MTF053 - Fluid Mechanics

2025-08-18 08.30 – 13.30

Approved aids:

- The formula sheet handed out with the exam (attached as an appendix)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Any calculator with cleared memory

Exam Outline:

- In total 6 problems each worth 10p

Grading:

number of points on exam (including bonus points)	24-35	36-47	48-60
grade	3	4	5

Problem 1 - Cannonball Jump (10 p.)

Emil and his friends spend a hot summer day at Delsjön. Emil wants to show off and jumps into the water from a cliff located 3 m above the water surface. The depth of the water under the cliff is 2.5 m and just after jumping off the cliff, Emil starts to worry that he will hit bottom of the lake and decides to go for a "cannonball jump". Emil breaks the water surface at a velocity of 5 m/s.

Emil weighs 80 kg and the density of a human is close to that of water (let's say 950 kg/m^3). For the the evaluation of the cannonball jump, Emil can be approximated as a sphere with $C_D = 1.0$ and if jumping into the water in a "standing position", $C_D A = 0.1\ m^2$.

Recall - the volume (V) of a sphere is $V = \dfrac{\pi D^3}{6}$

(a) Was it a wise move to go for a cannonball jump? i.e. will Emil hit the bottom? Would he hit the bottom if he would have jumped in a "standing position"? (8p)

Theory questions related to the topic:

(b) If you are going to do an experimental investigation of a problem including a number of important physical variables, why is it beneficial to divide the variables into non-dimensional groups? (1p)

(c) Describe the implication of buoyancy for immersed bodies and floating bodies, respectively. (1p)

Problem 2 - Hydraulically Smooth Pipe (10 p.)

A mechanical engineer gets the task to verify that a pipe is hydraulically smooth. Measurements shows that surface roughness of the pipe is 0.35 mm and the pressure drop for a D=0.1 m pipe of the length 5.0 m is 5.50 Pa when air @ 20 °C flows through the pipe at a velocity representative for the installation in which the pipe is to be used.

Hydraulically smooth means that the surface roughness does not affect the flow and thus it should be within the viscous sublayer. A rule of thumb is that the surface roughness should be **less than four viscous lengths**. The viscous length ℓ is defined as $\ell = \nu/u^*$ where ν is the kinematic viscosity of the fluid and u^* is the friction velocity.

(a) Evaluate if the pipe can be considered to be hydraulically smooth based on the provided data (8p)

Theory questions related to the topic:

(b) What does critical Reynolds number mean for a pipe flow? (1p)

(c) What is the effect of surface roughness on the friction factor? (1p)

Problem 3 - Thrust Requirement (10 p.)

An airplane with the weight 7500 kg flies at an altitude of 5500 m. The aircraft wings are of NACA 0009-type (without flaps) with an average chord ($\bar{c}$) of 4.0 m and the span (b) of each wing is 20 m.

The drag coefficient can be calculated as

$$C_D = C_{D\infty} + \frac{C_L^2}{\pi AR}$$

where $C_{D\infty}$ is the drag coefficient for an infinite-span wing (*the drag coefficient that you usually would get from tables or graphs*), C_L is the lift coefficient, and $AR = b/\bar{c}$ is the wing aspect ratio.

(a) Estimate the thrust needed to cruise with the airplane at a velocity of 400 km/h (8p)

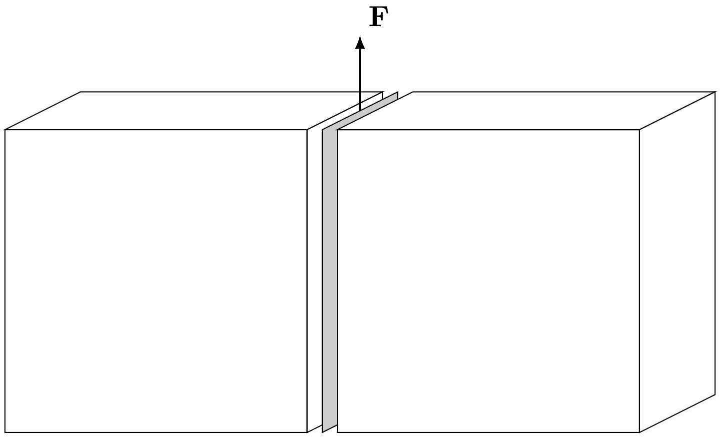
Theory questions related to the topic:

(b) The drag coefficient, C_D, can be divided into two components, which two components? What phenomena are associated with each of the two components? (2p)

Problem 4 - Viscous Force (10 p.)

