MTF053 - Fluid Mechanics

2024-01-05 08.30 – 13.30

Approved aids:

- The formula sheet handed out with the exam (attached as an appendix)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Any calculator with cleared memory

Exam Outline:

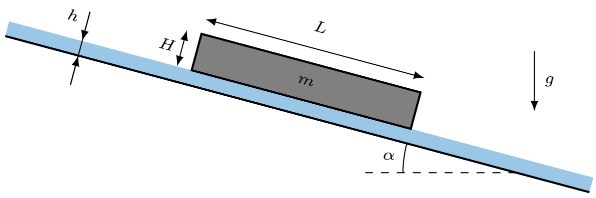
- In total 6 problems each worth 10p

Grading:

number of points on exam (including bonus points)	24-35	36-47	48-60
grade	3	4	5

Problem 1 - Going Down a Slippery Slope (10 p.)

A mass ($m = 5.0\ kg$) in the form of a rectangular block with the length $L = 3.0\ m$, width $B = 1.0\ m$ and height $H = 0.5\ m$ is sliding down an inclined plane ($\alpha = 15.0°$). There is an oil film with the thickness $h = 2.0\ mm$ between the rectangular block and the inclined plane. The oil film is built up from SAEW30 oil at $20.0°C$. The rectangular block can be assumed to move at constant velocity down the plane and the flow in the oil film is laminar.



(a) Find an expression describing the velocity distribution in the oil film (5.0 p.)
hint: Setup your coordinate system such that the x-direction is in the flow direction and the y-direction is aligned with the normal direction of the inclined plane. Don't forget that gravity affects the flow.

(b) Using the data provided above, calculate the velocity of the rectangular block. (2.0 p.)

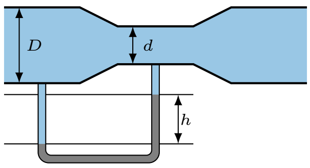
(c) If gravity is neglected in the derivation of the velocity distribution in the oil film, a simpler expression is obtained. Will the difference in the calculated velocity of the rectangular block be significant if doing so? (2.0 p.)

Theory questions related to the topic:

(d) Under what circumstances can the general formulation of the momentum equation be reduced to the Navier-Stokes equation? (1.0 p.)

Problem 2 - Venturimeter (10 p.)

A Venturimeter (a pipe with a contraction and a manometer) setup according to the illustration below is to be used to measure the flow velocity in a pipe. The fluid flowing through the pipe is air at $20.0°C$. The diameter of the pipe is $D = 20.0\ cm$ and in the contraction the tube diameter is reduced to $d = 10.0\ cm$.



(a) Find an expression for calculation of the flow velocity in the pipe as a function of the manometer reading h, densities of the involved fluids, and pipe diameters. (5.0 p.)

(b) Due to space constraints, the manometer reading h is limited to $10.0\ mm$. One would like to be able to measure pipe flow velocities up to $12.0\ m/s$. Find a suitable manometer fluid that fulfills the requirements. (3.0 p.)

Theory questions related to the topic:

(c) Show how the volume flow Q and mass flow $\dot{m}$ over a control volume surface can be calculated in a general way (1.0 p.)

(d) How can we simplify the continuity equation on integral form under the following circumstances (assuming that the control volume is fixed)? (1.0 p.)

$$\int_{cv} \frac{\partial \rho}{\partial t} d\mathcal{V} + \int_{cs} \rho \left(\mathbf{V} \cdot \mathbf{n}\right) dA \tag{0.21}$$

i. inlets and outlets can be assumed to be one-dimensional

ii. steady-state flow

iii. incompressible unsteady flow

Problem 3 - Pipe Design (10 p.)

As part of a construction water @ $20.0°C$ is to be transported through a $20.0\ m$ horizontal pipe at a flow rate of $1400.0\ L/min$. In a first design attempt, a cast iron pipe with an inner diameter of $15.0\ cm$ is used for this part of the construction.

(a) Calculate the pressure drop through the pipe due to friction (4.0 p.)

(b) For the next design iteration, the aim is to reduce the pressure loss of the $20.0\ m$ pipe section by 50%. Would it be possible to achieve the 50%-reduction by increasing the pipe diameter and reduce the surface roughness if the inner diameter of the pipe due to constrains set by surrounding devices is limited to $17.0\ cm$ and if the budget allow for the addition of a polishing operation that reduces wall roughness of the inner surface of the pipe to 80% of the nominal surface roughness given by tabulated data? (4.0 p.)

Theory questions related to the topic:

(c) What does critical Reynolds number mean for a pipe flow? (0.5 p.)

