MTF053 - Fluid Mechanics

2024-08-19 08.30 – 13.30

Approved aids:

- The formula sheet handed out with the exam (attached as an appendix)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Any calculator with cleared memory

Exam Outline:

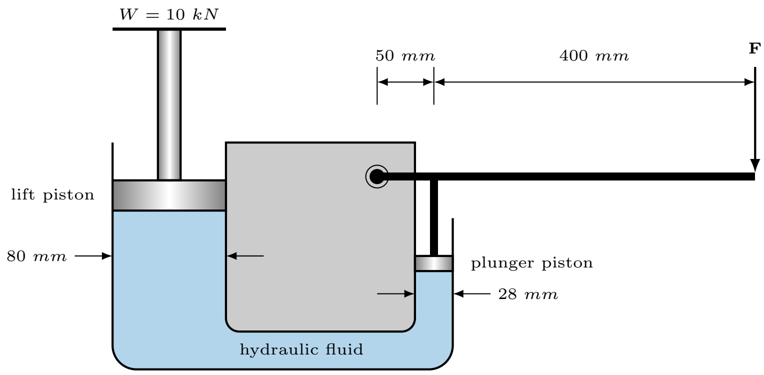
- In total 6 problems each worth 10p

Grading:

number of points on exam (including bonus points)	24-35	36-47	48-60
grade	3	4	5

Problem 1 - Hydraulic Jack (10 p.)

The hydraulic jack showed in the figure below has a lift-piston diameter of 80 mm and a plunger-piston diameter of 28 mm. The pivot of the jack is located 50 mm from the plunger shaft and the force is applied 400 mm from the plunger shaft. The hydraulic fluid has a density of 920 kg/m^3.



(a) Calculate the applied force **F** needed to lift a weight of 10 kN (6.0 p)

Theory questions related to the topic:

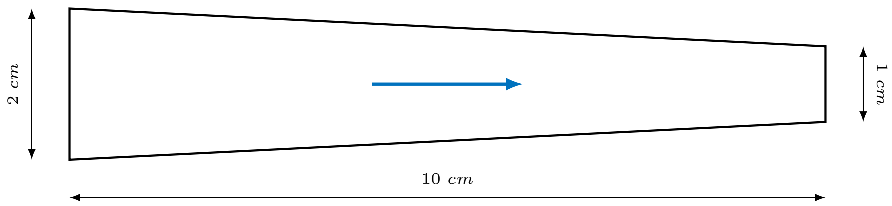
(b) Show that the normal of a constant-pressure surface must be aligned with the gravity vector in a fluid at rest (2.0 p)

(c) How does the hydrodynamic pressure distribution differ in liquids and gases? (1.0 p)

(d) How can we make use of Pascal's law when analyzing manometer tubes? (1.0 p)

Problem 2 - Garden Hose (10 p.)

The internal diameter of the nozzle of a garden hose reduces linearly from 2 cm to 1 cm over a length of 10 cm. The flow rate through the nozzle is 0.4 L/s.



(a) Derive an expression for the pressure gradient dp/dx in the nozzle (6.0 p)

(b) Calculate the pressure gradient at both ends of the nozzle (1.0 p)

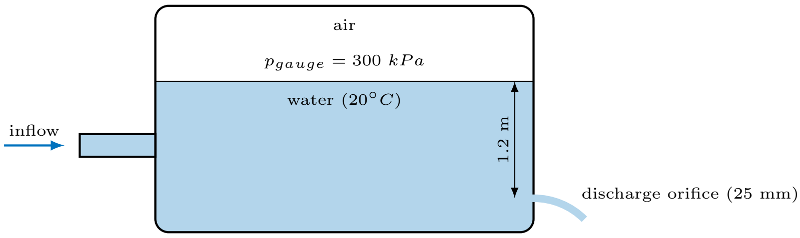
Theory questions related to the topic:

(c) Explain how a venturi meter works and derive the relation needed to estimate the velocity (2.0 p)

(d) What assumptions are made in the derivation of the Bernoulli equation? (1.0 p)

Problem 3 - Water Tank (10 p.)

Compressed air is used to force water through a 25-mm-diameter orifice in a large tank (see figure below). The water level in the tank is kept stable by adding water at the same rate at which water is being discharged through the orifice. The discharge coefficient of the orifice has been estimated to be 0.94, i.e. $Q_{real} = 0.94 Q_{ideal}$ where Q is the flow rate through the orifice in m^3/s.