A thin flat plate is to be pulled up through a vertical cavity filled with SAE50W oil @ 20 °C according to the figure below. The plate is pulled up at a constant velocity.
note: the figure shows a cut through the cavity, it continues above the plate

You do not have to account for end effects at the edges of the plate. You can also neglect the weight of the plate (the 850N is the applied force in addition to the weight of the plate), and the pressure can be assumed to be constant. The plate is 1.0 m wide and the length of the plate in the direction of the force is 5.0 m. You can neglect the thickness of the plate. The width of the cavity is 2.0 cm and the plate can be assumed to be perfectly centered in the cavity as it moves upwards.



(a) Calculate the constant velocity at which the plate is moving if the applied force is 850 N (8p)

Theory questions related to the topic:

(b) Explain the following concepts (2p):

- Steady-state flow
- Incompressible flow
- No-slip condition
- Newtonian fluid

Problem 5 - Pipe Flow (10 p.)

Oil @ 30 °C flows through a 25 m long pipe with a diameter of 0.15 m. Pressure measurements are done to determine the massflow through the pipe. A pitot tube placed at the center of the pipe gives a pressure reading of 250 kPa and a pressure sensor placed at the pipe wall at the same axial location as the pitot tube gives a pressure of 225 kPa. The oil density is $\rho_{oil} = 980\ kg/m^3$ and the kinematic viscosity is $\nu_{oil} = 1.0 \times 10^{-6}\ m^2/s$. The flow at the location where the pressure measurements are done can be considered to be fully developed.

(a) Calculate the oil massflow (7p)

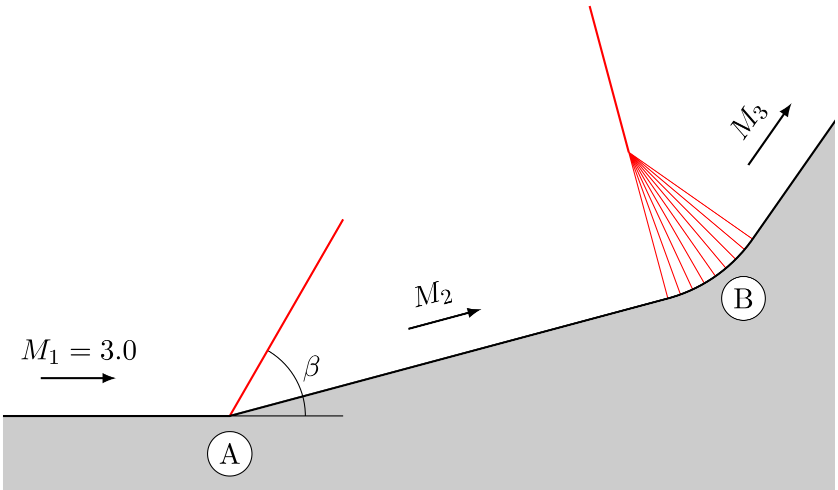
Theory questions related to the topic:

(b) Explain how to measure velocity using a Prandtl tube (Pitot-static tube) and derive the relation needed to estimate the velocity (2p)

(c) What does the concept *entrance length* mean? How does the flow velocity profile change over the entrance length? (1p)

PROBLEM 6 - SUPERSONIC FLOW DEFLECTION (10 P.)

A supersonic air flow is deflected at two locations according to the figure below. At the location denoted (A), there is a sharp bend i the surface and the flow is deflected by an oblique shock with a shock angle with the shock angle $\beta = 40°$. At location (B), the flow goes through a continuous deflection. The inclomming flow Mach number is $M_1 = 3.0$ and the air temperature is 20 °C.