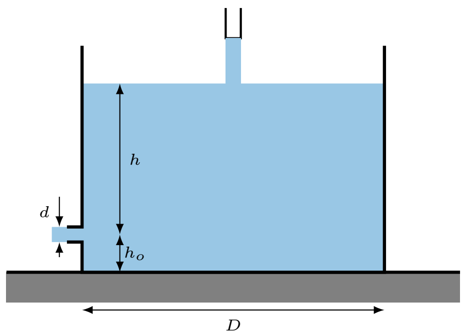
(d) What do we mean when we say that a pipe flow is *fully developed*? (0.5 p.)

(e) Give three examples of sources of local losses in a pipe system (1.0 p.)

Problem 4 - Water Tank (10 p.)

A cylindrical tank according to the illustration below is filled with water. The tank diameter is $D = 1.0\ m$ and the weight of the tank when empty is $W = 150.0\ N$. Water is tapped out from the tank through an outlet with the diameter $d = 9.0\ cm$ located at a position $h_o = 30.0\ cm$ above the bottom surface of the tank. The tank is filled continuously from above such that the water level remains constant at h above the outlet. The friction coefficient between the bottom of the tank and the surface on which the tank stands is $\mu_{friction} = 0.01$

Note!! Here μ is <u>not</u> a viscosity



(a) Calculate the water level h for which the tank will start to move (slide along the surface on which it stands) (5.0 p.)

For the water level calculated above

(b) Calculate the velocity of the water leaving the tank through the outlet (1.0 p.)

(c) Calculate the flow rate at which water has to be added to the tank from above in order to keep the water level constant (1.0 p.)

Theory questions related to the topic:

(d) Show that the normal of a constant-pressure surface must be aligned with the gravity vector in a fluid at rest. (1.0 p.)

(e) Derive the Bernoulli equation for steady-state, incompressible flow along a streamline (2.0 p.)

$$\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho} + \frac{1}{2}V_2^2 + gz_2 = const$$

PROBLEM 5 - BOUNDARY-LAYER FLOW (10 P.)

As a fluid flows past a flat plate, a boundary layer is developed where the flow velocity is gradually decelerated from the freestream velocity (U) a bit away from the flat plate to zero at the surface (if the flat plate is stationary). A very simple approximation of the velocity distribution $u(y)$ from the surface of the flat plate ($y = 0$) to the outer part of the boundary layer ($y = \delta$) is given by the linear relation below.

$$u(y) = U\frac{y}{\delta} \quad \text{for } y \text{ in the range } 0 \leq y \leq \delta$$

$$u = U \quad \text{for } y > \delta$$

Using the provided velocity profile:

(a) Derive a relation describing the non-dimensional boundary-layer thickness δ/x as a function of local Reynolds number Re_x (6.0 p.)
hint: the wall shear stress can be expressed in two ways

(b) Derive an expression for the overall drag coefficient C_D for a flat surface with the length L (2.0 p.)

Theory questions related to the topic:

(c) For laminar flow over a flat plate, the velocity profile is self-similar - what does that mean? (0.5 p.)

(d) Show how the velocity profile as well as its first and second derivative and the pressure gradient change in a boundary layer when the flow separates (1.0 p.)

(e) Why do dimpled golf balls have lower pressure drag than golf balls with smooth surfaces? (0.5 p.)

Problem 6 - Supersonic Santa (10 p.)

In order to be able to deliver packages to all children, Santa and his reindeers have to move at supersonic speed. Assume that a speed corresponding to $M = 4.0$ would be enough (although it is probably not even close) and that we can treat the gas as being calorically perfect (although, at the specified Mach number, we would be in the hypersonic flow regime).

(a) Calculate the temperature and pressure at the nose of the leading reindeer (4.0 p.)

(b) To make the working conditions for the reindeers somewhat better, Santa's helpers builds a 30.0° cone to be placed in front of the first reindeer that leads to the formation of an oblique shock. Calculate the pressure and temperature downstream of this shock (assume that 2D relations can be used) (3.0 p.)

Theory questions related to the topic:

(c) Which of the properties h_o, T_o, a_o, p_o, and ρ_o are constants in a flow if the flow is adiabatic and isentropic, respectively? (1.0 p.)

