MTF053 - Fluid Mechanics

2024-11-01 08.30 – 13.30

Approved aids:

- The formula sheet handed out with the exam (attached as an appendix)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Any calculator with cleared memory

Exam Outline:

- In total 6 problems each worth 10p

Grading:

number of points on exam (including bonus points)	24-35	36-47	48-60
grade	3	4	5

Problem 1 - Pipe Flow (10 p.)

A consultant working in the ventilation field has got a request from a customer to design a duct for horizontal transport of air in an office building. Since the duct will be placed inside an office building, it can be assumed that the average temperature of the air is $20°C$.

Specifications from the customer:

- Pipe material: galvanized iron (circular cross section)
- Air flow rate: $Q = 0.056\ m^3/s$
- Pipe length: $L = 30\ m$
- The pressure drop is limited to $3.4\ kPa$ (a constraint set by the fan installed to drive the flow in the ventilation system)

(a) Find the smallest pipe diameter that will fulfill the requirements (5.0 p)

(b) How much can the flow rate be increased if the inner surface of the pipe is polished (smooth surface) and the pressure drop is remained at $3.4\ kPa$ (4.0 p)

Theory questions related to the topic:

(c) Give three examples of sources of local losses in a pipe system (0.5 p)

(d) What do we mean when we say that a pipe flow is *fully developed*? (0.5 p)

Problem 2 - Human-powered Flight (10 p.)

In 1977, the designers of the Gossamer Condor (figure above) won a price in a competition as it was the first human-powered aircraft to complete a prescribed figure-of-eight course defined by two turning points 0.8 km apart. Shortly after that it crashed, but thats another story. During the competition, the average flight speed was estimated to be 4.6 m/s. The drag coefficient for the construction (based on the wing planform area) was 0.046. The average wing chord was 2.3 m and the total wing span (from one wing tip to the other) was 29.0 m. Tests showed that the efficiency of the power transmission (from human power to propulsion of the aircraft) was $\eta = 0.8$. During the competition, the total weight of the construction (including the hard-workin pilot) was 934 N.

hint: the wing planform area (the area that you would see from above) is calculated as $A = chord \times span$

(a) Estimate the lift coefficient (the finite wing span does not have to be taken into account at this low velocity) (3.0 p)

(b) Estimate the total drag (2.0 p)

(c) Estimate the pilot-generated power (1.0 p)

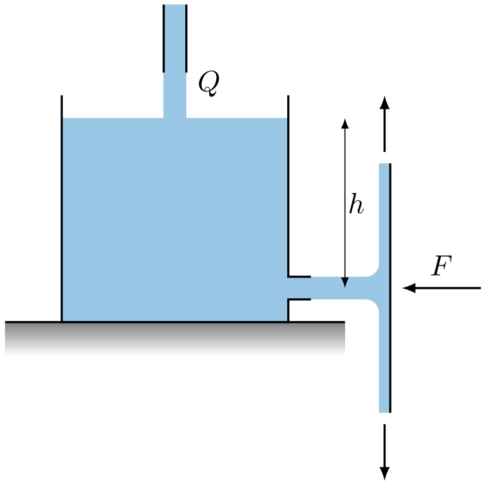
Theory questions related to the topic:

(d) The drag coefficient, C_D, can be divided into two components, which two components? What phenomena are associated with each of the two components? (1.0 p)

(e) Make a schematic representation of the pressure distribution around a cylinder for inviscid flow (potential flow), viscous flow with laminar and turbulent separation respectively. Explain why the pressure varies the way it does. Which of the three cases will give the lowest and highest pressure drag? (2.0 p)

(f) Show how the velocity profile as well as its first and second derivative change in a boundary layer when the flow separates. What is the relation between flow separation and the pressure gradient? (1.0 p)

Problem 3 - Water Tank (10 p.)



A water tank with the diameter $D = 5.0\ m$ is constantly filled from above such that the water level in the tank is constant at $h = 2.0\ m$ above the discharge orifice located in the lower part of the tank. The diemater of the discharge orifice is $d = 5.0\ cm$. The temperature of the water in the tank is $20°C$. The water jet emanating from the orifice hits a plate oriented normal to the jet flow.

(a) Calculated the flow velocity at the exit of the discharge orifice (3.0 p)

(b) Calculate the flow rate Q at which the tank is filled from above (1.0 p)

(c) Calculate the force needed to hold the plate downstream of the discharge orifice in place (the tank can be assumed to be well anchored to the ground) (3.0 p)

Theory questions related to the topic:

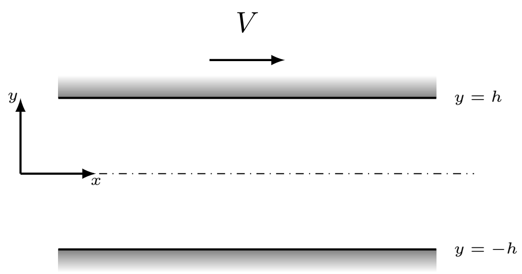
(d) What assumptions are made in the derivation of the Bernoulli equation? (1.0 p)

(e) What does it mean that inlets and outlets are one-dimensional? (1.0 p)

(f) Explain the physical meaning of each of the terms in the continuity equation on integral form (1.0 p)

$$\int_{cv} \frac{\partial \rho}{\partial t} d\mathcal{V} + \int_{cs} \rho \left(\mathbf{V_r} \cdot \mathbf{n} \right) dA = 0$$

Problem 4 - Couette-Poiseuille Flow (10 p.)



