightarrow$

40% INCREASE OF $Q \Rightarrow$

INCREASE OF $Re \Rightarrow f$ WILL NOT
CHANGE (INDEPENDENT OF Re)

CASE 2: $Q_2 = 1.4 Q$

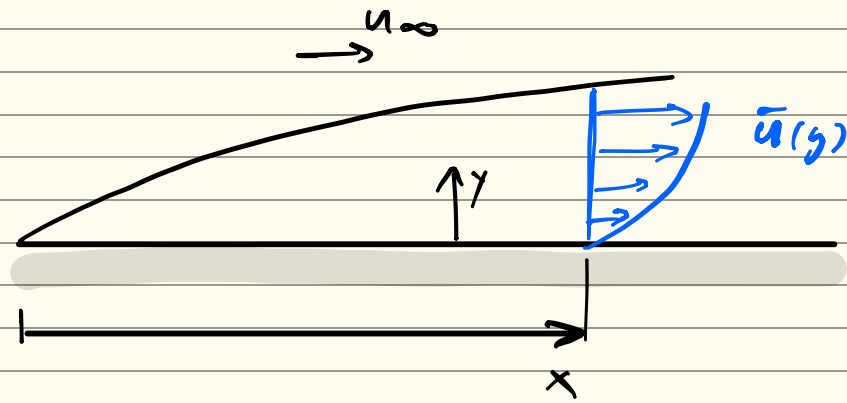
$$\Delta p_{f2} = f \frac{8 \rho L Q_2^2}{\pi^2 D^5} \Rightarrow$$

$$\Delta p_{f2} = 196 \text{ kPa}$$

$$P_1 + \rho g z_1 = P_2 + \Delta p_{f2} \Rightarrow$$

$$p_{new} = 221.2 \text{ kPa}$$

P₉



$$U_{\infty} = 15 \text{ m/s}$$

$$y^+ = 5 \Rightarrow \tau = 0.39 \text{ N/m}^2$$

$$80 < y^+ < 1000 : u^+ = 2.44 \ln(y^+) + 4.9$$

a) FIND THE WALL-SHEAR STRESS (τ_w) AT x

IN THE NEAR-WALL REGION $\tau = \text{const} = \tau_w$

$$\Rightarrow \tau_w = \tau_{y^+=5.0} = 0.39 \text{ N/m}^2$$

b) UPPER LIMIT OF LOG LAW ($y^+ = 1000$)

$$u^+ = 2.44 \ln(1000) + 4.9$$

$$u^+ = \frac{\bar{u}}{u^*} \quad (6.29)$$

$$u^* \equiv \sqrt{\frac{\tau_w}{\rho}}$$

$$\text{AIR} \Rightarrow \rho = 1.2$$

$$\Rightarrow \bar{u} = 12.4 \text{ m/s}$$

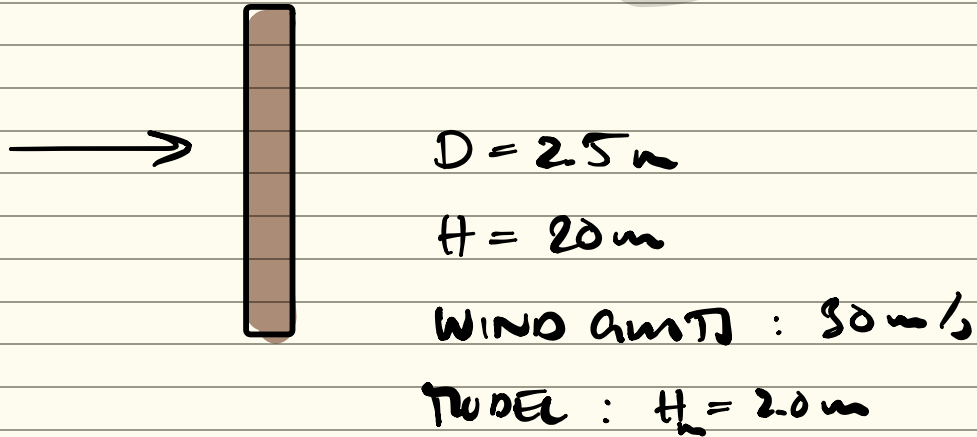
c)

$$\frac{\bar{u}}{u_{\infty}} = 82.7\%$$

THE BOUNDARY LAYER OUTER EDGE IS
LOCATED WHERE $\bar{u}/u_{\infty} = 99\%$.

