MTF053 - Fluid Mechanics

2025-01-09 08.30 – 13.30

Approved aids:

- The formula sheet handed out with the exam (attached as an appendix)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Any calculator with cleared memory

Exam Outline:

- In total 6 problems each worth 10p

Grading:

number of points on exam (including bonus points)	24-35	36-47	48-60
grade	3	4	5

Problem 1 - Boundary Layer Measurements (10 p.)

A group of students is supposed to make measurements to analyze the boundary layer over a flat plate in a wind tunnel as part of a lab in a fluid mechanics course. The measurements are supposed to be done at a location 3.0 m downstream of the leading edge of the flat plate. A hot-wire probe is used for the measurements and the velocity profile is supposed to be obtained by measuring the velocity at different vertical distances from the flat plate surface. The first measurement is done for a vertical distance of 3.0 mm from the flat plate surface where the average velocity is found to be 3.0 m/s. When the hot-wire probe is traversed to the next location, one of the students accidentally breaks the probe and no more measurements are possible. However, one of the students finds a way to calculate the boundary layer properties that are asked for using the single measurement point and formulas from the chapter on external boundary layers in their course book.

(a) Do the same exercise as the students and calculate: (8p)

1. the wall-shear stress (τ_w)
2. the freestream velocity (U_∞)
3. the boundary layer thickness (δ)

(The critical Reynolds number can be assumed to be 500000)

Theory questions related to the topic:

(b) Name two alternative ways to measure the boundary layer thickness other than δ. How can these measures be interpreted physically? (1p)

(c) The drag coefficient, C_D, can be divided into two components, which two components? What phenomena are associated with each of the two components? Which of the components dominates the total drag of a flat plate? (1p)

PROBLEM 2 - PIPE FLOW (10 P.)

The space between two very long and wide flat plates is filled with water. The vertical distance between the plates is $h = 5.0\ cm$. The upper plate moves at a constant velocity of 5.0 m/s in relation to the lower plate, which induces a **turbulent flow** in the water between the plates.

(a) Calculate the **wall-shear stress** τ_w (4p)

(b) Calculate the **Reynolds stress** component $\overline{u'v'}$ at a location midway between the two plates ($y = h/2$) (4p)
Hint: use the x-component of the RANS equations (Eqn. 6.21) to calculate $\overline{u'v'}$

Theory questions related to the topic:

(c) Turbulent flow is dissipative. What does that mean? (1p)

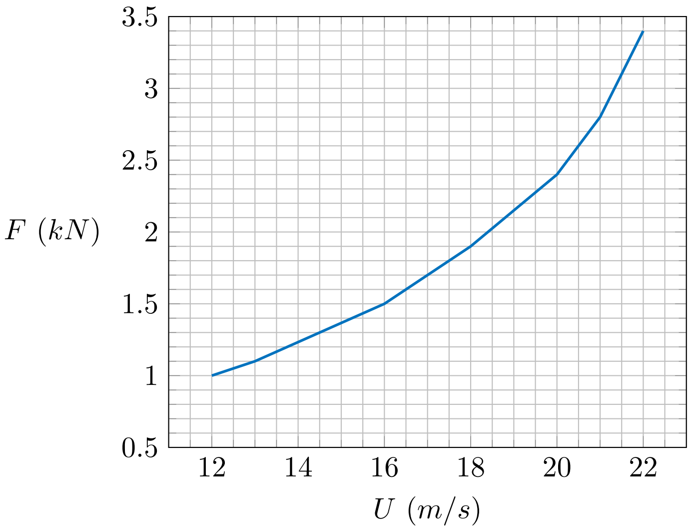
(d) In the Reynolds decomposition, the velocity components and pressure are divided into an average part and a fluctuating part as for example

$$u = \bar{u} + u'$$

Define the time average and show that the time average of the fluctuating component is identically equal to zero. (1p)

Problem 3 - Model-scale Test (10 p.)

A model-scale test of a vehicle is done in order to estimate the drag force on the full-scale vehicle at different velocities. The model vehicle is scaled down by a factor of 12 such that geometrical similarity with the prototype vehicle is established. In order to establish Reynolds similarity, the tests are done in a water tunnel (the water temperature is 20 degrees). Drag force as a function of freestream velocity from the model-scale test is given below.



(a) Estimate the increased power required to overcome the drag force if the velocity of the full-scale vehicle is increased from 20.0 m/s to 25.0 m/s (8p)

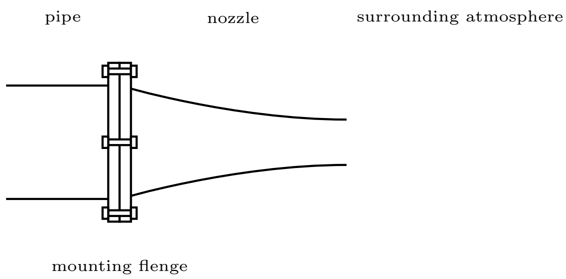
Theory questions related to the topic:

(b) If you are going to do an experimental investigation of a problem including a number of important physical variables, why is it beneficial to divide the variables into non-dimensional groups? (1p)

(c) Explain the concepts *geometric similarity*, *kinematic similarity*, and *dynamic similarity* (1p)

Problem 4 - Nozzle (10 p.)

Water at 20° degrees flows through a pipe at a flow rate of $Q = 0.2\ m^3/s$. At the end of the pipe, the water is accelerated through a convergent nozzle before exiting into the surrounding air as a water jet. The atmospheric pressure at the exit is 101325 Pa. The pipe cross-section area is 0.015 m^2 and the cross-section area at the exit of the nozzle is 0.0075 m^2.