(a) Find the deflection angle at location (B) such that the outgoing Mach number is sonic $M_3 = 1.0$ (8p)

Theory questions related to the topic:

(b) Which of the properties h_o, T_o, a_o, p_o, and ρ_o are constants in a flow if the flow is adiabatic and isentropic, respectively? (1p)

(c) The normal shock equation system has two solutions. How do we know which solution that is the correct one? (1p)

P1

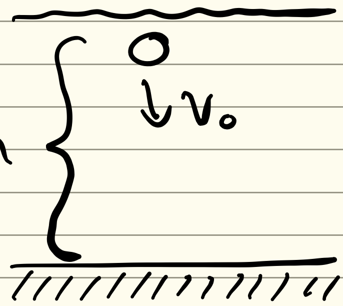
GIVEN:

$m = 80\ kg$

$\rho = 950\ kg/m^3$

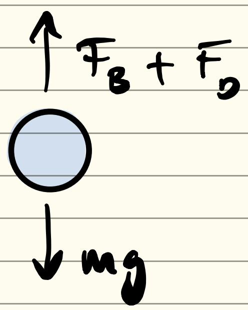
$U_0 = 5.0\ m/s$

$h = 2.5\ m$



CANNONBALL : $C_D = 1.0$

SPIKE : $C_D A = 0.1\ m^2$



FORCE BALANCE.

$F_B = \rho_{WATER} V g$ (WHERE V is THE VOLUME OF EMIL)

$F_D = \frac{1}{2} \rho_{WATER} U^2 C_D A$

NEWTON II:

$$\frac{d(mU)}{dt} = mg - \rho V g - \frac{1}{2} \rho U^2 C_D A$$

$U = \frac{dy}{dt} \Rightarrow dt = \frac{1}{U} dy \Rightarrow$

$$\Rightarrow \quad U \frac{d(mU)}{dy} = mg - \rho V g - \frac{1}{2}\rho U^2 C_D A$$

REWRITE L.H.S USING THE CHAIN RULE

$\Rightarrow$

$$U\left(m \frac{dU}{dy} + U \underbrace{\frac{dm}{dy}}_{=0}\right) =$$

$$= mg - \rho V g - \frac{1}{2}\rho U^2 C_D A$$

$\Rightarrow$

$$mU \frac{dU}{dy} = mg - \rho V g - \frac{1}{2}\rho U^2 C_D A$$

$$U \frac{dU}{dy} = \underbrace{g - \frac{\rho V g}{m}}_{b} - \underbrace{\frac{\rho U^2 C_D A}{2m}}_{-aU^2}$$

$$U \frac{dU}{dy} = aU^2 + b \quad \Rightarrow$$

$$\frac{U}{aU^2 + b} = dy$$

INTEGRATE:

$$\int_{U_0}^{0} \frac{U}{aU^2+b}\, dU = \int_0^h dy$$

$$\left[\frac{\ln(aU^2+b)}{2a}\right]_{U_0}^{0} = h$$

$$\frac{\ln(b)}{2a} - \frac{\ln(aU_0^2+b)}{2a} = h$$

$$\frac{1}{2a}\left(\ln\left(\frac{b}{aU_0^2+b}\right)\right) = h$$

a)

1: CANNONBALL:

$C_D = 1.0$

$V = \frac{\pi D^3}{6}$, $m = 80\,kg$, $\rho = 950\,kg/m^3$

$\Rightarrow a = -1.4517$, $b = -0.5163$

$$\Rightarrow \boxed{h = 1.46 < 2.5\,m}$$

2: SPIKE :

$C_D A = 0.1\ m^2$

$a = -0.6250, \quad b = -0.5163$

$\Rightarrow \boxed{h \approx 2.75 \quad > 2.5}$

Σ :

IF JUMPING A CANNONBALL JUMP, EMIL WILL NOT HIT THE BOTTOM OF THE LAKE BUT HE MIGHT HIT THE BOTTOM IF JUMPING IN A STANDING POSITION

$\Rightarrow$ IT WAS A WISE MOVE :)

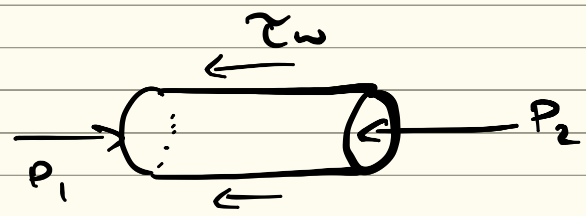
P_2

AIR @ 20°C : $\rho = 1.2\ kg/m^3$; $\nu = 1.5 \cdot 10^{-5}\ m^2/s$

SURFACE ROUGHNESS 0.35 mm

$D = 0.1\ m$

$$\left.\begin{array}{l}\Delta p = 5.50\ Pa \\ \Delta L = 5.0\ m\end{array}\right\} \Rightarrow \frac{\Delta p}{\Delta L} = 1.1\ Pa/m$$



$$(P_1 - P_2)\frac{\pi D^2}{4} = \pi D \Delta L \tau_w$$

$$\tau_w = \frac{\Delta p}{\Delta L}\frac{D}{4} = 0.0275\ N/m^2$$

$$u^* \equiv \sqrt{\frac{\tau_w}{\rho}} \Rightarrow u^* = 0.151\ m/s$$

$$\boxed{\ell = \frac{\nu}{u^*} = 0.1\ m}$$

$\Rightarrow$ 0.35 mm $< 4\ell \Rightarrow$

THE PIPE IS HYDRAULICALLY SMOOTH.