A lubrication film (SAE 30W oil at a temperature of 20°C) flows between two parallel plates separated a distance of $2h = 2.0\ mm$. The upper plate moves to the right at a constant velocity of $V = 0.5\ m/s$ and the lower plate is fixed. The flow between the plates is driven by the moving surface but is also affected by the presence of a constant pressure gradient. Thus, the flow is a combined Couette-Poiseuille flow. The flow can be assumed to be fully developed.

(a) Derive an expression describing the velocity distribution in the oil film between the plates (make sure to list all assumptions made and justify any simplifications of the flow equations) (3.0 p)

(b) Calculate the pressure gradient that results in that the average flow velocity is $V_{av} = 0$ (3.0 p)

(c) Calculate the shear stress at the upper and lower wall, respectively (use the pressure gradient calculated in 4b) (2.0 p)

Theory questions related to the topic:

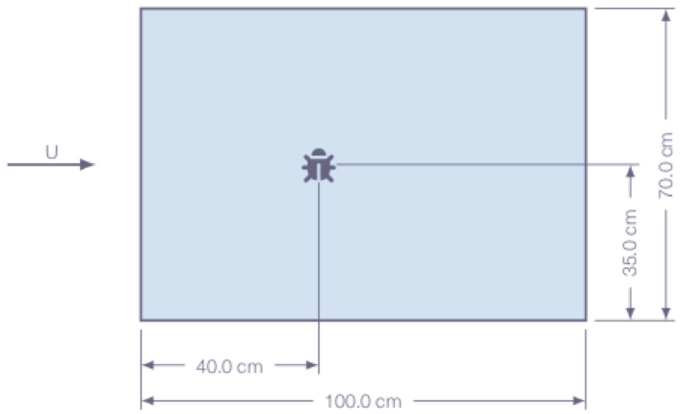
(d) Under what circumstances can the general formulation of the momentum equation be written on the form known as the Navier-Stokes equation? (1.0 p)

(e) When applying Reynolds decomposition to the flow equations as a step in the derivation of the Reynolds-Averaged Navier Stokes (RANS) equations, the velocity components and pressure are divided into an average part and a fluctuating part as for example

$$u = \bar{u} + u'$$

Define the time average and show that the time average of the fluctuating component is identically equal to zero (1.0 p)

Problem 5 - Windows with Bugs (Linux is just better) (10 p.)



A small bug has landed on the outside of a car side window (see figure above). The flow generated over the side window as the car moves forward can be approximated as flat plate flow with zero pressure gradient. A boundary layer is built up over the window surface starting at the left edge (the leading edge of the window). The small step between the window frame and the window surface can be assumed to trigger the onset of turbulent flow and thus the boundary layer will be turbulent directly at the leading edge.

(a) Calculate the minimum car speed at which the bug will not be able to stick to the window surface if the bug can resist a shear stress of $1.0 N/m^2$ (4.0 p)

(b) What is the total skin friction drag on the window surface at this speed? (2.0 p)

(c) calculate the boundary-layer thickness at the location of the bug ($x = 40\ cm$) and at the downstream end of the window ($x = 100\ cm$), respectively. (1.0 p)

Theory questions related to the topic:

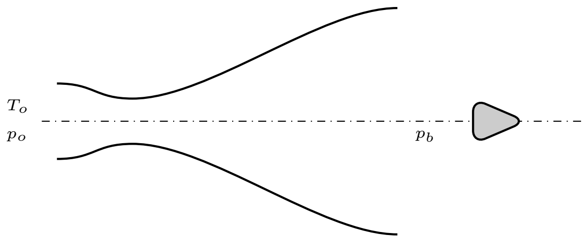
(d) What assumption is made to be able to derive the boundary layer equations and which are the main limitations of these equations? (1.0 p)

(e) For laminar flow over a flat plate, the velocity profile (the Blasius profile) is self-similar - what does that mean? (1.0 p)

(f) How is the transition from laminar to turbulent flow in an external boundary layer affected by (assume other properties to be constant) (1.0 p)

i. increased freestream velocity (U_∞)
ii. surface roughness (ε)
iii. increased intensity of freestream turbulence
iv. positive pressure gradient

Problem 6 - Re-entry Vehicle (10 p.)