(a) Calculate the flow rate (Q_{inflow}) at which water must be supplied to the tank if the gauge pressure of the air in the tank is 300 kPa, the water level is to be kept constant at 1.2 m above the center of the orifice, and the temperature of the water is 20°C (8.0 p)

Theory questions related to the topic:

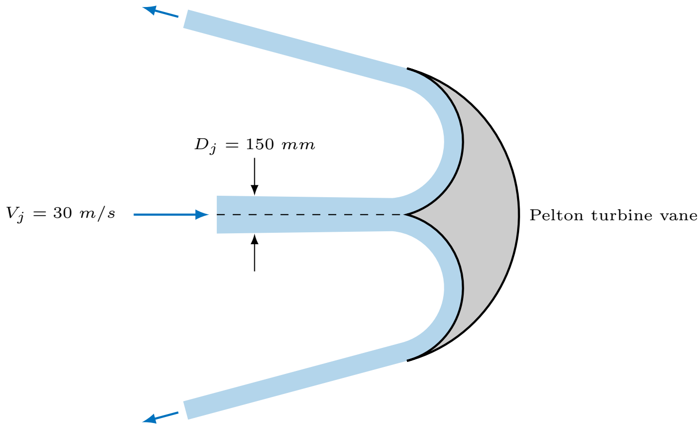
(b) How can the generic form of Reynolds transport theorem (the relation given below) be be simplified for a fix control volume? (1.0 p)

$$\frac{d}{dt}\left(B_{syst}\right) = \frac{d}{dt}\left(\int_{cv} \beta \rho d\mathcal{V}\right) + \int_{cs} \beta \rho \left(\mathbf{V}_r \cdot \mathbf{n}\right) dA$$

(c) What does it mean that inlets and outlets are one-dimensional? (1.0 p)

Problem 4 - Pelton Turbine (10 p.)

The rotor of a Pelton wheel turbine is driven by water jets impinging on vanes mounted on the periphery of the rotor. The incident water jet has a velocity relative to the vane of 30 m/s and the diameter of the jet can be approximated to be 150 mm. The vane deflects the incoming water jet by 165°. The water temperature can be assumed to be $20°C$.



(a) Calculate the force exerted by the water on the vane (6.0 p)

Theory questions related to the topic:

(b) Derive the momentum equation on differential form starting from the integral form (2.0 p)

$$\sum \mathbf{F} = \frac{d}{dt}\left(\int_{cv} \mathbf{V}\rho d\mathcal{V}\right) + \sum (\dot{m}_i \mathbf{V}_i)_{out} - \sum (\dot{m}_i \mathbf{V}_i)_{in}$$

(c) A fluid element is subjected to both body forces and surface forces. Give an example of a body force and name the two surface forces (1.0 p)

(d) Under what circumstances can the general formulation of the momentum equation be reduced to the Navier-Stokes equation? (1.0 p)

PROBLEM 5 - PIPE FLOW (10 P.)

Water at a temperature of $20°C$ flows at a flow rate of 150 L/s through a circular cast iron pipe with a diameter of 300 mm. The following empirical formula describes the velocity distribution in the pipe

$$v(r) = V_{av}\left[(1 + 1.326\sqrt{f}) - 2.04\sqrt{f}\log_{10}\left(\frac{R}{R-r}\right)\right]$$

where r is the radial coordinate from the pipe centerline, R is the radius of the pipe, f is the Darcy friction factor for this specific pipe flow, and V_{av} is the average velocity in the pipe.

Another way to describe the velocity profile mathematically is to use a power relation

$$v(r) = V_o\left(1 - \frac{r}{R}\right)^{1/n}$$

where V_o is the centerline velocity and n is a constant. The value of n can be adjusted to match a specific flow.

(a) Obtain a value of n such that both the expressions above give the same centerline velocity (V_o) and average velocity (V_{av}) (8.0 p)

hint:

$$\int \left(1 - \frac{x}{a}\right)^{1/n} x dx = \frac{n(x-a)\left(1 - \frac{x}{a}\right)^{1/n}(n(a+x)+x)}{(n+1)(2n+1)} + constant$$

Theory questions related to the topic:

(b) Why does the Moody chart not give reliable values in the Reynolds number range $2000 < Re < 4000$? (1.0 p)

(c) Compare the velocity profiles for fully developed laminar and turbulent flow, which of the flows gives the highest wall shear stress for a given mass flow and what is the reason for that? (1.0 p)

Problem 6 - Boundary-layer Flow (10 p.)