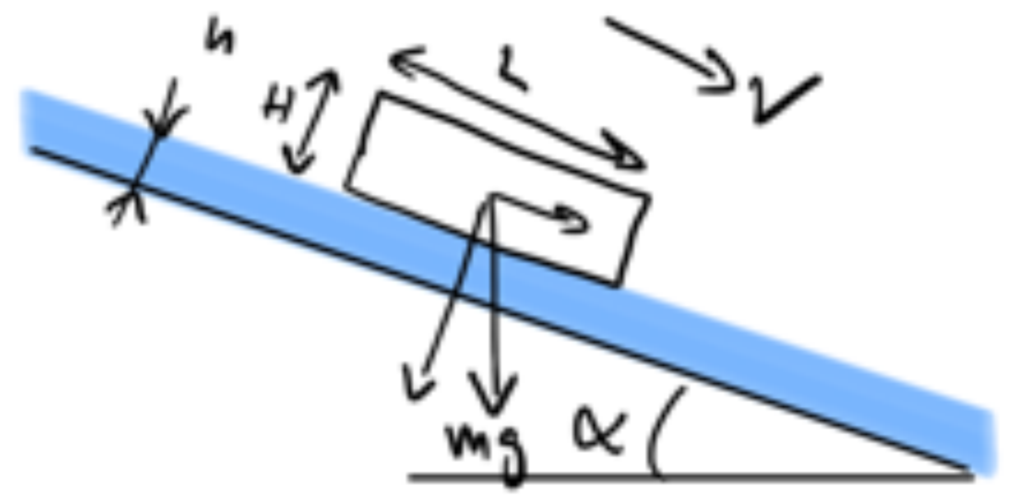
(d) Show schematically how the oblique shock formed ahead of a wedge traveling at supersonic speed if (1.0 p.)

   i. $\theta < \theta_{max}$

   ii. $\theta > \theta_{max}$

(e) Can the relations for total pressure and total temperature derived for a normal shock be used for an oblique shock? Explain why/why not. (1.0 p.)

P_1



A mass ($m = 5.0\,kg$) moves at constant velocity down a plane. There is an oil film (height h) between the block and the plane.

Given:

Block dimensions: $L = 3.0\,m$, $B = 1.0\,m$, $H = 0.5\,m$

Block mass: $5.0\,kg$

Oil film: SAE W50 @ 20°C
thickness $h = 2.0\,mm$

Inclined plane: $\alpha = 15.0°$

a) Find an expression describing the velocity distribution in the oil film.

A coordinate system aligned with the flow is used:

Continuity:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

incompressible $\Rightarrow$ $\rho = \text{const}$ $\Rightarrow$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$v = w = 0$ (only flow in x-direction) $\Rightarrow$

$$\boxed{\frac{\partial u}{\partial x} = 0}$$

Momentum (x-direction)

$$\rho\left(\underbrace{\frac{\partial u}{\partial t}}_{\substack{=0 \\ \text{steady state}}} + u\underbrace{\frac{\partial u}{\partial x}}_{=0} + \underbrace{v}_{=0}\frac{\partial u}{\partial y} + \underbrace{w}_{=0}\frac{\partial u}{\partial z}\right) =$$

$$= -\underbrace{\frac{\partial p}{\partial x}}_{=0} + \rho g_x + \mu\left(\underbrace{\frac{\partial^2 u}{\partial x^2}}_{=0} + \frac{\partial^2 u}{\partial y^2} + \underbrace{\frac{\partial^2 u}{\partial z^2}}_{=0}\right)$$

$$\Rightarrow \left.\begin{aligned} \frac{\partial^2 u}{\partial y^2} &= -\frac{1}{\mu}\rho g_x \\ g_x &= g \sin\alpha \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \boxed{\frac{\partial^2 u}{\partial y^2} = -\frac{\rho g}{\mu}\sin\alpha}$$

Integrate:

$$\frac{\partial u}{\partial y} = -\frac{\rho g}{\mu}\sin\alpha\, y + C_1$$

$$u(y) = -\frac{\rho g}{2\mu}\sin\alpha\, y^2 + C_1 y + C_2$$

Apply boundary conditions:

$u(0) = 0$ (no slip at wall) $\Rightarrow$

$C_2 = 0$

$u(h) = V$ (no slip at the block surface) $\Rightarrow$

$$C_1 = \frac{V}{h} + \frac{\rho g h}{2\mu}\sin\alpha$$

$$\boxed{\begin{aligned} u(y) &= V\frac{y}{h} + \frac{\rho g}{2\mu}\sin\alpha\,(yh - y^2) \\ \frac{\partial u}{\partial y} &= \frac{V}{h} + \frac{\rho g}{2\mu}\sin\alpha\,(h - 2y) \end{aligned}}$$

b) Calculate the velocity of the block:

To get the velocity we will set up a force balance for the block.

Since the block moves at a constant velocity, the net force must be zero..

$$\sum F_x = 0 \Rightarrow mg\sin\alpha = \tau_w B \cdot L$$

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=h} = \frac{\mu V}{h} + \frac{\rho g \sin\alpha}{2}(h - 2h)$$

$$\tau_w = \frac{\mu V}{h} - \frac{\rho g \sin\alpha}{2} h$$

$$\Rightarrow \boxed{mg\sin\alpha = \left(\frac{\mu V}{h} - \frac{\rho g \sin\alpha}{2} h\right) BL}$$

Since we have numerical values for all quantities, we can calculate V.