A model of a re-entry vehicle (a very blunt object) is to be tested in a supersonic wind tunnel at a freestream Mach number of 4.0. The test section (where the model is placed) is located inside of a vaccum tank and during the test, the pressure in the tank, the pressure downstream of the nozzle exit (p_b), is reduced to 10% of the atmospheric pressure. The nozzle-exit cross-section area is 1.0 m^2

(a) Calculate nozzle throat area given that the nozzle flow is perfectly expanded, i.e. that there are no shocks inside or outside of the nozzle and no expansion fans generated at the nozzle exit (3.0 p)

(b) Calculate the total pressure upstream of the nozzle (1.0 p)

(c) Calculate the stagnation pressure and temperature at the surface of the re-entry vehicle at the centerline given that the temperature upstream of the nozzle is 600 K (2.0 p)

Theory questions related to the topic:

(d) Which of the properties h_o, T_o, a_o, p_o, and ρ_o are constants in a flow if the flow is adiabatic and isentropic, respectively? (1.0 p)

(e) Using the continuity equation and energy equation on differential form together with the definition of speed of sound, the following relation can be derived

$$\frac{dV}{V}\left(M^2-1\right) = -\frac{dp}{\rho V^2}\left(M^2-1\right) = \frac{dA}{A}$$

i. Show, using the relation given above, how the velocity and pressure changes in a flow through a divergent or convergent duct for initially subsonic flow or initially supersonic flow (1.0 p)

ii. What can we tell about sonic flow from the relation and how does that relate to the flow in a convergent-divergent nozzle? (1.0 p)

(f) Show schematically how the oblique shock formed ahead of a wedge traveling at supersonic speed for the two cases below (1.0 p)

i. $\theta < \theta_{max}$

ii. $\theta > \theta_{max}$

P1

PIPE FLOW

GIVEN:

air @ 20°C

$Q = 5.6 \cdot 10^{-2}\ m^3/s$

$\Delta P_{max} = 3.4\ kPa\ /\ 80\ m$

PIPE MATERIAL = GALVANIZED IRON ⇒

⇒ $\varepsilon = 0.15\ mm$

a) FIND A PIPE DIAMETER THAT MEETS THE SPECIFIED REQUIREMENTS..

ASSUME STEADY-STATE FLOW:

(3.73) NO PUMP OR TURBINE.

$$\left(\frac{P}{\rho g} + \frac{V_{av}^2}{2g} + z\right)_{in} = \left(\frac{P}{\rho g} + \frac{V_{av}^2}{2g} + z\right)_{out} + h_f$$

HORIZONTAL PIPE ⇒ $z_{in} = z_{out}$

ASSUME FULLY-DEVELOPED PIPE FLOW ⇒

$V_{av\,in} = V_{av\,out}$

$$\left.\begin{aligned} &\Rightarrow P_{in} = P_{out} + \rho g h_f \\ &(6.10):\ h_f = f\frac{L}{D}\frac{V_{av}^2}{2g} \end{aligned}\right\} \Rightarrow$$

$$P_{in} = P_{out} + f\frac{L}{D}\frac{\rho V_{av}^2}{2}$$

WE SHOULD FIND A DIAMETER THAT MEETS WITH THE SPECIFICATIONS

$P_{in} - P_{out} = \Delta p_{max}$

$L = 30 m$

1. $V_{av} = \frac{4Q}{\pi D^2} \Rightarrow D = \left(\frac{8 f L \rho Q^2}{\pi^2 \Delta p_{max}} \right)^{1/5}$

2. $Re_D = \frac{\rho V_{av} D}{\mu}$

3. (6.48) $\frac{1}{\sqrt{f}} = -2.0 \log_{10} \left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{Re_D \sqrt{f}} \right)$

GUESS A VALUE FOR f AND ITERATE 1 → 3 UNTIL CONVERGED..

$\Rightarrow f = 2.67 \cdot 10^{-2}$

$D = 5.7$ cm

$Re_D = 8.3 \cdot 10^4$

$\Rightarrow$ TURBULENT OK..

b) HOW MUCH CAN THE FLOW RATE BE INCREASED IF THE PIPE IS POLISHED SUCH THAT IT IS SMOOTH?

FROM a) WE KNOW THAT

$$p_{in} = p_{out} + \rho g h_f$$

$$h_f = f \frac{L}{D} \frac{V_{av}^2}{2g}$$

$$\Rightarrow V_{av} = \sqrt{\frac{2 \Delta p_{max} D}{f L \rho}}$$

$(D = 5.7 \text{ cm}, \ L = 30 \text{ m}, \ \Delta p = 3.4 \text{ kPa})$

1. $$V_{av} = \sqrt{\frac{2 \Delta p_{max} D}{f L \rho}}$$

2. $$Re_D = \frac{\rho V_{av} D}{\mu}$$

3. (6.38) $$\frac{1}{\sqrt{f}} = 2.0 \log_{10} (Re_D \sqrt{f}) - 0.8$$

GUESS f, ITERATE 1–3 UNTIL CONVERGED..