Air at 10°C flows at 18 km/h over a flat surface that is 1 m long and 2 m wide.

Calculate:

(a) The shear stress at the downstream end of the surface (4.0 p)

(b) The average shear stress on the surface (2.0 p)

(c) The total drag force on the surface (1.0 p)

Theory questions related to the topic:

(d) What assumption is made to be able to derive the boundary layer equations? (1.0 p)

(e) For laminar flow over a flat plate, the velocity profile is self-similar - what does that mean? (1.0 p)

(f) Name two alternative ways to measure the boundary layer thickness other than δ. How can these measures be interpreted physically? (1.0 p)

P1

GIVEN:

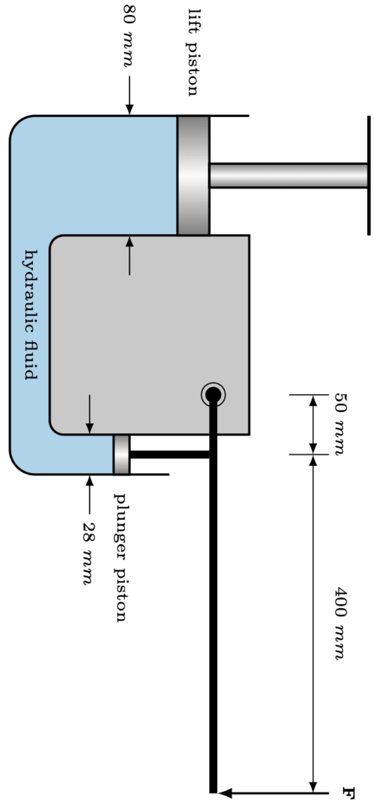
DIMENSIONS OF HYDRAULIC JACK ACCORDING TO FIGURE BELOW.

DENSITY OF HYDRAULIC FLUID

$\rho = 920\ kg/m^3$

TASK:

CALCULATE THE APPLIED FORCE F NEEDED TO LIFT A WEIGHT OF 10 kN



$L_1 = 50\ mm$

$L_2 = 400\ mm$

$$A_1 = \frac{\pi D_1^2}{4} = \frac{\pi \cdot 0.08^2}{4}$$

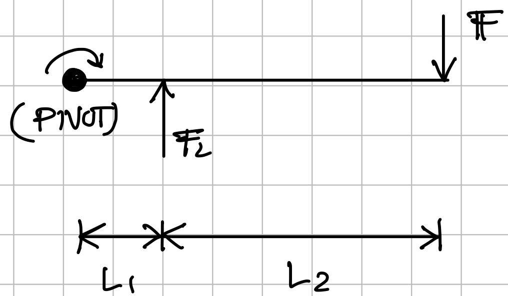
$$A_2 = \frac{\pi D_2^2}{4} = \frac{\pi \cdot 0.028^2}{4}$$

$$P_1 = \frac{W}{A_1}$$

$$P_2 = P_1 \Rightarrow F_2 = P_1 A_2$$

$$\Rightarrow F_2 = 1.225\ kN$$

MOMENT ABOUT THE PIVOT:



$$F_2 L_1 = F(L_1 + L_2)$$

$$\Rightarrow F = \frac{F_2 L_1}{L_1 + L_2} \Rightarrow$$

$$\underline{F = 0.1361\ kN}$$

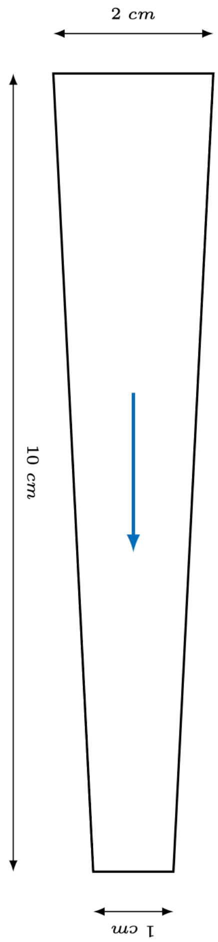
P_2

GIVEN:

INTERNAL NOZZLE DIAMETER REDUCES LINEARLY FROM $D_1 = 0.02\,m$ TO $D_2 = 0.01\,m$ OVER THE LENGTH $L = 0.1\,m$

THE FLOW RATE THROUGH THE NOZZLE IS $Q = 0.4\,L/s$

GARDEN HOUSE ⇒ WATER @ ~ 20°C ⇒ $\rho = 998\,kg/m^3$



TASK:

(a) DERIVE AN EXPRESSION FOR THE PRESSURE GRADIENT IN THE NOZZLE

(b) CALCULATE THE PRESSURE GRADIENT AT BOTH ENDS OF THE NOZZLE

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a) THE BERNOULLI EQUATION

$$\frac{p}{\rho} + \frac{1}{2}V^2 + gz = \text{const}$$

MULTIPLY BY DENSITY

$$p + \frac{1}{2}\rho V^2 + \rho g z = \text{const}$$

DIFFERENTIATE IN FLOW DIRECTION

$$\frac{dp}{dx} + \rho V \frac{dV}{dx} + \rho g \frac{dz}{dx} = 0$$

ASSUME MINOR DIFFERENCE IN ELEVATION FROM INLET TO OUTLET

$z_1 \simeq z_2 \Rightarrow \frac{dz}{dx} = 0 \Rightarrow$

$$\frac{dp}{dx} = -\rho V \frac{dV}{dx}$$

WE NEED TO FIND $V(x)$

AND $\frac{dV}{dx}$ ...

INCOMPRESSIBLE FLOW (WATER)

$\Rightarrow \quad Q_1 = Q_2 = Q = \text{const}$

$$V(x) = \frac{Q}{A(x)}$$

$$D(x) = D_1 + \underbrace{\frac{D_2 - D_1}{L}}_{C} x =$$

$$= D_1 + Cx$$

$$A(x) = \frac{\pi}{4} D^2(x) = \frac{\pi}{4}(D_1 + Cx)^2$$

$$V(x) = \frac{Q}{A(x)} = \frac{4Q}{\pi (D_1 + Cx)^2}$$

$$\frac{dV}{dx} = -\frac{8CQ}{\pi (D_1 + Cx)^3}$$

$$\frac{dp}{dx} = -\rho V \frac{dV}{dx} =$$

$$= -\rho \frac{4Q}{\pi (D_1 + Cx)^2} \frac{-8CQ}{\pi (D_1 + Cx)^3}$$

$$= \frac{32 \rho C Q^2}{\pi^2 (D_1 + Cx)^5}$$

So :

$$\boxed{\frac{dp}{dx} = \frac{32 \rho C Q^2}{\pi^2 (D_1 + Cx)^5}}$$

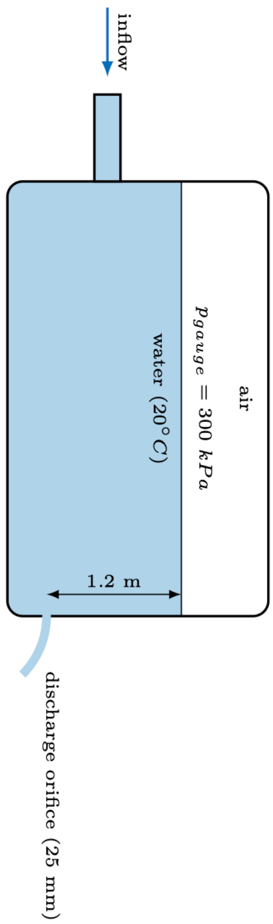
b)

$$\left.\frac{dp}{dx}\right|_{x=0} = \frac{32 \rho C Q^2}{\pi^2 D_1^5} = -16.2 \text{ kPa/m}$$

$$\left.\frac{dp}{dx}\right|_{x=L} = \frac{32 \rho C Q^2}{\pi^2 D_2^5} =$$

$$= -517.7 \text{ kPa/m}$$

P3



GIVEN:

DIMENSIONS ACCORDING TO FIGURE

PRESSURIZED TANK WITH

A GAUGE PRESSURE OF 300 kPa

TANK FILLED WITH WATER @ 20°C

($\rho = 998\ kg/m^3$) TO A CONSTANT

LEVEL AT 1.2 m ABOVE THE

DISCHARGE ORIFICE

DISCHARGE COEFFICIENT OF

THE ORIFICE IS 0.94

TASK:

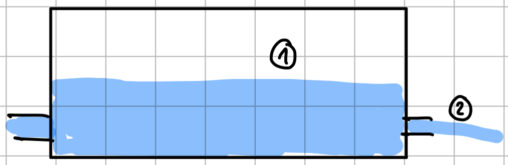
CALCULATE THE INFLOW FLOW-RATE NEEDED TO KEEP THE WATER LEVEL CONSTANT

A CONSTANT WATER LEVEL MEANS THAT THE INCOMING FLOW RATE MUST EQUAL THE DISCHARGE FLOW RATE

(WATER IS CONSIDERED TO BE AN INCOMPRESSIBLE FLUID)

FIRST CALCULATE THE IDEAL FLOW RATE

THE OUTLET VELOCITY IS CALCULATED USING THE BERNOULLI EQUATION



$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2$$

$$V_1 = 0$$

$$(P_1 - P_2) + \rho g (z_1 - z_2) = \frac{1}{2}\rho V_2^2$$

$$\Rightarrow \quad V_2 = \sqrt{2g\left(\frac{P_1 - P_2}{\rho g} + z_1 - z_2\right)}$$

$$P_1 - P_2 = P_{gauge}$$

$$z_1 - z_2 = h$$

$$V_2 = \sqrt{2g\left(\frac{P_{gauge}}{\rho g} + h\right)} = 25 \, m/s$$

$$Q_{IDEAL} = A_2 V_2$$

$$Q_{REAL} = C_D A_2 V_2$$

$$Q_{IN} = Q_{REAL}$$

$$Q_{IN} = 0{,}94 \, \frac{\pi}{4} D_2^2 \sqrt{2g\left(\frac{P_{gauge}}{\rho g} + h\right)}$$

$$= 11.5 \, L/s$$

P4

GIVEN:

WATER @ 20°C $\Rightarrow \rho = 998 \, kg/m^3$

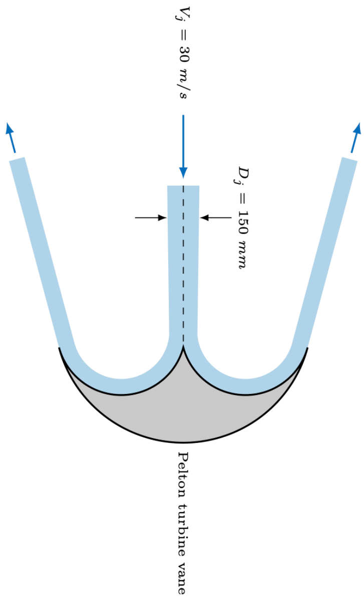
$V_j = 30 \, m/s$

$D_j = 150 \, mm$

WATER DEFLECTION ANGLE $= 165°$

TASK:

CALCULATE THE FORCE EXERTED BY THE WATER ON THE VANE.



$$A_j = \frac{\pi D_j^2}{4}$$

$$\dot{m} = \rho V_j A_j = 529.1 \text{ kg/s}$$

CONSERVATION OF LINEAR MOMENTUM $\Rightarrow$

$$F = \frac{d}{dt}\left(\int_{CV} V \rho \, dV\right) + \int_{CS} V\rho \,(V_r \cdot n)\, dA$$

STEADY-STATE

ONE INLET AND TWO OUTLETS

$$F_x = 2\left(\frac{\dot{m}}{2}\right) V_j \cos\theta - \dot{m} V_j$$

$$\Rightarrow F_x = \dot{m} V_j (\cos\theta - 1)$$

$$= -31.2 \text{ kN}$$

NOTE: AS WE HAVE NOT BEEN GIVEN ANY INFORMATION ABOUT WHEEL ROTATION, IT IS ASSUMED THAT THE VANE IS NOT MOVING.

P5

GIVEN:

WATER @ 20°C ($\rho = 998 \text{ kg/m}^3$) FLOWS AT A FLOW RATE OF $Q = 150 \text{ m}^3/\text{s}$ THROUGH A CAST IRON PIPE WITH A DIAMETER OF 300 mm. THE FOLLOWING EMPIRICAL FORMULA DESCRIBES THE VELOCITY DISTRIBUTION

$$u(r) = V_{av}\left[(1 + 1.326\sqrt{f}) - 2.04\sqrt{f}\log_{10}\left(\frac{R}{R-r}\right)\right]$$

ONE CAN ALSO USE THE FOLLOWING EXPRESSION

$$u(r) = V_0\left(1 - \frac{r}{R}\right)^{1/n}$$

WHERE V_0 IS THE CENTERLINE VELOCITY.