So $y^+ = 1000$ IS BELOW THE BOUNDARY LAYER
EDGE ...

P5



a) Why is A WIND TUNNEL TEST NOT POSSIBLE?

THE REYNOLDS NUMBER MUST BE THE SAME IN THE TEST

$$H_m = 2.0 \Rightarrow D_m = D \frac{H_m}{H}$$

$$Re = \frac{\rho U D}{\mu} = \frac{\rho U_m D_m}{\mu}$$

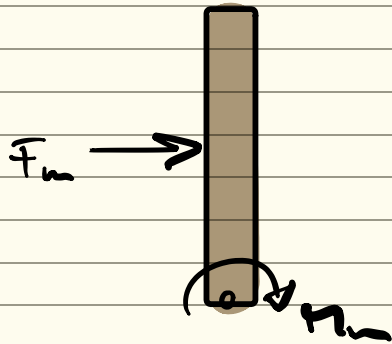
$$\Rightarrow U D = U_m D_m = U_m D \frac{H_m}{H}$$

$$\Rightarrow U_m = \frac{H}{H_m} U = 300 \text{ m/s}$$

A WIND TUNNEL SPEED OF 300 m/s IS OF COURSE NOT REALISTIC. FURTHER IT WOULD IMPLY THAT THE TEST MACH NUMBER WOULD BE 0.87 !

b) WATER TUNNEL TEST

BENDING MOMENT: 50 kNm



MODEL:

$$M_m = F_m H_m \frac{1}{2}$$

$$F_m = C_D \frac{1}{2} \rho_{\text{water}} U_m^2 A_p$$

$$A_p = H_m D_m$$

FULL SCALE:

$$M = F H \frac{1}{2}$$

$$F = C_D \frac{1}{2} \rho_{\text{air}} U^2 A_p$$

$$A_p = H D$$

$$M = C_D \frac{1}{4} \rho_{\text{air}} U^2 H^2 D$$

$$\Rightarrow C_D = \frac{M}{\frac{1}{4} \rho_{\text{air}} U^2 H^2 D}$$

$$C_D = \frac{M_m}{\frac{1}{4} \rho_{\text{water}} U_m^2 H_m^2 D_m}$$

$\Rightarrow$

$$\frac{M}{\frac{1}{4} \rho_{\text{air}} U^2 H^2 D} = \frac{M_m}{\frac{1}{4} \rho_{\text{water}} U_m^2 H_m^2 D_m}$$

$\Rightarrow$

$$M = M_m \frac{\rho_{\text{air}} U^2 H^2 D}{\rho_{\text{water}} U_m^2 H_m^2 D_m}$$

$$Re = Re_m$$

$$\frac{\rho_{air} U D}{\mu_{air}} = \frac{\rho_{water} U_m D_m}{\mu_{water}}$$

Assume $T = 20^\circ C$:

$$\rho_{air} = 1.2 \text{ kg/m}^3$$

$$\mu_{air} = 1.8 \cdot 10^{-5} \text{ kg/(m s)}$$

$$\rho_{water} = 998 \text{ kg/m}^3$$

$$\mu_{water} = 1.005 \cdot 10^{-3} \text{ kg/(m s)}$$

$$H_m = 2.0$$

$$D_m = D \frac{H_m}{H}$$

$$\Rightarrow \begin{array}{l} U_m = 20.1 \text{ m/s} \\ \eta = 137 \text{ kNm} \end{array}$$