$\Rightarrow f = 1.83 \cdot 10^{-2}$

$Q = 6.2 \cdot 10^{-2} \text{ m}^3/\text{s}$

| $Re_D = 9.2 \cdot 10^4 \Rightarrow$ TURBULENT OK.

P2

HUMAN-POWERED FLIGHT.

GIVEN:

FLIGHT SPEED = 4.6 m/s
DRAG COEFFICIENT (C_D) = 0.046
CHORD = 2.3 m
WINGSPAN = 29.0 m
EFFICIENCY (η) = 0.8
TOTAL WEIGHT = 934 N

a) ESTIMATE THE LIFT COEFFICIENT (C_L)

THE LIFT FORCE IS EQUAL TO THE TOTAL WEIGHT

$$(7.66) \qquad \frac{F_L}{\frac{1}{2}\rho V^2 A_p} = C_L$$

$F_L = 984\,N$

ASSUME AIR @ 20°C $\Rightarrow \rho = 1.23\ kg/m^3$

A_p = CHORD × WING SPAN = 2.3 × 29

$V = 4.6\ m/s$

$\Rightarrow C_L = 1.076$

b) ESTIMATE THE TOTAL DRAG

(7.66)
$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A_p}$$

$$\Rightarrow F_D = 39.9\ N$$

C) CALCULATE THE PILOT-GENERATED POWER.

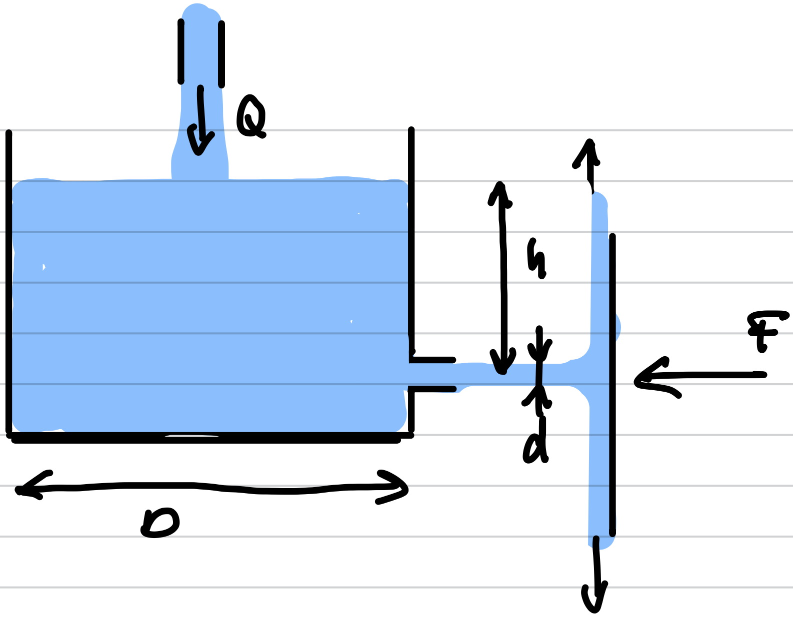
THE POWER REQUIRED TO PROPELL THE AIRCRAFT IS

$$P = F_D \cdot V$$

THE POWER GENERATE BY THE PILOT IS THUS:

$$P_{PILOT} = \frac{F_D \cdot V}{\eta} = 229.6\ W$$

P3



GIVEN:

WATER @ 20°C

$h = 2.0\ m$

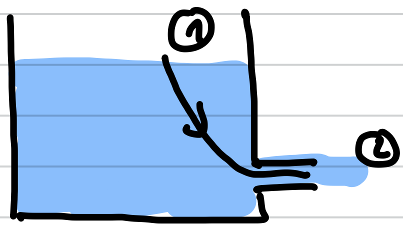
$D = 5.0\ m$

$d = 5.0\ cm \quad (0.05\ m)$

a) CALCULATE THE VELOCITY THE DISCHARGE ORIFICE..

(h = const = 2.0 m)

SET UP BERNOULLI FOR A STREAMLINE STARTING AT THE WATER SURFACE AND ENDING AT THE OUTLET



(3.54)