(a) Calculate the total force in the joint keeping the nozzle attached to the upstream pipe (8p)

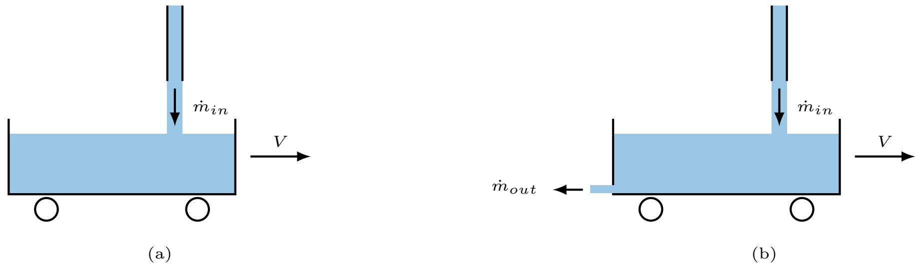
Theory questions related to the topic:

(b) Derive the Bernoulli equation for steady-state, incompressible flow along a streamline (2p)

$$\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho} + \frac{1}{2}V_2^2 + gz_2 = const$$

Problem 5 - Moving Water Tank (10 p.)

A big container on wheels moves at a constant velocity of $V = 1.5\ m/s$ while at the same time being filled from above with 50.0 kg of water per second.



(a) Calculate the extra force required to keep the container moving at a constant velocity related to the filling process. (4p)

(b) A evacuating valve located at the bottom of the container is opened. Calculate the force required to keep the container moving at a constant velocity with the evacuating valve opened. The diameter outlet at the bottom of the tank is circular with a diameter of 9.0 cm and the water level in the tank is 3.0 m above the outlet. (4p)

Theory questions related to the topic:

(c) Give examples of when it is appropriate to use fixed control volume, moving control volume, and deformable control volume, respectively. (1p)

(d) Derive the continuity equation on integral form for a fixed control volume using Reynolds transport theorem (1p)

$$\frac{d}{dt}\left(B_{syst}\right) = \frac{d}{dt}\left(\int_{cv} \beta\rho d\mathcal{V}\right) + \int_{cs} \beta\rho\left(\mathbf{V}_r \cdot \mathbf{n}\right) dA$$

Problem 6 - Supersonic Lift and Drag (10 p.)

A flat plate is placed in a supersonic freestream at an angle of attack of $\alpha = 6°$. The length of the flat plate is 3.0 m and the plate is very wide (the flow around the plate can be assumed to be two-dimensional).

(a) Calculate the lift and drag force (per meter) if the freestream Mach number is 2.5 and the static pressure upstream of the flat plate is 100.0 kPa (7p)

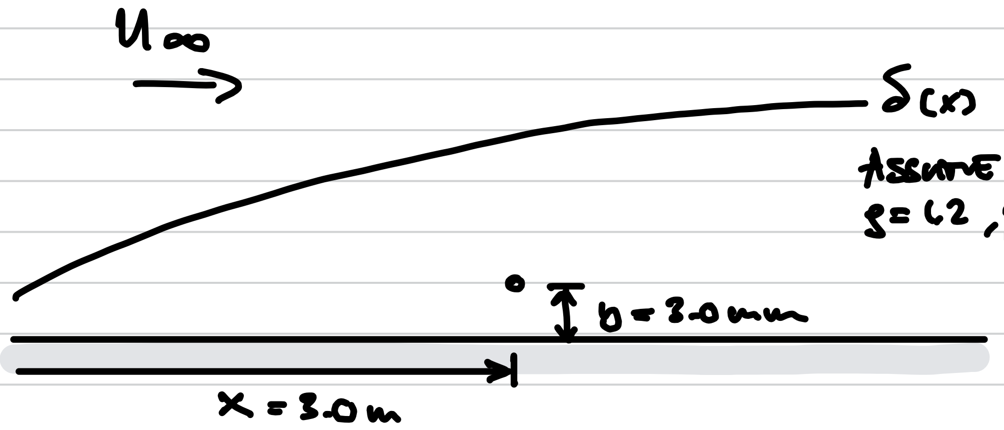
Theory questions related to the topic:

(b) Show schematically how the velocity (normal velocity component, tangential velocity component, and the total velocity) changes over an oblique shock. Indicate the shock angle, β, and the deflection angle, θ (1p)

(c) What is required for a process to be isentropic? (1p)

(d) How does pressure (p), temperature (T), density (ρ), Mach number (M), total pressure (p_o), and total temperature (T_o) change over an expansion region? (1p)

P1



ASSUME AIR @ 20°C
$\rho = 1.2$, $\mu = 1.8 \cdot 10^{-5}$

HOTWIRE MEASUREMENT OF VELOCITY IN A BOUNDARY LAYER OVER A FLAT PLATE.

ONE MEASUREMENT POINT: $x = 3.0$ m, $y = 3.0$ mm

$\bar{u} = 3.0$ m/s

CALCULATE U_∞, τ_w AND $\delta(x = 3.0\text{ m})$

WE DO NOT KNOW IF THE BOUNDARY LAYER IS LAMINAR OR TURBULENT, BUT WE ARE PROVIDED WITH A TRANSITION REYNOLDSNUMBER

$Re_{x,cr} = 500\,000$

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a) THE REYNOLDSNUMBER BASED ON $\bar{u}$ IS

$\frac{\bar{u}\,x}{\nu} = 600000 > Re_{x_{cr}} \Rightarrow$ THE BOUNDARY LAYER IS TURBULENT ($U_\infty > \bar{u}$)

$$(7.39) \quad \left(\frac{\bar{u}}{U_\infty}\right) = \left(\frac{y}{\delta}\right)^{1/7} \qquad (1)$$

(7.42) $$\frac{\delta}{x} = \frac{0.16}{Re_x^{1/7}} \quad (2)$$

δ AND U_∞ UNKNOWN

$$(1) \Rightarrow \delta = y\left(\frac{U_\infty}{\bar{u}}\right)^7 \quad (3)$$

$$(3) \text{ IN } (2) \Rightarrow \frac{y}{x}\left(\frac{U_\infty}{\bar{u}}\right)^7 = \frac{0.16}{\left(\frac{U_\infty x}{\nu}\right)^{1/7}}$$

$$\Rightarrow U_\infty^{50/7} = \frac{0.16\,\bar{u}^7 x^{6/7} \nu^{1/7}}{y} = 4.68 \text{ m/s}$$

$$Re_x = \frac{U_\infty x}{\nu} = 935761 > 500\,000$$

$$(1) \Rightarrow \delta = y\left(\frac{U_\infty}{\bar{u}}\right)^7 = 0.067 \text{ m}$$