$\Rightarrow V = 0.045 \text{ m/s}$

c.) If gravity is neglected, the x-component of the momentum eqn reduces to:

$$\frac{\partial^2 u}{\partial y^2} = 0 \Rightarrow$$

$$\frac{\partial u}{\partial y} = C_1 \quad ; \quad u(y) = C_1 y + C_2$$

The same boundary conditions as before $\Rightarrow$

$u(0) = 0 \Rightarrow C_2 = 0$

$u(h) = V \Rightarrow C_1 = \frac{V}{h}$

$$\boxed{u(y) = \frac{V}{h} y \qquad \frac{\partial u}{\partial y} = \frac{V}{h}}$$

Force balance in x-direction $\Rightarrow$

$mg \sin\alpha = \tau_w BL$

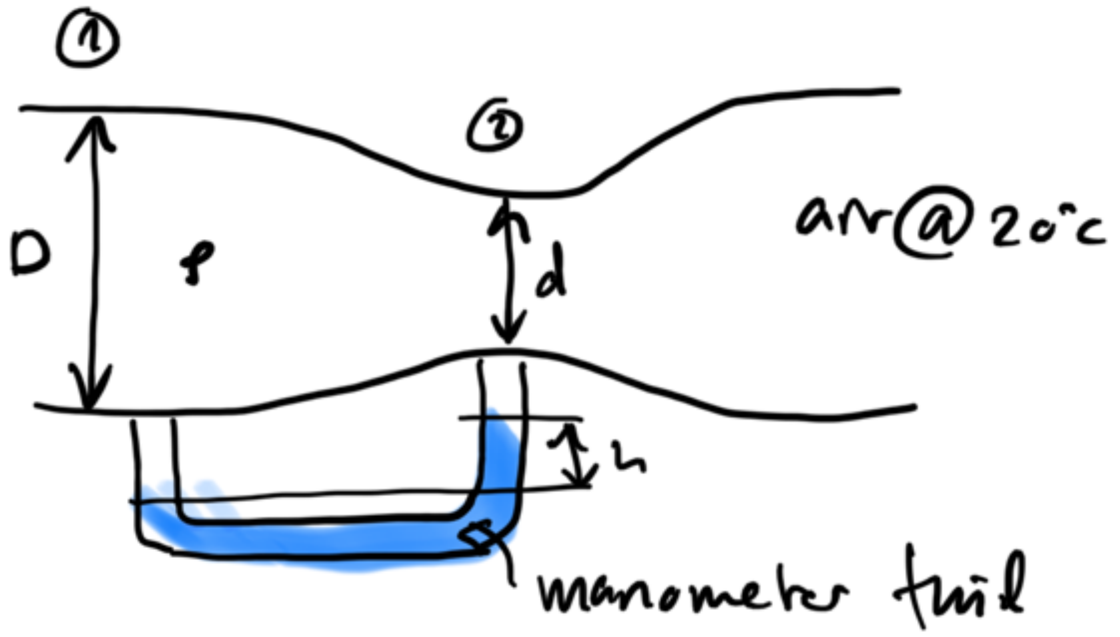
$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=h} = \mu \frac{V}{h}$

$$\Rightarrow \boxed{V = \frac{h}{\mu BL} mg \sin\alpha}$$

$\Rightarrow V = 0.029 \text{ m/s}$

(error : 37.8%)

P2



$D = 20\,cm$
$d = 10\,cm$

a) Find an expression for calculation of the flow velocity as a function of h (the manometer reading)

Bernoulli with $z_1 = z_2$

$$P_1 + \frac{1}{2}\rho u_1^2 = P_2 + \frac{1}{2}\rho u_2^2$$

continuity: $\rho u_1 \frac{\pi D^2}{4} = \rho u_2 \frac{\pi d^2}{4}$

$$\Rightarrow u_2 = u_1 \left(\frac{D}{d}\right)^2$$

We can find a relation between P_1 and P_2 using the manometer.

$$P_1 + \rho g h - \rho_{man} g h = P_2$$

$$(P_1 - P_2) = (\rho_{man} - \rho) g h \qquad (1)$$

Bernoulli gives:

$$P_1 - P_2 = \frac{1}{2}\rho (u_2^2 - u_1^2)$$

inserting continuity $\Rightarrow$

$$P_1 - P_2 = \frac{1}{2}\rho u_1^2 \left(\left(\frac{D}{d}\right)^4 - 1\right) \qquad (2)$$

(1) in (2) $\Rightarrow$

$$(\rho_{oil} - \rho) g h = \frac{1}{2}\rho u_1^2 \left(\left(\frac{D}{d}\right)^4 - 1\right)$$

$$\Rightarrow \boxed{u_1 = \sqrt{\frac{\left(\frac{\rho_{man}}{\rho} - 1\right) 2gh}{\left((D/d)^4 - 1\right)}}}$$

b) h is restricted to 10.0 mm and velocities should be able to get up to 12.0 m/s. Find a suitable manometer fluid.

$$\left(\frac{\rho_{man}}{\rho} - 1 \right) = \frac{1}{2gh} \left(\left(D/d \right)^4 - 1 \right) u_1^2$$

$\Rightarrow \rho_{man} > 1102 \ kg/m^3$

$\Rightarrow$ Mercury would be a good alternative.