P3

GIVEN: $m = 7500$ kg

$h = 5500$ m

WING PROFILE: NACA 0009

AVERAGE CHORD: $\bar{c} = 4.0$ m

WING SPAN: $\ell = 20$ m (ONE WING)

$U = 400$ km/h

CALCULATE DRAG (THRUST REQUIREMENT)

TABLE A.6:

$h = 5500 \text{ m} \Rightarrow \rho = 0.6970 \text{ kg/m}^3$

$$\frac{F_D}{2} = C_D \frac{1}{2} \rho U^2 A_p \qquad (7.66b)$$

$$\frac{F_L}{2} = C_L \frac{1}{2} \rho U^2 A_p \qquad (7.66a)$$

$F_L = mg \quad ; A_p = b\bar{c} \quad \Rightarrow C_L = 0.2158$

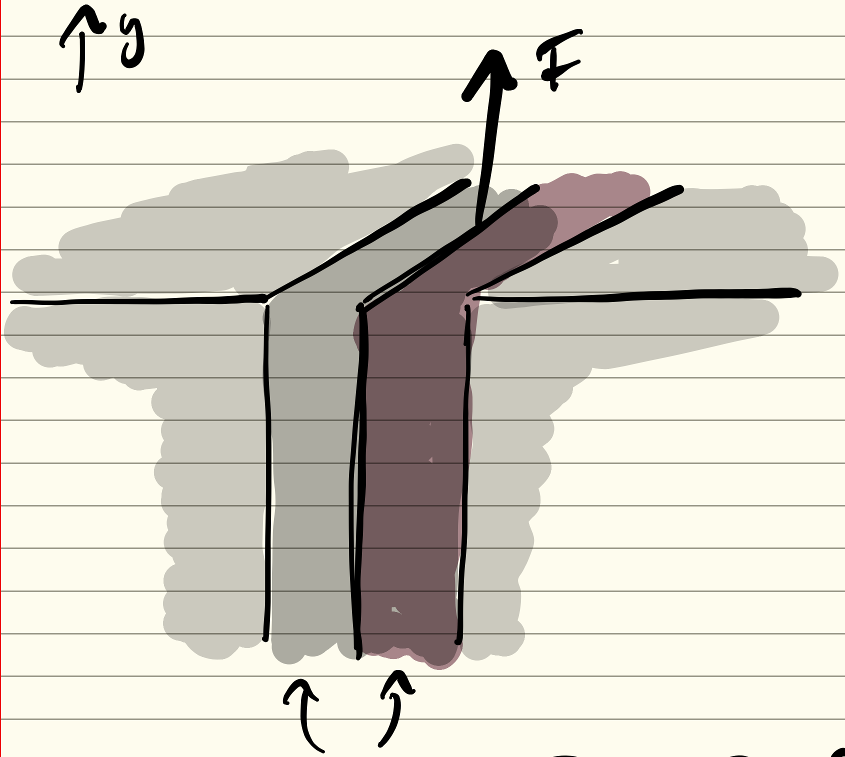
Fig 7.25 $\Rightarrow C_{D\infty} \approx 0.006$

$$C_D = C_{D\infty} + \frac{C_L^2}{\pi AR}$$

WHERE $AR = b/\bar{c}$

$\Rightarrow C_D \approx 0.01 \Rightarrow \boxed{F_D = 4.63 \text{ kN}}$

P4



$F = 850N$

$V = ?$

SAE50W OIL @ 20°C

PLATE : WIDTH = 1.0 m (b)

LENGTH = 5.0 m (L)

WIDTH OF CAVITY: 2cm

(1cm ON EACH SIDE OF PLATE)