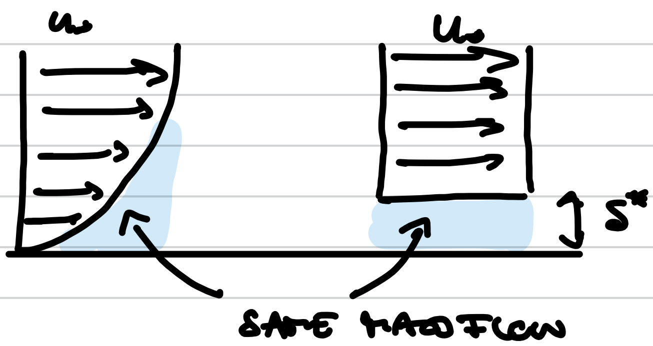
(7.44) $$\tau_w = \frac{0.0135\,\mu^{1/7} \rho^{6/7} U_\infty^{13/7}}{x^{1/7}} = 0.050 \text{ Pa}$$

b) TWO ALTERNATIVE WAYS TO MEASURE THE BOUNDARY-LAYER THICKNESS:

1) DISPLACEMENT THICKNESS (δ^*)

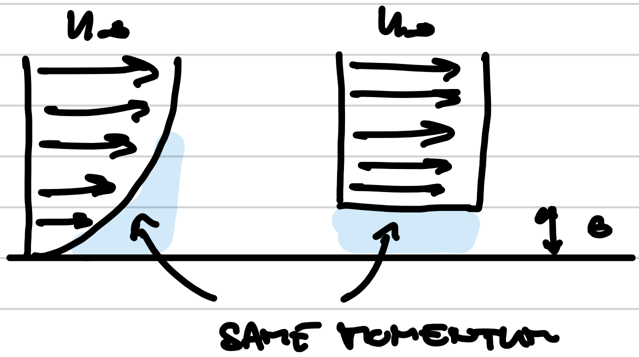
PHYSICAL INTERPRETATION:

THE DISTANCE A CONSTANT-VELOCITY PROFILE WOULD HAVE TO BE TRANSLATED FROM THE WALL TO GET THE SAME DEFICIT OF MASSFLOW AS THE BOUNDARY LAYER PROFILE.



2) MOMENTUM THICKNESS (θ)

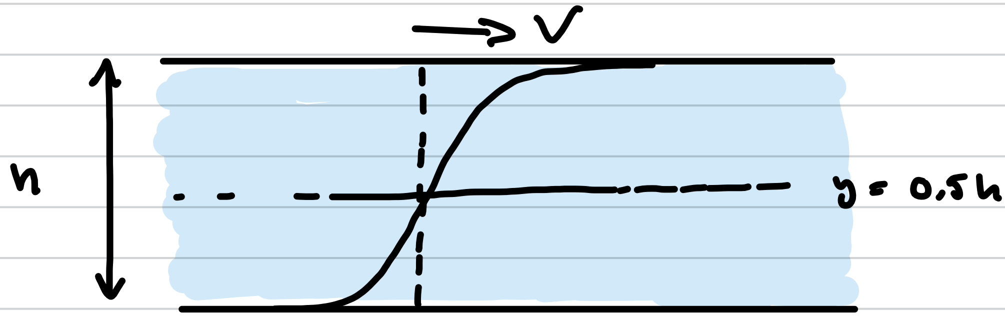
SAME AS δ^* BUT FOR MOMENTUM.



c) DRAG CAN BE DIVIDED INTO PRESSURE DRAG AND FRICTION DRAG.

FOR A FLAT PLATE, DRAG IS DOMINATED BY FRICTION DRAG.

P2



$h = 5\,cm$, $V = 5\,m/s$

DUE TO SYMMETRY, ASSUME $\bar{u}(h/2) = \frac{1}{2}V$

ASSUME WATER @ 20°C $\Rightarrow$ $\rho = 998\ kg/m^3$
$\mu = 0.001\ kg/ms$

a)

TURBULENT FLOW $\Rightarrow$

@ $y = h/2$:

$$\frac{\bar{u}}{u^*} = \frac{1}{\kappa}\ln\left(\frac{h u^*}{2\nu}\right) + B \qquad (6.28)$$

$(\kappa = 0.42,\ B = 5.0)$

$\bar{u} = V/2 \Rightarrow u^* = 0.102\ m/s$

$$u^* \equiv \sqrt{\frac{\tau_w}{\rho}} \Rightarrow \tau_w = \rho u^{*2} = 10.3\ Pa$$

b) CALCULATE $\overline{u'v'}$ @ $y = h/2$

USING 6.21.

(6.21)

$$\rho\left(\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{w}\frac{\partial \bar{u}}{\partial z}\right) =$$

$$= -\frac{\partial \bar{p}}{\partial x} + \rho g_x + \frac{\partial}{\partial x}\left(\mu\frac{\partial \bar{u}}{\partial x} - \rho\overline{u'^2}\right) +$$

$$+ \frac{\partial}{\partial y}\left(\mu\frac{\partial \bar{u}}{\partial y} - \rho\overline{u'v'}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial \bar{u}}{\partial z} - \rho\overline{u'w'}\right)$$

ASSUME FULLY DEVELOPED FLOW IN TWO DIMENSIONS AND STEADY STATE

$\frac{\partial}{\partial x}(\ ) = 0$: FULLY DEVELOPED.

$\frac{\partial}{\partial z}(\ ) = 0$, $\bar{w} = 0$: 2D FLOW

$\frac{\partial}{\partial t}(\ ) = 0$: STEADY STATE.