TASK:

FIND n SUCH THAT BOTH

EXPRESSIONS GIVE THE SAME

V_0 AND V_{av}

WE CAN CALCULATE V_0 USING

THE FIRST EXPRESSION

$$V_0 = U(0) = V_{av}(1 + 1.326\sqrt{f})$$

TO BE ABLE TO CONTINUE WE

MUST HAVE A VALUE FOR THE

FRICTION FACTOR (f)

CAST IRON $\Rightarrow \varepsilon \approx 0.26$ mm

$V_{av} = Q/A = 2.1$ m/s

$$Re_D = \frac{\rho V_{av} D}{\mu} = 6.35 \cdot 10^5$$

COLEBROOK (OR MOODY) $\Rightarrow$

$f = 0.019$

CALCULATE THE AVERAGE VELOCITY

USING THE SECOND EXPRESSION

$$V_{av} = \frac{1}{\pi R^2} \int_0^R v(r)\, 2\pi r\, dr$$

$$V_{av} = \frac{V_0}{\pi R^2} \int_0^R \left(1 - \frac{r}{R}\right)^{1/n} 2\pi r\, dr$$

$$= \frac{2V_0}{R^2} \int_0^R \left(1 - \frac{r}{R}\right)^{1/n} r\, dr$$

$$= \frac{2V_0}{R^2} \left[\frac{n(r-R)\left(1-\frac{r}{R}\right)^{1/n}(n(R+r)+r)}{(n+1)(2n+1)} \right]_0^R$$

$$= 2V_0 \left(\frac{n^2}{(n+1)(2n+1)} \right)$$

FROM BEFORE: $V_0 = (1 + 1.326\sqrt{f})V_{av}$

$$V_{av} = 2(1 + 1.326\sqrt{f})V_{av}\left(\frac{n^2}{(n+1)(2n+1)}\right)$$

$$\Rightarrow \frac{n^2}{(n+1)(2n+1)} = \underbrace{\frac{1}{2(1+1.326\sqrt{f})}}_{C}$$

$$n^2 = C(n+1)(2n+1)$$

$$n^2 = C(2n^2 + 3n + 1)$$

$$(2C - 1)n^2 + 3Cn + C = 0$$

$$\Rightarrow n \approx 8.42$$

P6

GIVEN:

AIR @ 10°C FLOWS AT 18km/h OVER A FLAT SURFACE THAT IS 1m LONG AND 2m WIDE.

TASK:

ⓐ CALCULATE THE SHEAR STRESS AT THE DOWNSTREAM END

ⓑ CALCULATE THE AVERAGE SHEAR STRESS

ⓒ CALCULATE THE TOTAL DRAG.

$$U_\infty = 18\ km/h = 5.0\ m/s$$

$$Re_L = \frac{\rho U_\infty L}{\mu}$$

$$\rho = 1.246\ kg/m^3$$
$$\mu = 0.0177 \cdot 10^{-3}$$

VALUES FOR 20°C WILL NOT CHANGE MUCH FOR 10°C

$$Re_L \approx 3.52 \cdot 10^5$$

THIS IS ASSUMED TO BE IN THE LAMINAR FLOW RANGE ⇒ LAMINAR FLOW ALL THE WAY TO THE DOWNSTREAM END. ⇒

a)

$$\tau_w = 0.332 \frac{\rho U_\infty^2}{Re_x^{1/2}} \Rightarrow$$

$$\tau_w(L) = 0.332 \frac{\rho U_\infty^2}{Re_L^{1/2}} = 17.4\ mPa$$

b)

$$\overline{\tau}_w = \frac{b \int_0^L \tau_w\, dx}{Lb} = C_D \frac{1}{2} \rho U_\infty^2 =$$

$$= \frac{1.328}{Re_L^{1/2}} \frac{1}{2} \rho U_\infty^2 = 34.9\ mPa$$

c)

$$F_{drag} = \overline{\tau}_w L b = 69.7\ mN$$