P3 Water is transported through a 20.0 m horizontal pipe at a flow rate of $Q = 1400.0$ L/min

First design attempt:
cast iron pipe with an inner diameter of 15.0 cm

a) calculate the pressure drop due to friction:

to get a relation between the pressure drop and friction we can for example use (3.75)

$$\left(\frac{p}{\rho g} + \frac{\alpha V_{av}^2}{2g} + z\right)_1 = \left(\frac{p}{\rho g} + \frac{\alpha V_{av}^2}{2g} + z\right)_2 +$$

$$+ \cancel{h_{turbine}} - \cancel{h_{pump}} + h_{friction}$$

fully-developed flow $\Rightarrow \alpha = \text{const}$
$V_{av} = \text{const}$.

horizontal pipe $\Rightarrow z_1 = z_2$

$$\Rightarrow \frac{P_1 - P_2}{\rho g} = h_{friction}$$

so we need to calculate h_f

$$(6.10) \Rightarrow h_f = f \frac{L}{d} \frac{V^2}{2g}$$

Where V is the average velocity:

$$V = Q \Big/ \left(\frac{\pi D^2}{4}\right) = 1.3 \text{ m/s}$$

To calculate the friction factor (f) we need to check if the flow is laminar or turbulent.

$$Re_D = \frac{\rho V D}{\mu} \sim 2.0 \cdot 10^5$$

$\Rightarrow$ turbulent flow.

For turbulent flow, we can use eqn 6.48 (or the Moody chart)

Eqn 6.48 : (cast iron => $\varepsilon/d = 1.73 \cdot 10^{-3}$)
$\varepsilon = 0.26$ mm

$$\frac{1}{\sqrt{f}} = -2.0 \log_{10}\left(\frac{\varepsilon/d}{3.7} + \frac{2.51}{Re_D \sqrt{f}}\right)$$

Solve for f => $f \simeq 2.55 \cdot 10^{-2}$

=> $h_f = f \frac{L}{D} \frac{V^2}{2g} = 0.3$ m

and the pressure loss

$$\Delta p = \rho g h_f = 2.7 \text{ kPa}$$

b) Design attempt 2:

reduced pressure loss by 50%

roughness reduced by 20%

pipe diameter increased to $\leq$ 17cm

$\varepsilon = 0.8 \cdot 0.26$ mm

$D = 17$cm

=> $V = 1.0$ m/s

$Re_D = 1.7 \cdot 10^5$ (turbulent)

(6.48) => $f = 2.2 \cdot 10^{-2}$

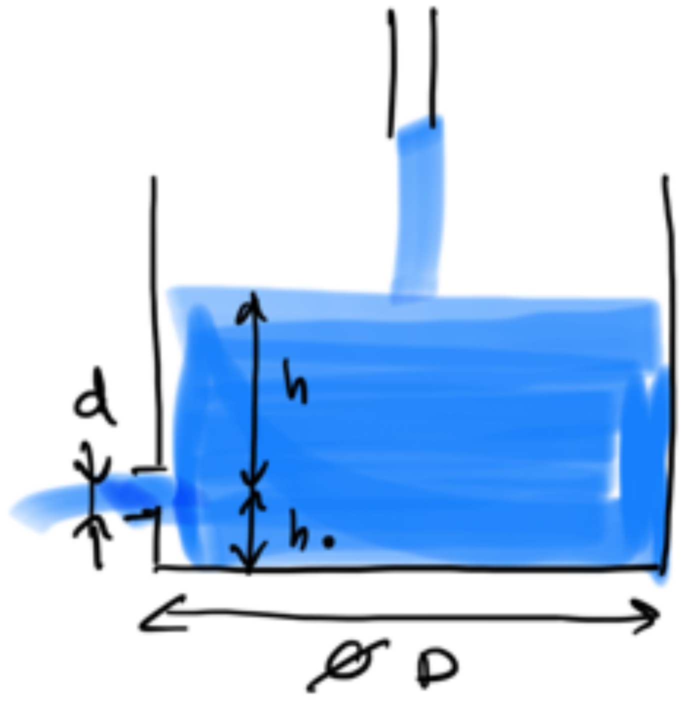
$h_f = 0.1$ m

=> $\Delta p = 1.4$

which is ~ 50% of the

pressure loss of design 1

P4



$D = 1.0\ m$

W_{tank} (empty) $= 150.0\ N$

$d = 9.0\ cm$

$h_0 = 30.0\ cm$

Steady state : filled continuously such that the water level is kept constant in the tank.

Friction coeff. $\mu = 0.01$

a) For which water level (h) will the tank start to move?

We need to calculate the velocity with which fluid is flowing out from the tank.