(6.20) CONTINUITY:

$$\underbrace{\frac{\partial \bar{u}}{\partial x}}_{=0} + \frac{\partial \bar{v}}{\partial y} + \underbrace{\frac{\partial \bar{w}}{\partial z}}_{=0} = 0 \Rightarrow \frac{\partial \bar{v}}{\partial y} = 0 \Rightarrow \bar{v} = 0$$

(6.21)

$$\rho\Big(\underbrace{\frac{\partial \bar{u}}{\partial t}}_{=0} + \bar{u}\underbrace{\frac{\partial \bar{u}}{\partial x}}_{=0} + \underbrace{\bar{v}}_{=0}\frac{\partial \bar{u}}{\partial y} + \underbrace{\bar{w}}_{=0}\underbrace{\frac{\partial \bar{u}}{\partial z}}_{=0}\Big) =$$

$$-\frac{\partial \bar{p}}{\partial x} + \rho g_x + \frac{\partial}{\partial x}\Big(\mu\underbrace{\frac{\partial \bar{u}}{\partial x}}_{=0} - \rho\overline{u'^2}\Big) +$$

$$+ \frac{\partial}{\partial y}\Big(\mu\frac{\partial \bar{u}}{\partial y} - \rho\overline{u'v'}\Big) + \frac{\partial}{\partial z}\Big(\mu\underbrace{\frac{\partial \bar{u}}{\partial z}}_{=0} - \rho\overline{u'w'}\Big)$$

$$\Rightarrow 0 = -\frac{\partial \bar{p}}{\partial x} + \rho g_x + \frac{\partial}{\partial x}\left(-\rho\overline{u'^2}\right) +$$

$$+ \frac{\partial}{\partial y}\Big(\mu\frac{\partial \bar{u}}{\partial y} - \rho\overline{u'v'}\Big) + \frac{\partial}{\partial z}\left(-\rho\overline{u'w'}\right)$$

ASSUME CONSTANT PRESSURE,

NEGLECT GRAVITY,

IN A BOUNDARY LAYER:

$\overline{u'v'} \gg \overline{u'^2}$, $\overline{u'v'} \gg \overline{u'w'}$

$$\Rightarrow \frac{\partial}{\partial y}\Big(\frac{1}{\mu}\frac{\partial \bar{u}}{\partial y} - \rho\overline{u'v'}\Big) = 0$$

or

$$\frac{\partial}{\partial y}\Big(\nu\frac{\partial \bar{u}}{\partial y} - \overline{u'v'}\Big) = 0$$

INTEGRATE $\Rightarrow$

$$\nu \frac{\partial \bar{u}}{\partial y} - \overline{u'v'} = C$$

AT THE WALL $(y=0)$

$$u' = 0 \; , \; v' = 0 \qquad (u' \to 0, \; v' \to 0 \text{ as } y \to 0)$$

$$\nu \left. \frac{\partial \bar{u}}{\partial y} \right|_{y=0} = \frac{\tau_w}{\rho} = u^{*2}$$

$$\Rightarrow C = u^{*2}$$

$$\nu \frac{\partial \bar{u}}{\partial y} - \overline{u'v'} = u^{*2}$$

$$@ \; y = \frac{h}{2} \; , \; \bar{u}(h/2) = V/2$$

$$\Rightarrow \left. \frac{\partial \bar{u}}{\partial y} \right|_{y=h/2} \approx \frac{V}{2}\frac{2}{h} = \frac{V}{h}$$

$$\overline{u'v'}\,(y = h/2) = u^{*2} - \nu \frac{V}{h} = -0{,}0109 \, m^2/s^2$$

c) A TURBULENT FLOW IS DISSIPATIVE, WHAT DOES THAT MEAN?

ENERGY IS CONTINUOUSLY TRANSFERRED FROM THE TURBULENT FLOW TO INTERNAL ENERGY OF THE FLUID WHICH MEANS THAT UNLESS ENERGY IS ADDED TO THE FLOW, TURBULENCE LEVELS WILL DECAY UNTIL THERE IS NO TURBULENT STRUCTURES IN THE FLOW.

d) DEFINE THE AVERAGE USED IN THE REYNOLDS DECOMPOSITION AND SHOW THAT THE AVERAGE OF A FLUCTUATION IS ZERO.

$$\bar{u} = \frac{1}{T}\int_0^T u \, dt$$

$\bar{u}$ IS AN ENSAMBLE AVERAGE

(IF THE SAMPLING FREQUENCY IS CONSTANT AN ENSAMBLE AVERAGE IS THE SAME AS A TIME AVERAGE)

$$\overline{u'} = \frac{1}{T}\int_0^T u' \, dt =$$

$$= \frac{1}{T}\int_0^T (u - \overline{u}) \, dt =$$

$$= \underbrace{\frac{1}{T}\int_0^T u \, dt}_{=\overline{u}} - \underbrace{\frac{1}{T}\int_0^T \overline{u} \, dt}_{=\overline{u}}$$

$$= \overline{u} - \overline{u} = 0$$

P3

A MODEL-SCALE VEHICLE IS TESTED IN A WATER-TUNNEL. FORCE VS. VELOCITY IS PRESENTED. (ASSUME AIR @ 20°C FOR THE PROTOTYPE)

WATER @ 20°C:

$\rho = 998\ kg/m^3$

$\mu = 0.001\ kg/ms$

AIR @ 20°C:

$\rho = 1.2\ kg/m^3$

$\mu = 1.8 \cdot 10^{-5}\ kg/ms$

a) ESTIMATE THE INCREASED POWER REQUIRED TO OVERCOME THE DRAG FORCE IF THE VELOCITY OF THE FULL-SCALE VEHICLE IS INCREASED FROM 20 m/s TO 25 m/s

WE WILL ASSUME DYNAMIC SIMILARITY:

- GEOMETRIC SIMILARITY: ALL LENGTHS OF THE MODEL ARE SCALED WITH THE SAME SCALE FACTOR (1/12)
- KINEMATIC SIMILARITY: HOMOLOGOUS EVENTS @ HOMOLOGOUS POINTS TAKES PLACE AT HOMOLOGOUS TIMES (REQUIRES REYNOLDS NUMBERS TO BE THE SAME).