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_1$$

$$P_1 = P_2$$

(3.22) (STEADY-STATE FLOW) $\Rightarrow$

$$\rho_1 \frac{\pi}{4} D^2 V_1^2 = \rho_2 \frac{\pi}{4} d^2 V_2^2$$

$$\rho_1 = \rho_2 \quad \Rightarrow V_1 = \left(\frac{d}{D}\right)^2 V_2$$

$$z_1 + \frac{1}{2}\rho \left(\frac{d}{D}\right)^4 V_2^2 = \frac{1}{2}\rho V_2^2 + z_2$$

$$z_1 - z_2 = h \quad \Rightarrow$$

$$2gh = \rho V_2^2 \left(1 - (d/D)^4\right)$$

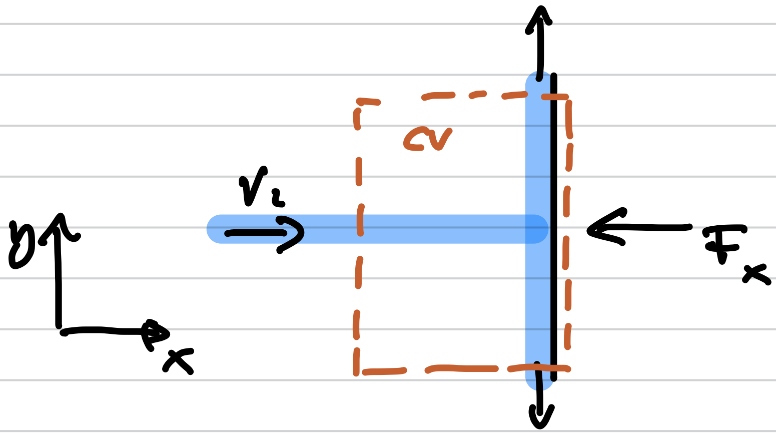
$$\Rightarrow V_2 = 6.8 \text{ m/s}$$

b) CALCULATE THE FLOW RATE AT WHICH THE TANK IS FILLED..

TO KEEP THE WATER LEVEL CONSTANT, THE TANK MUST BE FILLED AT THE SAME RATE AS THE FLOW RATE AT THE EXIT.

$$Q = \frac{\pi d^2}{4} V_2 = 1.23 \cdot 10^{-2} \; m^3/s$$

c) CALCULATE THE FORCE NEEDED TO HOLD THE PLATE DOWNSTREAM OF THE EXIT IN PLACE



(3.40)

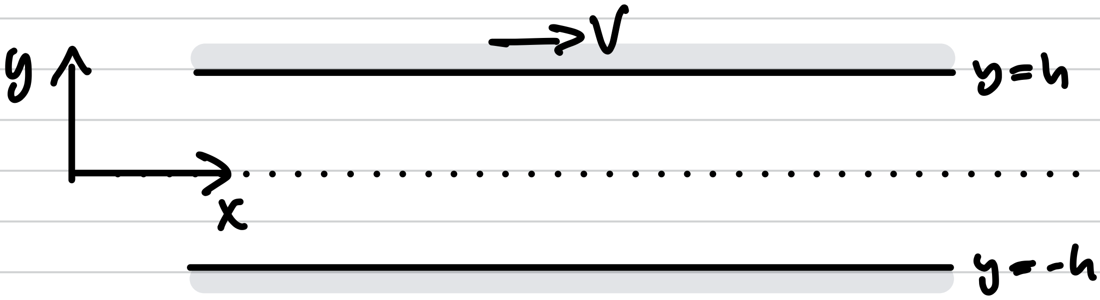
$$\sum F = \frac{d}{dt}\left(\int_{CV} V \rho \, dV\right) + \sum_i (\dot{m}_i V_i)_{out} + - \sum_i (\dot{m}_i V_i)_{in}$$

FOR STEADY-STATE FLOW AND FORCE IN X-DIRECTION WE GET

$$F_x = -\dot{m}_2 V_2 = -\rho \frac{\pi}{4} d^2 V_2^2 = -76.9 N$$

P4

COUETTE – POISEUILLE FLOW



SAE-30W OIL @ 20°C

$h = 1.0 \, mm$

$V = 0.5 \, m/s$

FLOW IS DRIVEN BY A MOVING SURFACE AND A CONSTANT PRESSURE GRADIENT.

ASSUMPTIONS:

- STEADY-STATE FLOW
- 2D FLOW $\Rightarrow$ $v = w = 0$
- FULLY DEVELOPED
- INCOMPRESSIBLE

(4.7) STEADY-STATE + INCOMPRESSIBLE

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$v = w = 0 \Rightarrow \frac{\partial u}{\partial x} = 0$$

(4.38) x – DIRECTION :

$$\rho\left(\underbrace{\cancel{\frac{\partial u}{\partial t}}}_{S.S.} + u\underbrace{\cancel{\frac{\partial u}{\partial x}}}_{C.E.} + \underbrace{\cancel{v}}_{v=0}\frac{\partial u}{\partial y} + \underbrace{\cancel{w}}_{w=0}\frac{\partial u}{\partial z}\right) =$$

$$= -\frac{\partial p}{\partial x} + \mu\left(\underbrace{\cancel{\frac{\partial^2 u}{\partial x^2}}}_{C.E.} + \frac{\partial^2 u}{\partial y^2} + \underbrace{\cancel{\frac{\partial^2 u}{\partial z^2}}}_{2D-FLOW}\right)$$

$$\Rightarrow \mu\frac{\partial^2 u}{\partial y^2} = \frac{\partial p}{\partial x}$$