$$c) \quad S_t = \frac{f D}{u} = \text{const.}$$

$$\frac{f D}{u} = \frac{f_m D_m}{U_m}$$

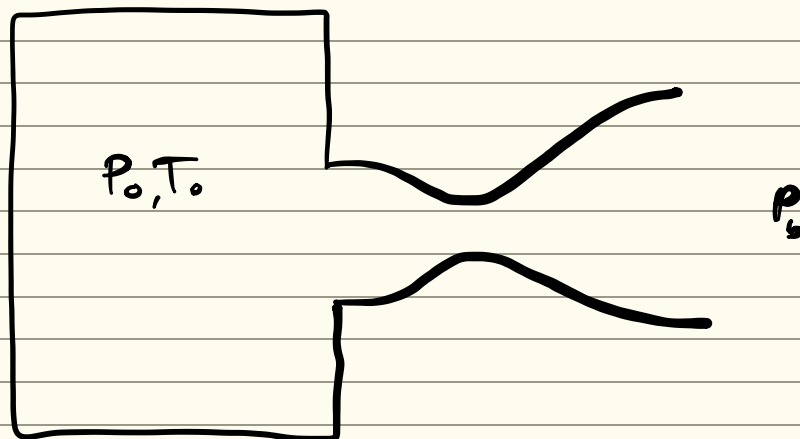
$$Re = \frac{\rho_{air} U D}{\mu_{air}} = \frac{\rho_{water} U_m D_m}{\mu_{water}}$$

$$u_m = 12 \text{ m/s} \quad ; \quad f_m = 10.56 \text{ Hz}$$

$$\Rightarrow u = 17.9 \text{ m/s}$$

$$f = 1.58 \text{ Hz}$$

P_c



$$P_0 = 2.0 \text{ MPa}$$

$$T_0 = 27^\circ\text{C} \quad (300\text{K})$$

$$P_b = 100 \text{ kPa}$$

$$\text{DESIGN } \text{NPR} = 10 \quad (P_0/P_c = 10)$$

$$A_e = 10 \text{ cm}^2$$

- a) CALCULATE THE NOZZLE FLOW IF THE PRESSURE DOWNSTREAM OF THE EXIT IS 100 kPa (P_b)

DESIGN CONDITIONS (SUPERCRITICAL)

$$\text{NPR} = 10$$

$$P_0 = 2.0 \text{ MPa} \Rightarrow P_c = 200 \text{ kPa}$$

$$P_b = 100 \text{ kPa} < 200 \text{ kPa} \Rightarrow$$

UNDEREXPANDED

CHOKED NOZZLE FLOW, DESIGN MACH NUMBER AT EXIT..

$$\frac{p_0}{p_e} = \left(1 + \frac{\gamma-1}{2} M_e^2 \right)^{\gamma/(\gamma-1)} \quad (9.28)$$

ASSUME: Air @ 20°C $\Rightarrow \gamma = 1.4$

$$1 + \frac{\gamma-1}{2} M_e^2 = \left(\frac{p_0}{p_e} \right)^{(\gamma-1)/\gamma}$$

$$M_e = \sqrt{\left(\left(\frac{p_0}{p_e} \right)^{(\gamma-1)/\gamma} - 1 \right) \left(\frac{2}{\gamma-1} \right)}$$

$$\Rightarrow M_e = 2.16$$

AREA - MACH NUMBER RELATION:

$$(9.44) \left(\frac{A_e}{A^*} \right)^2 = \frac{1}{M_e^2} \left(\frac{2 + (\gamma-1)M_e^2}{\gamma+1} \right)^{(\gamma+1)/(\gamma-1)}$$

$$\Rightarrow A^* = 5.18 \text{ cm}^2$$

CHOKED MASS FLOW:

$$\dot{m} = \frac{p_0 A^*}{\sqrt{T_0}} \sqrt{\frac{\gamma}{k}} \left(\frac{2}{\gamma+1} \right)^{(\gamma+1)/(\gamma-1)}$$

$$\Rightarrow \dot{m} = 2.4 \text{ kg/s}$$

b) CALCULATE THE EXIT VELOCITY.

$$(9.2c) \quad \frac{T_0}{T_e} = 1 + \frac{\gamma-1}{2} \eta_e^2$$

$$(9.16) \quad a_e = \sqrt{\gamma R T_e}$$

$$\eta_e = \frac{u_e}{a_e}$$

$$\Rightarrow u_e = 539 \text{ m/s}$$

c)

~~SUBCRITICAL~~

~~CRITICAL~~

CHOKED

~~INTERNAL SHOCK~~

~~SHOCK AT EXIT~~

~~OVEREXPANDED~~

~~SUPERCRITICAL~~

UNDEREXPANDED

ISENTROPIC INTERNAL FLOW

~~NON-ISENTROPIC INTERNAL FLOW~~