$\Rightarrow$ FORCES CAN BE SCALED.

$$Re_m = Re_p$$

$$\frac{U_m D_m \rho_m}{\mu_m} = \frac{U_p D_p \rho_p}{\mu_p}$$

$\rho_m = \rho_{water}$ $\qquad$ $\rho_p = \rho_{air}$

$\mu_m = \mu_{water}$ $\qquad$ $\mu_p = \mu_{air}$

$D_m = 1/12\ D_p$

$U_p = 20\ m/s \Rightarrow U_m = 16\ m/s$

$U_p = 25\ m/s \Rightarrow U_m = 20\ m/s$

FROM THE PROVIDED DATA:

$U_m = 16\ m/s \Rightarrow F_m = 1.5\ kN$

$U_m = 20\ m/s \Rightarrow F_m = 2.4\ kN$

$$F_m = \frac{1}{2} \rho_m A_m C_D U_m^2 \qquad (7.66)$$

$$\left.\begin{aligned} C_D &= \frac{2 F_m}{\rho_m A_m U_m^2} \\ C_D &= \frac{2 F_p}{\rho_p A_p U_p^2} \end{aligned}\right\} \Rightarrow$$

$$F_p = F_m \frac{\rho_p A_p U_p^2}{\rho_m A_m U_m^2}$$

(Note! $A_p = 12^2 A_m$)

$$\Rightarrow \quad F_p(20\,m/s) = 404.2\ N$$

$$F_p(25\,m/s) = 646.7\ N$$

POWER:

$$P_p = F_p U_p$$

$$P_p(25\,m/s) - P_p(20\,m/s) = 8.1\,kW$$

b) WHY IS IT BENEFICIAL TO GROUP VARIABLES IN DIMENSIONAL GROUPS.

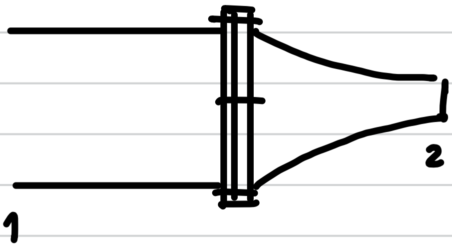
1) YOU CAN REDUCE THE NUMBER OF MEASUREMENTS SIGNIFICANTLY.

2) IT IS EASIER TO PRESENT DATA IN (FEWER GRAPHS AND TABLES)

3) IT IS POSSIBLE TO COMPARE RESULTS OBTAINED AT DIFFERENT CONDITIONS.

c) SEE 3a.

P4



$Q = 0.2\ m^3/s$

$P_{amb} = 101325\ Pa$

$A_1 = 0.015\ m^2$

$A_2 = 0.0075\ m^2$

WATER @ 20°C => $\rho = 998\ kg/m^3$

a) CALCULATE THE FORCE KEEPING THE NOZZLE ATTACHED TO THE PIPE.

ASSUME: STEADY STATE, INCOMPRESSIBLE.

CONTINUITY:

$$(3.22)\qquad \underbrace{\int_{cv} \frac{\partial \rho}{\partial t}\, dV}_{\substack{=0 \\ \text{(steady state)}}} + \sum_i (\rho_i A_i V_i)_{out} - \sum_i (\rho_i V_i A_i)_{in} = 0$$

$$\Rightarrow \rho_1 A_1 V_1 = \rho_2 A_2 V_2 \Rightarrow (\rho_1 = \rho_2) \Rightarrow$$

$$\Rightarrow A_1 V_1 = A_2 V_2 = Q$$

$$\Rightarrow V_1 = \frac{Q}{A_1}, \quad V_2 = \frac{Q}{A_2}$$

BERNOULLI:

$$(3.54)\qquad \frac{P_1}{\rho} + \frac{1}{2}V_1^2 + \cancel{g z_1} = \frac{P_2}{\rho} + \frac{1}{2}V_2^2 + \cancel{g z_2}$$

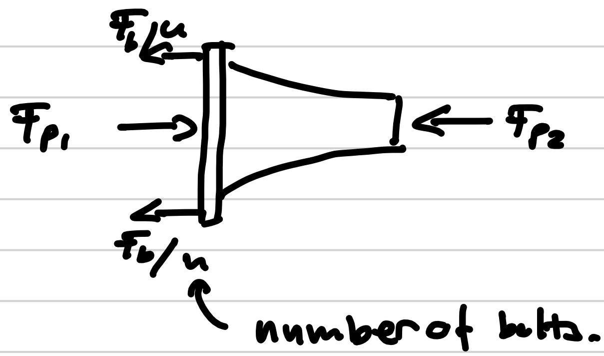
$$\Rightarrow P_1 = P_2 - \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)$$

CONSERVATION OF LINEAR MOMENTUM:

(3.40)

$$\sum F = \underbrace{\frac{d}{dt}\left(\int_{cv} V \rho \, dV\right)}_{=0 \text{ (steady state)}} + \sum_i (\dot{m}_i V_i)_{out} + - \sum_i (\dot{m}_i V_i)_{in}$$

$$\Rightarrow \sum F_x = \dot{m}_2 V_2 - \dot{m}_1 V_1$$



$F_{p_1} = P_1 A_1$

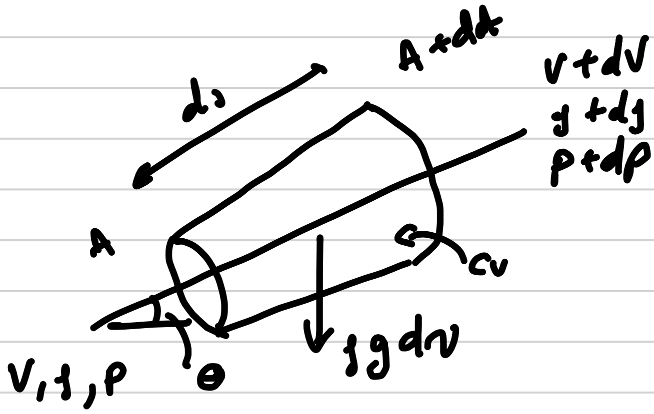
$F_{p_2} = P_2 A_2$

$$\sum F_x = F_{p_1} - F_{p_2} - F_b$$

$$-F_b = \dot{m}_2 V_2 - \dot{m}_1 V_1 + P_2 A_2 - P_1 A_1$$

$$\Rightarrow F_b = 2090 \text{ N}$$

b) DERIVE THE BERNOULLI EQUATION FOR STEADY-STATE, INCOMPRESSIBLE FLOW ALONG A STREAMLINE.