(4.38) y – DIRECTION:

$$\rho\left(\underbrace{\cancel{\frac{\partial v}{\partial t}}}_{S.S.} + u\underbrace{\cancel{\frac{\partial v}{\partial x}}}_{V=0} + \cancel{v}\underbrace{\cancel{\frac{\partial v}{\partial y}}}_{V=0} + \cancel{w}\underbrace{\cancel{\frac{\partial v}{\partial z}}}_{W=V=0}\right) =$$

$$= -\frac{\partial p}{\partial y} + \mu\left(\underbrace{\cancel{\frac{\partial^2 v}{\partial x^2}}}_{V=0} + \underbrace{\cancel{\frac{\partial^2 v}{\partial y^2}}}_{V=0} + \underbrace{\cancel{\frac{\partial^2 v}{\partial z^2}}}_{V=0}\right)$$

$$\Rightarrow \frac{\partial p}{\partial y} = 0$$

IN THE SAME WAY (4.38) z – DIRECTION $\Rightarrow$

$$\frac{\partial p}{\partial z} = 0$$

$$\frac{\partial p}{\partial y} = \frac{\partial p}{\partial z} = 0 \Rightarrow p = p(x)$$

$$\frac{\partial p}{\partial x} = \frac{dp}{dx}$$

AND THUS :

$$\mu \frac{\partial^2 u}{\partial y^2} = \frac{dp}{dx}$$

INTEGRATE $\Rightarrow$

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{dp}{dx} y + C_1$$

$$u(y) = \frac{1}{2\mu}\frac{dp}{dx} y^2 + C_1 y + C_2$$

BOUNDARY CONDITIONS (NO SLIP)

$$u(h) = V \quad , \quad u(-h) = 0$$

$$u(-h) = \frac{1}{2\mu}\frac{dp}{dx} h^2 - C_1 h + C_2 = 0$$

$$+ \quad u(h) = \frac{1}{2\mu}\frac{dp}{dx} h^2 + C_1 h + C_2 = V$$

$$\frac{1}{\mu}\frac{dp}{dx} h^2 + 2C_2 = V$$

$$\Rightarrow C_2 = -\frac{1}{2\mu}\frac{dp}{dx} + h^2 + \frac{V}{2}$$

$$u(-h) = \frac{1}{2\mu}\frac{dp}{dx}h^2 - C_1 h + C_2 = 0$$

$$-\quad u(h) = \frac{1}{2\mu}\frac{dp}{dx}h^2 + C_1 h + C_2 = V$$

$$-2C_1 h = -V \Rightarrow C_1 = \frac{V}{2h}$$

$$\boxed{\begin{aligned} u(y) &= \frac{1}{2\mu}\frac{dp}{dx}(y^2 - h^2) + \frac{V}{2}\left(1 + \frac{y}{h}\right) \\ \frac{\partial u}{\partial y} &= \frac{1}{\mu}\frac{dp}{dx}y + \frac{V}{2h} \end{aligned}}$$

b) CALCULATE THE PRESSURE GRADIENT SUCH THAT THE AVERAGE VELOCITY IS ZERO.

$$V_{av} = \frac{1}{2h}\int_{-h}^{h} u(y)\,dy = 0$$

$$\frac{1}{2h}\left[\frac{1}{2\mu}\frac{dp}{dx}\left(\frac{y^3}{3}-h^2y\right)+\frac{V}{2}\left(y+\frac{y^2}{2h}\right)\right]_{-h}^{h}$$

$$=\frac{1}{2h}\left[\frac{1}{2\mu}\frac{dp}{dx}\left(\frac{h^3}{3}-h^3\right)+\frac{V}{2}\left(h-\frac{h^2}{2h}\right)+\right.$$

$$\left.-\frac{1}{2\mu}\frac{dp}{dx}\left(\frac{-h^3}{3}+h^3\right)-\frac{V}{2}\left(-h+\frac{h^2}{2h}\right)\right]$$

$$=\frac{1}{2h}\left[-\frac{2}{3}\frac{h^3}{\mu}\frac{dp}{dx}+Vh\right]$$

$$V_{av}=0 \Rightarrow Vh=\frac{2}{3}\frac{h^3}{\mu}\frac{dp}{dx} \Rightarrow$$

$$\Rightarrow \boxed{\frac{dp}{dx}=\frac{3\mu V}{2h^2}}$$

SAE-30W OIL @ 20°C $\Rightarrow \mu = 2.9\cdot 10^{-1}$

$$\Rightarrow \frac{dp}{dx}=2.175\cdot 10^{5}\ \mathrm{Pa/m}$$

(A VERY HIGH VALUE FOR THE PRESSURE GRADIENT)

c) CALCULATE THE STRESS AT THE UPPER AND LOWER WALL..