Bernoulli between the surface (1) and the outlet (2) $\Rightarrow$

$$P_1 + \frac{1}{2}\rho u_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho u_2^2 + \rho g z_2$$

$$P_1 = P_2$$

$$z_1 - z_2 = h$$

$$\Rightarrow \frac{1}{2}\rho u_1^2 + \rho g h = \frac{1}{2}\rho u_2^2$$

Continuity $\Rightarrow$

$$u_1 A_1 = u_2 A_2$$

$$\Rightarrow u_1 = u_2\left(\frac{d}{D}\right)^2$$

$$\Rightarrow \frac{1}{2}\rho u_2^2\left(\frac{d}{D}\right)^4 + h = \frac{1}{2}\rho u_2^2$$

$$u_2 = \sqrt{2gh/\left(1-\left(\frac{d}{D}\right)^4\right)}$$

$$D \gg d \Rightarrow u_2 \approx \sqrt{2gh}$$

With the velocity calculated, we can go on to look at conservation of linear momentum.

$$\sum F = \frac{d}{dt}\left(\int_{cv} V\rho\, dV\right) + \sum_i (\dot m_i V_i)_{out} - \sum_i (\dot m_i V_i)_{in}$$

$$\dot{m}_{out} = \rho u_2 \frac{\pi D_2^2}{4}$$

We are only interested in the x-component of the momentum eqn

$$F_x = \mu F_N = \mu \left(W + \frac{\pi D^2}{4}(h + h_0) \rho g \right)$$

(friction force)

momentum eqn ->

$$\mu \left(W + \frac{\pi D^2}{4}(h + h_0) \rho g \right) = u_2^2 \frac{\pi d^2}{4} \rho$$

$$= 2gh \frac{\pi d^2}{4} \rho$$

$$\Rightarrow \boxed{h = \frac{\mu \left(W + \frac{\pi D^2}{4} h_0 \rho g \right)}{\rho g \frac{\pi}{4} \left(2d^2 - \mu D^2 \right)}}$$

b) Calculate the outlet flow velocity:

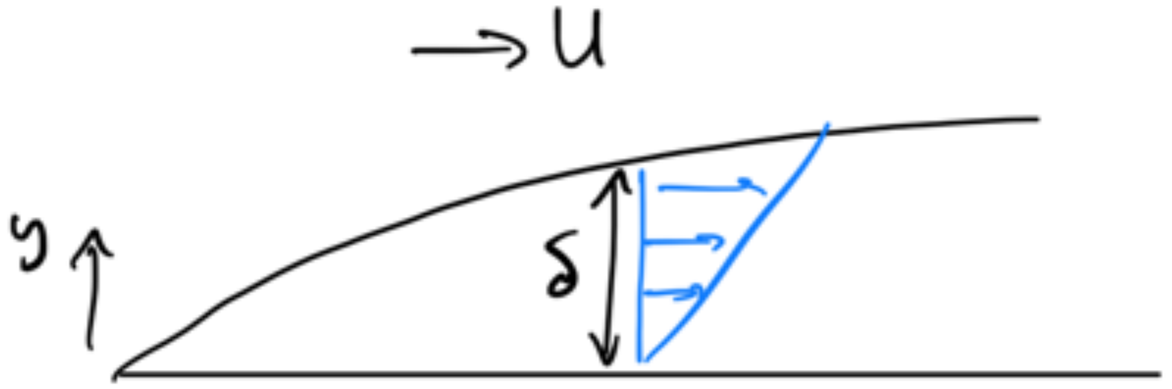
$$u_2 = \sqrt{2gh} = 3.18 \text{ m/s}$$

c) Calculate the flow rate at which fluid must be added to the tank to keep the level constant.

$$Q_{in} = Q_{out} = u_2 A_2 =$$

$$= \sqrt{2gh} \frac{\pi d^2}{4} = 0{,}02 \frac{m^3}{s}$$

(P5)



$$\begin{cases} u(y) = U\frac{y}{\delta} & y \in (0, \delta] \\ u(y) = U & y > \delta \end{cases}$$

a) derive an expression for δ/x as a function of the local Reynoldsnumber Re_x

$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_{y=0} \quad ; \quad \tau_w = \rho U^2 \frac{d\theta}{dx}$$

$$\theta = \int_0^{\delta(x)} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$\Rightarrow \theta = \int_0^{\delta(x)} \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy =$$

$$= \left[\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right]_0^\delta = \left(\frac{\delta}{2} - \frac{\delta}{3}\right) =$$

$$= \frac{\delta}{6}$$

$$\Rightarrow \theta(x) = \frac{\delta(x)}{6} \Rightarrow$$

$$\Rightarrow \frac{d\theta}{dx} = \frac{1}{6}\frac{d\delta(x)}{dx}$$

$$\left.\begin{array}{l} \tau_w = \rho U^2 \frac{d\theta}{dx} = \frac{\rho U^2}{6}\frac{d\delta}{dx} \\ \frac{\partial u}{\partial y}\Big|_{y=0} = \frac{U}{\delta} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \mu \frac{U}{\delta} = \frac{\rho U^2}{6}\frac{d\delta}{dx}$$