CONSERVATION OF MASS:

$$\frac{d}{dt}\left(\int_{cv} \rho \, dV\right) + \dot{m}_{out} - \dot{m}_{in} = 0$$

$$\approx \frac{\partial \rho}{\partial t} dV + d\dot{m} \quad \text{(FOR A SMALL CV)}$$

WHERE $dV = A\,ds$, $d\dot{m} = -\frac{\partial \rho}{\partial t} A\,ds$

CONSERVATION OF LINEAR MOMENTUM:

$$\sum dF_s = \frac{d}{dt}\left(\int_{cv} V \rho \, dV\right) + (\dot{m}V)_{out} - (\dot{m}V)_{in}$$

$$\approx \frac{\partial}{\partial t}(\rho V) A\,ds + d(\dot{m}V)$$

INVISCID FLOW ALONG A STREAMLINE $\Rightarrow$

ONLY PRESSURE AND GRAVITY FORCES

$$dF_{s,p} \approx \frac{1}{2}\, dp\, dA - (A + dA)\, dp \approx -A\, dp$$

$$dF_{s,g} \approx -\rho g A\, ds \sin\theta = -\rho g A\, dz$$

$$\sum F_s = -\rho g A\, dz - A\, dp - \frac{\partial}{\partial t}(\rho V) A\, ds + d(\dot{m} V)$$

$$-\rho g A\, dz - A\, dp = \frac{\partial \rho}{\partial t} V A\, ds + \frac{\partial V}{\partial t}(\rho A)\, ds + \underbrace{\dot{m}\, dV}_{=\rho A V dV} + V\, d\dot{m}$$

$$V\left[\frac{\partial \rho}{\partial t} A\, ds + d\dot{m}\right] = 0$$

$$\Rightarrow \frac{\partial V}{\partial t} \rho A\, ds + A\, dp + \rho g A\, dz + \rho A V\, dV = 0$$

DIVIDE BY ρA $\Rightarrow$

$$\frac{\partial V}{\partial t}\, ds + \frac{dp}{\rho} + V\, dV + g\, dz = 0$$

STEADY STATE $\rightarrow$ $\dfrac{\partial V}{\partial t} = 0$

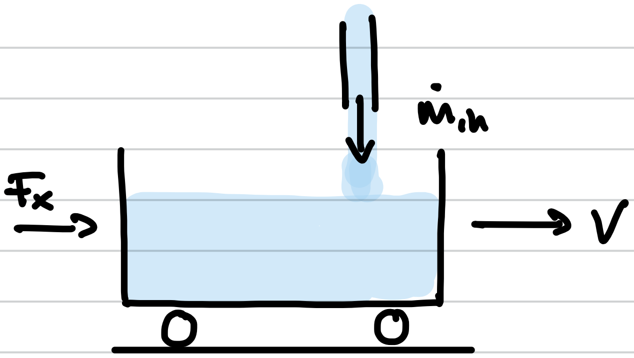
INTEGRATE :

$$\int_1^2 \frac{dp}{\rho} + \int_1^2 V\,dV + \int_1^2 g\,dz = 0$$

INCOMPRESSIBLE (ρ = CONST) $\Rightarrow$

$$p_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2$$

P5



$V = 1.5$ m/s
$\dot{m}_{in} = 50$ kg/s

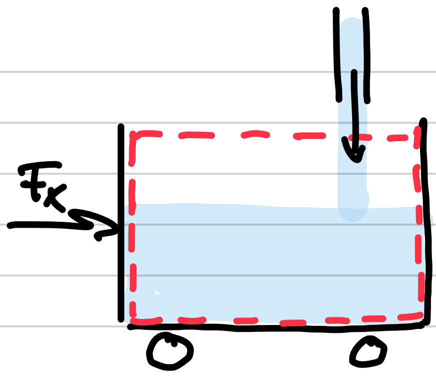
a) CALCULATE THE EXTRA FORCE REQUIRED TO KEEP THE CONTAINER MOVING AT A CONSTANT VELOCITY RELATED TO THE FILLING PROCESS

CONSERVATION OF LINEAR MOMENTUM:

(3.40)

$$\sum F = \frac{d}{dt}\left(\int_{cv} V_y \, dV\right) + \sum_i (\dot{m}_i V_i)_{out} - \sum_i (\dot{m}_i V_i)_{in}$$

IN ORDER TO ESTABLISH A SITUATION WHERE THE ACCUMULATION OF MOMENTUM WITHIN THE CONTROL VOLUME (THE MOVING CONTAINER) IS ZERO, EXTRA FORCE F_x MUST BALANCE THE NET FLUX OF MOMENTUM OVER THE CONTROL VOLUME SURFACE.



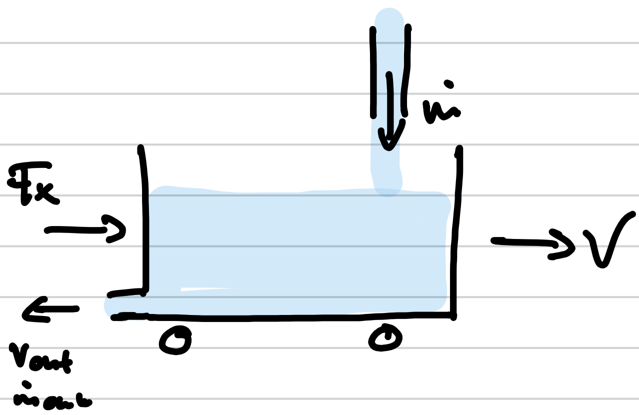
CONTROL VOLUME FOLLOWING THE CONTAINER

FOR THIS CASE, THE MOMENTUM EQN BECOMES:

$$F_x = -\dot{m}_{in} V_{in}$$

SINCE THE CONTROL VOLUME MOVES TO THE RIGHT WITH THE VELOCITY V, THE VELOCITY OF THE INCOMING FLUID RELATIVE TO THE CONTROL VOLUME IS $-V$ $\Rightarrow$ $F_x = \dot{m} V = 75N$

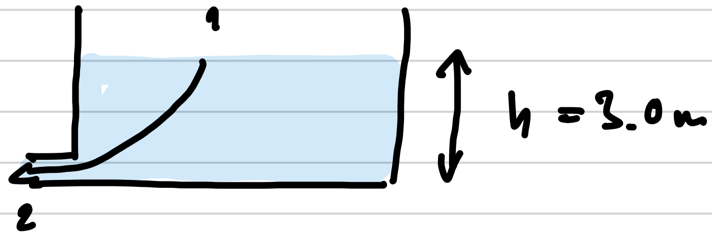
b)



AN OUTLET IS ADDED TO THE TANK.