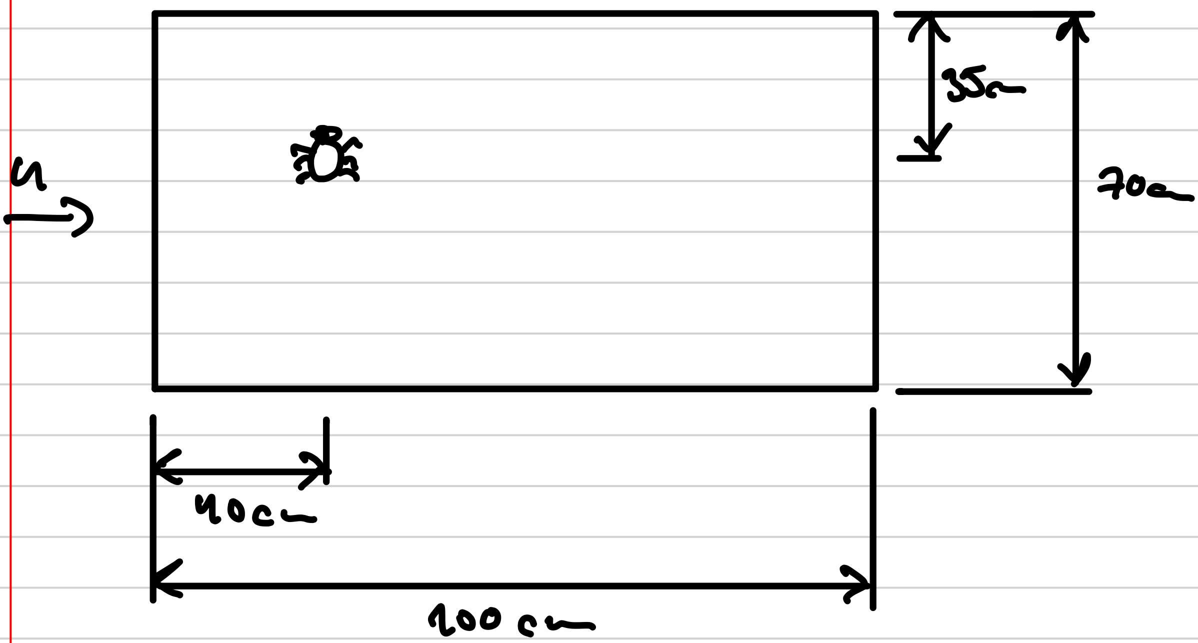
$$\tau = \mu \frac{\partial u}{\partial y} \Rightarrow$$

$$\tau_{upper} = \mu \frac{\partial u}{\partial y}\bigg|_{y=h} = 290\ N/m^2$$

$$\tau_{lower} = \mu \frac{\partial u}{\partial y}\bigg|_{y=-h} = -145\ N/m^2$$

P5

WINDOWS WITH BUGS.



ASSUME:

AIR @ 20°C ($\rho = 1.2\ kg/m^3$, $\mu = 1.8 \cdot 10^{-5}\ Ns/m^2$)

TURBULENT BOUNDARY LAYER FROM LEADING EDGE.

a) CALCULATE THE MINIMUM CAR SPEED FOR WHICH THE BUG WILL NOT BE ABLE TO STICK TO THE SURFACE IF THE BUG CAN RESIST A SHEAR STRESS OF $1\ N/m^2$

TURBULENT B.L.

(7.44) $$\tau_w = \frac{0.0135\,\mu^{1/7}\,\rho^{6/7}\,U^{13/7}}{x^{1/7}}$$

$\Rightarrow U = 20.17\ m/s$

b) CALCULATE THE SKIN FRICTION DRAG AT THIS SPEED.

(7.45) $$C_D = \frac{0.031}{Re_L^{1/7}}$$

$$Re_L = \frac{\rho U L}{\mu}$$

$$D = \frac{1}{2}\rho U^2 C_D A_p$$

$$A_p = bL = 0.7 \times 1.0$$

$\Rightarrow D = 0.71\ N$

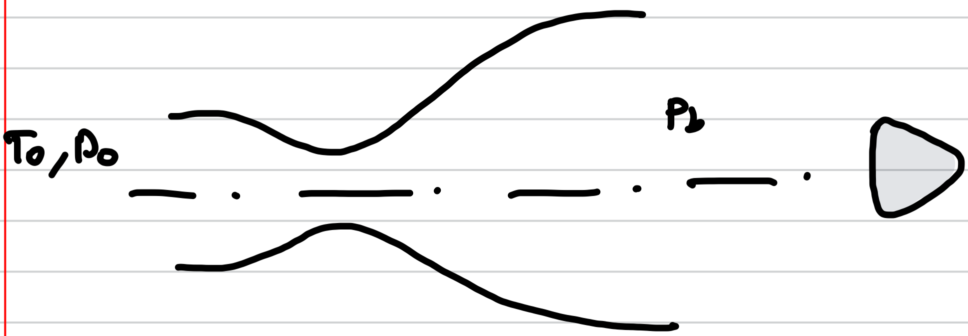
c) CALCULATE THE BOUNDARY-LAYER THICKNESS AT $x = 0.4$ AND $x = 1.0$

(7.42) $$\frac{\delta}{x} = \frac{0.16}{Re_x^{1/7}}, \quad Re_x = \frac{\rho U x}{\mu}$$

$\delta(x=0.4) = 9.7\,mm$, $\delta(x=1.0) = 21.3\,mm$

P6

RE-ENTRY VEHICLE.