$$\Rightarrow \frac{6\mu}{\rho U} dx = \delta \, d\delta$$

Integrate:

$$\frac{6\mu x}{\rho U} = \frac{1}{2}\delta^2 \Rightarrow \delta = \sqrt{12\frac{\mu x}{\rho U}}$$

$$\Rightarrow \frac{\delta}{x} = \frac{2}{x}\sqrt{3\frac{\mu x}{\rho U}} = \frac{2\sqrt{3}}{\sqrt{Re_x}}$$

$$\therefore \boxed{\frac{\delta}{x} = \frac{2\sqrt{3}}{\sqrt{Re_x}}}$$

b) Derive an expression for the overall drag coefficient for a flat surface with length L.

Drag force:

$$F_D = b\int_0^L \tau_w(x)\,dx$$

$$\left.\begin{aligned} \tau_w &= \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu \frac{U}{\delta} \\ \delta(x) &= 2x\sqrt{\frac{3}{Re_x}} \end{aligned}\right\} \Rightarrow$$

$$\tau_w(x) = \mu \frac{U}{2x\sqrt{3/Re_x}}$$

$$\Rightarrow \tau_w(x) = \frac{\mu^{1/2} U^{3/2} \rho^{1/2}}{\sqrt{12}\, x^{1/2}}$$

$$F_D = \frac{b\mu^{1/2}U^{3/2}\rho^{1/2}}{\sqrt{12}}\int_0^L x^{-1/2}\,dx$$

$$= \frac{b\mu^{1/2}U^{3/2}\rho^{1/2}}{\sqrt{12}}\left[2x^{1/2}\right]_0^L =$$

$$= \frac{2\mu^{1/2}U^{3/2}\rho^{1/2}L^{1/2}b}{\sqrt{12}}$$

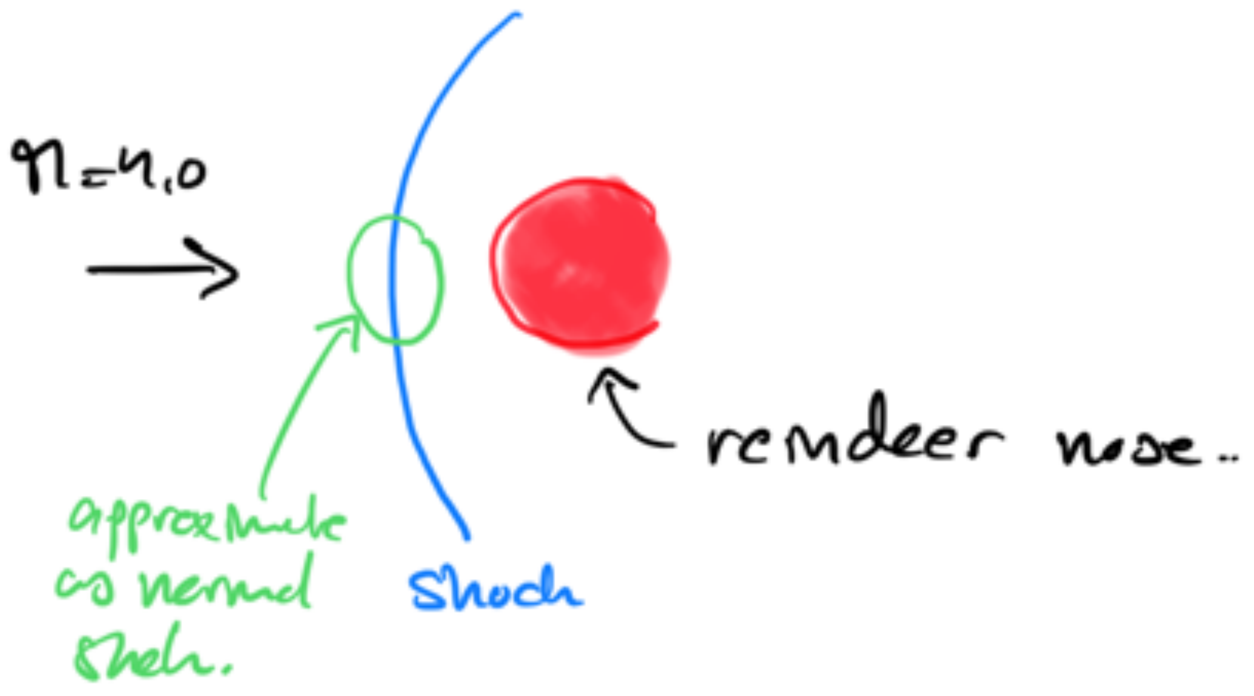
$$C_D = F_D \Big/ \frac{1}{2}\rho U^2 bL$$

$$\Rightarrow \boxed{C_D = \frac{2}{\sqrt{3}\,Re_L^{1/2}}}$$

P6

$M = 4.0$

calorically perfect gas



we will use the normal shock relations for temperature and pressure.