WE NEED THE OUTLET VELOCITY:

USE BERNOULLI BETWEEN THE SURFACE AND THE OUTLET.



(3.54)

$$\frac{P_1}{\rho} + \frac{1}{2}V_1^2 + g z_1 = \frac{P_2}{\rho} + \frac{1}{2}V_2^2 + g z_2$$

$$P_1 = P_2 \quad , \quad z_1 - z_2 = h$$

ASSUME THAT THE SURFACE AREA IS LARGE COMPARED TO THE OUTLET AREA $\Rightarrow$

$$V_1 \ll V_2$$

$$\Rightarrow \quad V_2 = \sqrt{2gh} = 7.67 \text{ m/s}$$

$$\dot{m}_2 = \rho A V_2$$

WATER @ 20°C : $\rho = 998$ kg/m³

$$A = \frac{\pi d^2}{4} \quad \{ d = 9.0 \text{ cm} \}$$

$$\Rightarrow \dot{m}_2 = 48.7 \text{ kg/s}$$

CONSERVATION OF LINEAR MOMENTUM:

$$F_x = \dot{m}_{out} V_{x\,out} - \dot{m}_{in} V_{x\,in}$$

$$= \dot{m}_2 \left(-V_2 - V\right) - \dot{m}_{in} V_{x\,in}$$

$$= 48.7 \,(-7.67) - 50\,(-1.5)$$

$$= -299 \text{ N}$$

(A FORCE THAT HOLDS THE CONTAINER BACK)

c) GIVE AN EXAMPLE OF WHEN TO USE A FIXED, MOVING AND DEFORMABLE CONTROL VOLUME

- FIXED OBJECT (THE NOZZLE IN P4)
- FOLLOWING A MOVING OBJECT (THE CONTAINER IN THIS PROBLEM, A CAR, A BOAT, ...)
- A DEFORMABLE CONTROL VOLUME COULD BE USED FOR THE ANALYSIS OF THE FLOW IN THE CYLINDER OF A COMBUSTION ENGINE.

d) DERIVE THE CONTINUITY EQUATION ON INTEGRAL FORM FOR A FIXED CONTROL VOLUME USING REYNOLDS TRANSPORT THEOREM

$$\frac{d}{dt}\left(B_{sys}\right) = \frac{d}{dt}\left(\int_{CV} \beta \rho \, dV\right) + \int_{CS} \beta \rho (V_r \cdot n) \, dS$$

CONTINUITY: $B = m$, $\beta = \frac{dB}{dm} = 1$

$\frac{d}{dt}\left(m_{sys}\right) = 0$ BY DEFINITION $\Rightarrow$

$$0 = \frac{d}{dt}\left(\int_{CV} \rho \, dV\right) + \int_{CS} \rho (V_r \cdot n) \, dS$$

FOR A FIXED CONTROL VOLUME:

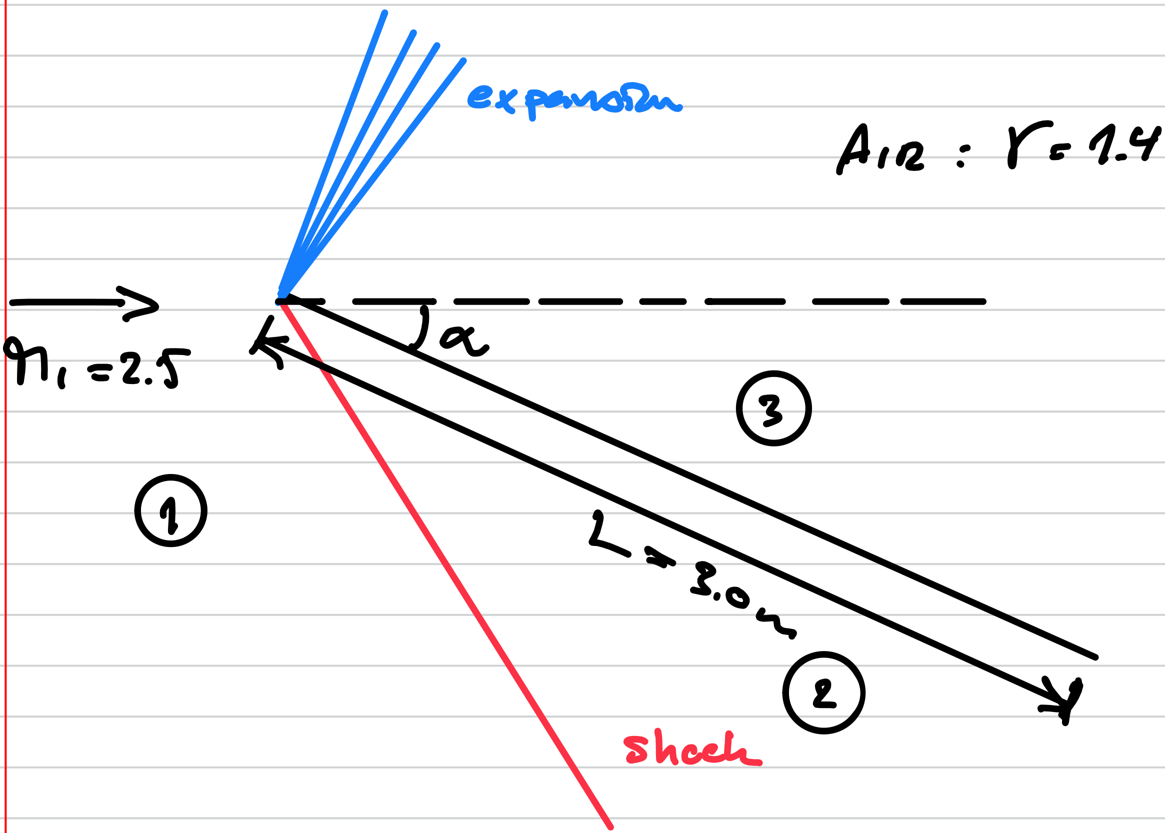
$$\frac{d}{dt}\left(\int_{CV} (\;) \, dV\right) = \int_{CV} \frac{\partial}{\partial t}(\;) \, dV$$