SAE 50W OIL @ 20°C

TABLE A.3 : $\rho = 902 \ kg/m^3$

$\mu = 0.86 \ kg/ms$

$\nu = 9.537 \cdot 10^{-4} \ m^2/s$

$$F = -\tau_w \cdot A = -\tau_w \cdot 2 \cdot L \cdot b \Rightarrow$$

$$\tau_w = \frac{-F}{2Lb} = -85 \ N/m^2$$

NAVIER-STOKES (y-DIRECTION)

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} =$$

$$= -\frac{1}{\rho}\frac{\partial p}{\partial y} + g_y +$$

$$+ \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

STEADY STATE $\Rightarrow$ $\frac{\partial v}{\partial t} = 0$

NO PRESSURE GRADIENT $\Rightarrow$ $\frac{\partial p}{\partial y} = 0$

FULLY DEVELOPED : $u = 0$, $w = 0$,

$\frac{\partial}{\partial y}() = 0$, $\frac{\partial}{\partial z}() = 0$, $\frac{\partial^2}{\partial y^2}() = 0$,

$\frac{\partial^2}{\partial z^2}() = 0$

$$\Rightarrow \quad \frac{\partial^2 v}{\partial x^2} = -\frac{g_y}{\nu}$$

INTEGRATE:

$$\frac{\partial v}{\partial x} = -\frac{g_y}{\nu}x + C_1$$

$$v(x) = -\frac{g_y}{2\nu}x^2 + C_1 x + C_2$$

BOUNDARY CONDITIONS:

$$x = 0 \Rightarrow \mu \frac{\partial v}{\partial x} = \tau_w$$

$$\Rightarrow \frac{\tau_w}{\mu} = -\frac{g_y}{\nu} \cdot 0 + C_1 \Rightarrow$$

$$\boxed{C_1 = \frac{\tau_w}{\mu}}$$

$$x = h \Rightarrow v = 0$$

$$0 = -\frac{g_y}{2\nu} h^2 + \frac{\tau_w}{\mu} h + C_2$$

$$\Rightarrow \boxed{C_2 = h^2 \left(\frac{g_y}{2\nu} - \frac{\tau_w}{\mu h} \right)}$$

$$\boxed{v(x) = -\frac{g_y}{2\nu} x^2 + \frac{\tau_w}{\mu} x + h^2 \left(\frac{g_y}{2\nu} - \frac{\tau_w}{\mu h} \right)}$$

NOW, LET'S CALCULATE THE VELOCITY OF THE MOVING PLATE...

$$V(0) = h^2\left(\frac{g_y}{2\nu} - \frac{\tau_w}{\mu h}\right) = 0.97\ m/s$$

$\therefore$ THE CONSTANT PLATE VELOCITY

IS $\boxed{V = 0.97\ m/s}$

P_5

GIVEN:

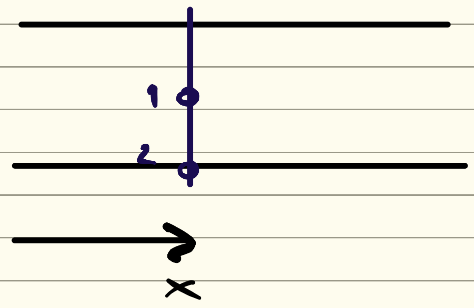
$L = 25\,m$

$D = 0.15\,m$

OIL @ $30°C$

$P_{01} = 250\,kPa$

$P_2 = 225\,kPa$



$\rho = 980\,kg/m^3$

$\nu = 1.0 \cdot 10^{-6}\,m^2/s$

FULLY-DEVELOPED FLOW.

CALCULATE THE MASSFLOW

THE PRESSURE MEASURED IN (1) IS THE TOTAL PRESSURE AND THE PRESSURE MEASURED AT (2) IS THE STATIC PRESSURE

$$\Rightarrow \quad P_2 + \frac{1}{2}\rho u_1^2 = P_{01}$$

$$\Rightarrow \boxed{u_1 = 7.14\,m/s}$$

NOW, WE MUST CHECK IF THE FLOW IS LAMINAR OR TURBULENT

LET'S FIRST ASSUME LAMINAR FLOW:

LAMINAR FLOW $\Rightarrow$ $U_{AV} = \frac{1}{2} u_1$

$$Re_D = \frac{U_{AV} D}{\nu} = 535714 \gg 2300$$

$\Rightarrow$ THE FLOW IS TURBULENT

FOR TURBULENT FLOWS (EQN: 6.32)

$$\frac{u(r)}{u^*} \approx \frac{1}{\kappa} \ln \frac{(R-r)u^*}{\nu} + B$$

$\kappa = 0.42$; $B = 5.0$

$u(0) = u_1$ $\Rightarrow$

$$\boxed{u_1 = \frac{u^*}{\kappa} \ln\left(\frac{u^* R}{\nu}\right) + B u^*}$$

FIND u^* :

NEWTON - RAPHSON $\Rightarrow$

$u^* = 0.25$ m/s

THE AVERAGE VELOCITY IS OBTAINED AS:

$$\bar{u} = \frac{Q}{A} =$$

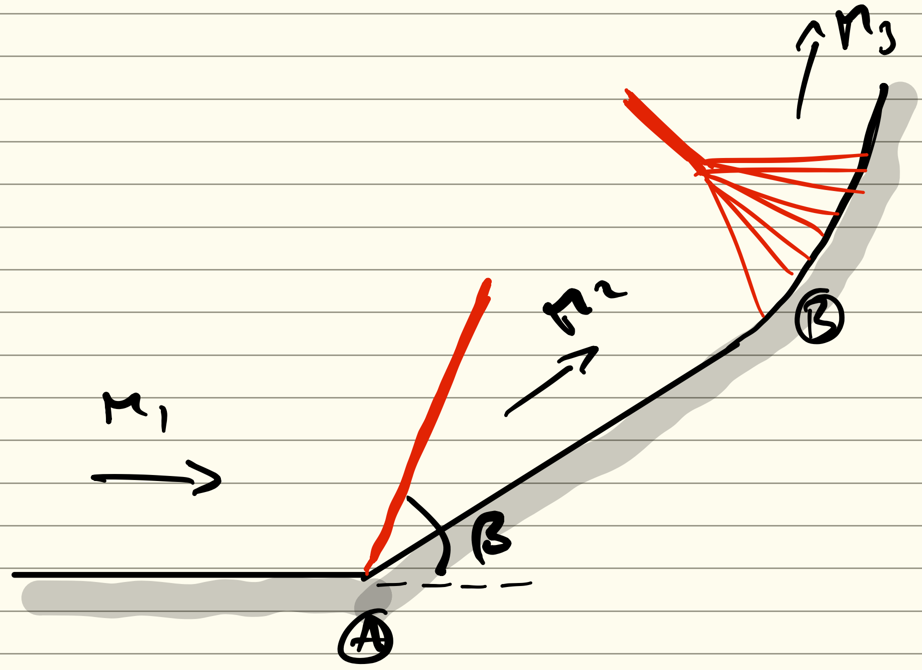
$$= \frac{1}{\pi R^2} \int_0^R u^* \left(\frac{1}{\chi} \ln\left(\frac{(R-r)\,u^*}{\nu} \right) + B \right) 2\pi r\, dr$$

$$= \boxed{\frac{1}{2} u^* \left(\frac{2}{\chi} \ln\left(\frac{R u^*}{\nu} \right) + 2B - \frac{3}{\chi} \right) = 6.25\ \text{m/s}}$$

$$\left(\bar{u} / u_1 = 0.87 > 0.5 \right)$$

$$\boxed{\dot{m} = \bar{u} \frac{\pi D^2}{4} \rho = 108.2\ \text{kg/s}}$$

P_6



AIR @ 20°C

$M_1 = 3.0$

$M_3 = 1.0$

$\beta = 40°$

FIND FLOW DEFLECTION ANGLE IN (B)
SUCH THAT $M_3 = 1.0$

OBLIQUE SHOCK IN (A)

$M_{1n} = M_1 \sin\beta = 1.93$ (EQN 9.82)

EQN. 9.57 ⇒

$$M_{n_2}^2 = \frac{(\gamma - 1) M_{n_1}^2 + 2}{2\gamma M_{n_1}^2 - (\gamma - 1)} = 0.59$$

$\theta - \beta - M \Rightarrow \theta \simeq 21.8^\circ$

EQN 9.82 $\Rightarrow$

$$M_2 = \frac{M_{2n}}{\sin(\beta - \theta)} = 1.89$$

TABLE B.5 (PRANDTL - MEYER):

$\omega(M_2) = 23.42^\circ$

$\omega(M_3 = 1.0) = 0.$

$$\Rightarrow \boxed{\Delta\theta_B = \omega_2 - \omega_3 = 23.42^\circ}$$