$P_b = 0.1\ P_{atm}$

$A_{exit} = 1.0\,m^2$

$M_{exit} = 4.0$

a) CALCULATE THE THROAT AREA GIVEN THAT THE FLOW THROUGH THE NOZZLE IS PERFECTLY EXPANDED — NO SHOCKS INSIDE THE NOZZLE, NO SHOCK AT THE NOZZLE-EXIT PLANE, NO OBLIQUE SHOCKS OR EXPANSIONS DOWNSTREAM OF THE EXIT (RELATED TO THE EXPANSION).

$$(9.44)\quad \left(\frac{A_e}{A^*}\right)^2 = \frac{1}{M_e^2}\left[\frac{2+(\gamma-1)M_e^2}{\gamma+1}\right]^{\frac{\gamma+1}{\gamma-1}}$$

ASSUME CALORICALLY PERFECT AIR $\Rightarrow$

$\gamma = 1.4 \Rightarrow A_e/A^* = 10.72$

THE FLOW IS SUPERSONIC AT THE EXIT AND THUS THE THROAT AREA IS

$$A_t = A^* = 0.093\ m^2$$

b) CALCULATE THE TOTAL PRESSURE UPSTREAM OF THE NOZZLE.

NO SHOCKS $\Rightarrow$ TOTAL PRESSURE IS CONSTANT THROUGH THE NOZZLE (ISENTROPIC FLOW)

$$\frac{p_o}{p_e} = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\gamma/(\gamma-1)} \qquad (9.28)$$

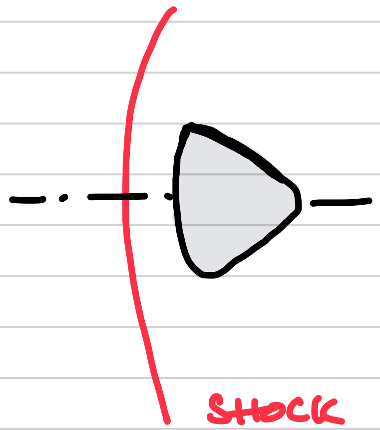
$p_e = p_b$ (NO SHOCKS OR EXPANSIONS AT THE EXIT..)

$\Rightarrow P_o = 15.2$ atm

(1538.5 kPa IF $P_{atm} = 101325$ Pa)

c) CALCULATE THE STAGNATION PRESSURE AND TEMPERATURE AT THE SURFACE OF THE TEST OBJECT.

THERE WILL BE A SHOCK UPSTREAM OF THE TEST OBJECT

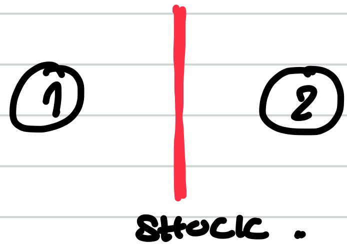


AT THE CENTERLINE, THE SHOCK CAN BE APPROXIMATED AS A NORMAL SHOCK..

THE SHOCK IS AN ADIABATIC PROCESS ⇒ T_0 WILL NOT CHANGE OVER THE SHOCK AND THUS THE STAGNATION TEMPERATURE WILL BE THE SAME AS UPSTREAM OF THE NOZZLE.

SO IF THE UPSTREAM TEMPERATURE IS, AS AN EXAMPLE, $T_0 = 600K$, THE STAGNATION TEMPERATURE AT THE SURFACE OF THE TEST OBJECT IS ALSO $T_0 = 600K$.

THE STAGNATION PRESSURE WILL, HOWEVER, CHANGE OVER THE SHOCK



(9.55)
$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right)$$

$P_1 = P_b$, $M_1 = 4.0$

(9.57)
$$M_2^2 = \frac{(\gamma-1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma-1)}$$

$M_1 = 4.0$

(9.28)
$$\frac{P_{02}}{P_2} = \left(1 + \frac{\gamma-1}{2}M_2^2\right)^{\gamma/(\gamma-1)}$$

$\Rightarrow P_{02} = 1.92 \text{ Patm}$

SO, AT THE SURFACE OF THE TEST OBJECT THE STAGNATION PRESSURE AND TEMPERATURE IS $T_0 = 600\ K$, $P_0 = 1.92 \text{ Patm}$.