$\Rightarrow$

$$0 = \int_{CV} \frac{\partial \rho}{\partial t} \, dV + \int_{CS} \rho (V_r \cdot n) \, dS$$

P6

FLAT PLATE AT AN ANGLE OF ATTACK OF $\alpha = 6°$ IN A SUPERSONIC FREESTREAM:



a) CALCULATE THE LIFT AND DRAG IF THE PRESSURE IN THE FREESTREAM IS 100.0 kPa

WE NEED THE PRESSURE ON THE UPPER AND LOWER SIDE.

LOWER SIDE:

OBLIQUE SHOCK FOR $M_1 = 2.5$ AND A FLOW DEFLECTION OF $\alpha = 6°$

(9.86)

$\theta - \beta - M$ (Fig 9.23) $\Rightarrow \beta = 28.3°$

(9.82) $M_{n1} = M_1 \sin\beta$

(9.55) with $M_1 = M_{n1}$:

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n1}^2 - 1\right)$$

$\Rightarrow$ $p_2 = 146.8$ kPa

UPPER SIDE:

EXPANSION $\alpha = 6°$

(9.99) $\Rightarrow$

$$\omega(M_1) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}\left(M_1^2 - 1\right)} +$$
$$- \tan^{-1}\sqrt{M_1^2 - 1}$$

$$\omega(M_3) = \omega(M_1) + \alpha$$

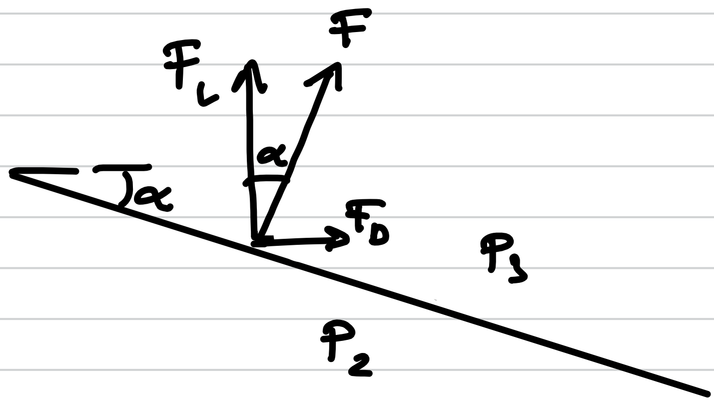
(9.99) $\Rightarrow$ $M_3 = 2.8$

(9.28a) $$\frac{P_{01}}{P_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)$$

$$\frac{P_{03}}{P_3} = \left(1 + \frac{\gamma - 1}{2} M_3^2\right)$$

$P_{02} = P_{03}$ (THE EXPANSION IS ISENTROPIC)

$\Rightarrow P_3 = 65.9$ kPa



$$F = (P_3 - P_2) L b$$

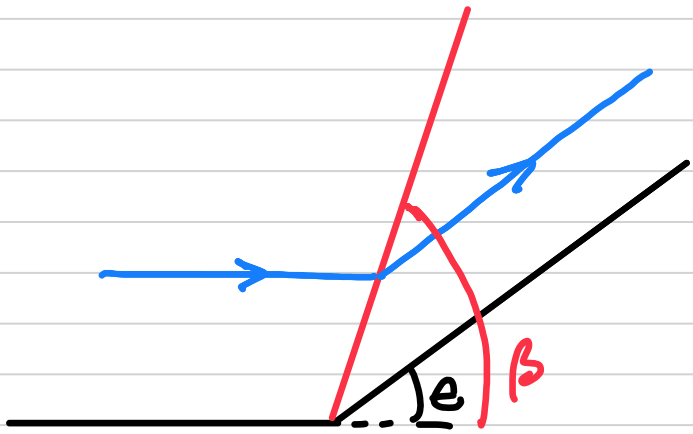
$$F_L = F \cos\alpha = (P_3 - P_2) L b \cos\alpha$$

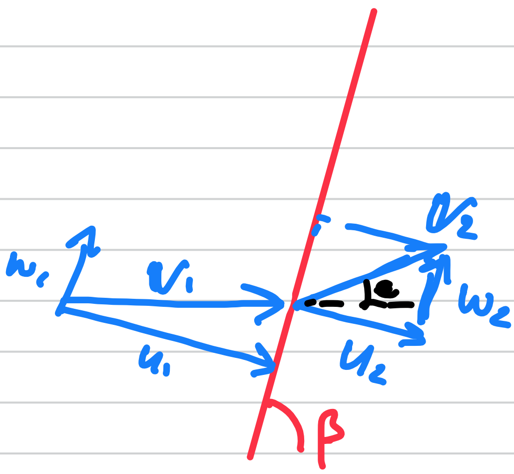
$$F_D = F \sin\alpha = (P_3 - P_2) L b \sin\alpha$$

$F_L = 241.4$ kN/m

$F_D = 25.4$ kN/m

b) SHOW SCHEMATICALLY HOW THE VELOCITY (NORMAL, TANGENTIAL, AND TOTAL) CHANGES OVER AN OBLIQUE SHOCK, INDICATE SHOCK ANGLE (β) AND DEFLECTION ANGLE (θ)





SHOCK-NORMAL DIRECTION:

$u_2 < u_1$

SHOCK-TANGENTIAL DIRECTION:

$w_1 = w_2$

c) WHAT IS REQUIRED FOR A PROCESS TO BE ISENTROPIC?

THE PROCESS MUST BE ADIABATIC AND REVERSIBLE.

d) HOW DOES P, T, ρ, M, P_0, AND T_0 CHANGE OVER AN EXPANSION REGION?

$P \downarrow$

$T \downarrow$

$\rho \downarrow$

$M \uparrow$

P_0 CONSTANT

T_0 CONSTANT.