(9.55) $$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)$$

(9.58) $$\frac{T_2}{T_1} = (2+(\gamma-1)M_1^2) \cdot \left(\frac{2\gamma M_1^2 - (\gamma-1)}{(\gamma+1)^2 M_1^2}\right)$$

Behind the shock the subsonic flow is slowed down to zero velocity before reaching the nose $\Rightarrow$ we can calculate the total temperature and pressure.

(9.26) $$\frac{T_{02}}{T_2} = 1 + \frac{\gamma-1}{2} M_2^2$$

(9.28a) $$\frac{p_{02}}{p_2} = \left(\frac{T_{02}}{T_2}\right)^{\gamma/(\gamma-1)}$$

we need the Mach number downstream of the shock (M_2)

(9.57) $$M_2^2 = \frac{(\gamma-1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma-1)}$$

$M_2 = 0.43$

$p_2/p_1 = 18.5$

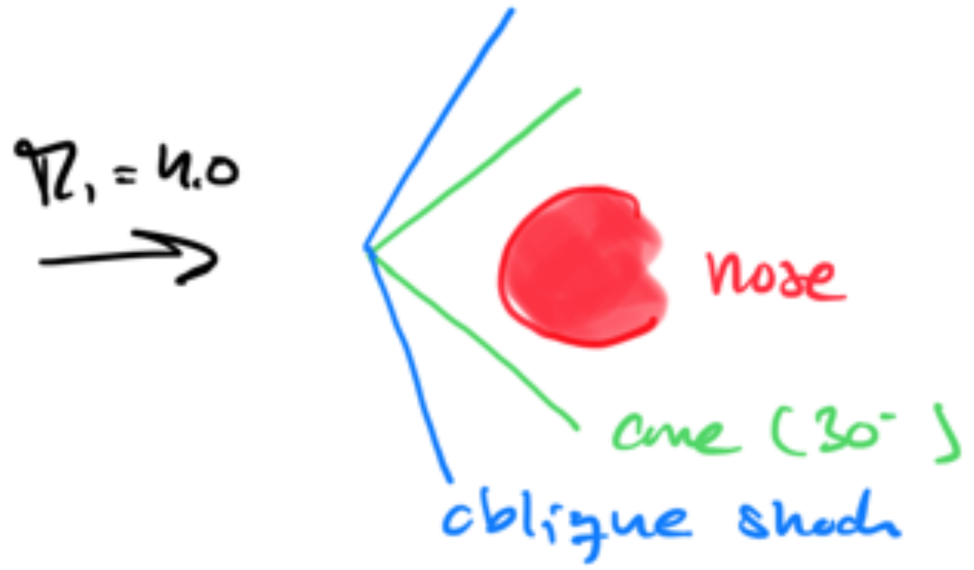
$T_2/T_1 = 4.05$

if the pressure and temperature upstream of the shock are:

$p_1 = 101325$ Pa and $T_1 = 288$ K

$$\Rightarrow \begin{cases} p_2 = 1.87 \text{ MPa} \\ T_2 = 1165 \text{ K} \\ p_{02} = 2.13 \text{ MPa} \\ T_{02} = 1209.6 \text{ K} \end{cases}$$

b) cone in front of nose



flow deflection : $\theta = 15°$

θ-β-M relation ($M = 4.0$; $\theta = 15°$)

$\Rightarrow \beta = 27.06°$

Shock-normal Mach number

$$M_{n_1} = M_1 \sin\beta$$

(9.57) $$M_{n_2} = \frac{(\gamma-1)M_{n_1}^2 + 2}{2\gamma M_{n_1}^2 - (\gamma-1)}$$

$$M_{n_2} = M_2 \sin(\beta - \theta) \Rightarrow$$

$$M_2 = 2.93$$

(9.55)
$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right)$$

$$\Rightarrow \frac{P_2}{P_1} = 3.70$$

(9.58)
$$\frac{T_2}{T_1} = \left(2 + (\gamma-1)M_{n_1}^2\right)\frac{2\gamma M_{n_1}^2 - (\gamma-1)}{(\gamma+1)^2 M_{n_1}^2}$$

$$\Rightarrow \frac{T_2}{T_1} = 1.55$$

Again, assume the upstream conditions to be $P_1 = 101385$ Pa and $T_1 = 288$ K $\Rightarrow$

$$\begin{cases} P_2 = 374.6 \text{ kPa} \\ T_2 = 445.4 \text{ K} \end{cases}$$