MTF053 - Fluid Mechanics

2023-01-05 08.30 – 13.30

Approved aids:

- The formula sheet handed out with the exam (attached as an appendix)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Graph drawing calculator with cleared memory

Exam Outline:

- In total 6 problems each worth 10p

Grading:

number of points on exam (including bonus points)	24-35	36-47	48-60
grade	3	4	5

Problem 1 - Boundary-Layer Flow (10 p.)

As a fluid flows past a flat plate, a boundary layer is developed where the flow velocity is gradually decelerated from the freestream velocity (U) a bit away from the flat plate to zero at the surface (if the flat plate is stationary). A very simple approximation of the velocity distribution $u(y)$ from the surface of the flat plate ($y = 0$) to the outer part of the boundary layer ($y = \delta$) is given by the linear relation below.

$$u(y) = U\frac{y}{\delta} \quad \text{for } y \text{ in the range } 0 \leq y \leq \delta$$

$$u = U \quad \text{for } y > \delta$$

(a) Using the linear velocity distribution given above, determine the wall shear stress $\tau_w(x)$ using the momentum thickness equation

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

(6p)

(b) Compare your result with the shear stress calculated using the Blasius velocity profile. How off is the shear stress calculated using the very simple velocity profile given above? (2p)

(c) What is the wall boundary condition used in this problem called and what does it imply? (0.5p)

(d) What assumptions are made in the derivation of the boundary-layer equations, i.e. the boundary-layer formulation of the Navier-Stokes equations? (0.5p)

(e) For laminar flow over a flat plate, the velocity profile is self-similar - what does that mean? (1p)

Problem 2 - Flow Acceleration and Continuity (10 p.)

Steady-state flow through a converging nozzle can be approximated by a one-dimensional velocity distribution $u = u(x)$ where x is the coordinate direction aligned with the nozzle centerline. Let's assume that the velocity varies linearly from $u = u_o$ at the entrance ($x = 0$) to $u = 3u_o$ at the exit ($x = L$).

(a) Derive an expression for the flow acceleration through the nozzle as a function of x (4p)

(b) What is the flow acceleration at the nozzle entrance (0.5p)

(c) What is the flow acceleration at the nozzle exit (0.5p)

(d) Explain the physical meaning of the local acceleration term and the convective acceleration term (1p)

(e) Derive the continuity equation on differential form starting from the integral form for a fixed control volume

$$\int_{cv} \frac{\partial \rho}{\partial t} d\mathcal{V} + \sum_i (\rho_i A_i V_i)_{out} - \sum_i (\rho_i A_i V_i)_{in} = 0$$

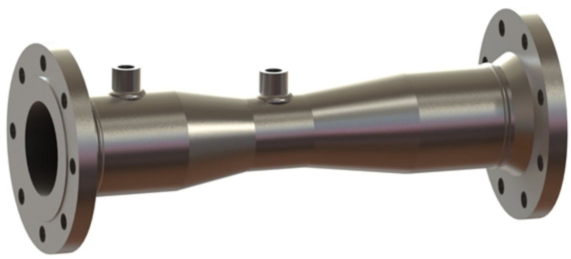
and let the size of the control volume reduce to infinitesimal size (3p)

(f) How can we simplify the continuity equation on differential form under the following circumstances? (1p)

- steady-state flow
- incompressible flow

Problem 3 - Pipe Flows (10 p.)

A venturi meter is a device used for tube flow rate measurements. It is a simple construction where the flow is passing through a tube section with locally reduced cross-section area. The pressure difference between a location upstream of the contraction and the location in the contraction where the tube diameter is the smallest is measured. The measured pressure difference can then be used to calculate the flow rate.



(a) Derive a general expression for the flow rate (Q) through a venturi meter as a function of fluid density (ρ), pressure and cross-section area upstream of the contraction (p_1 and A_1), and pressure and cross-section area in the contraction (p_2 and A_2). (6p.)

(b) For the range of flow rates given in the venturi meter specification below, determine the range of pressure differences that any connected pressure measurement device needs to be able to handle. (2p.)

Venturi meter specification:

Fluid	Kerosene
Flow rate range	$0.005\ m^3/s \leq Q \leq 0.050 m^3/s$
Tube diameter before and after contraction	$0.1\ m$
Tube diameter in the contraction	$0.06\ m$

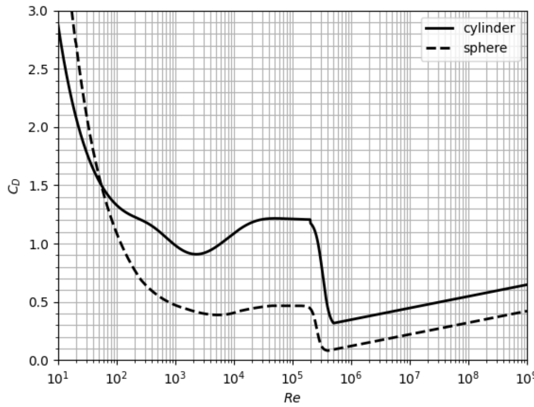
(c) The Moody diagram can be used for estimation of pressure losses in tubes. Why does the Moody chart not give reliable values in the Reynolds number range $2000 < Re < 4000$? (0.5p)

(d) What does fully developed pipe flow mean? (0.5p)

(e) Is the wall shear stress, τ_w, in general higher or lower in a fully developed laminar pipe flow than in a fully developed turbulent pipe flow? Explain why. (1p)

Problem 4 - Water Tower (10 p.)

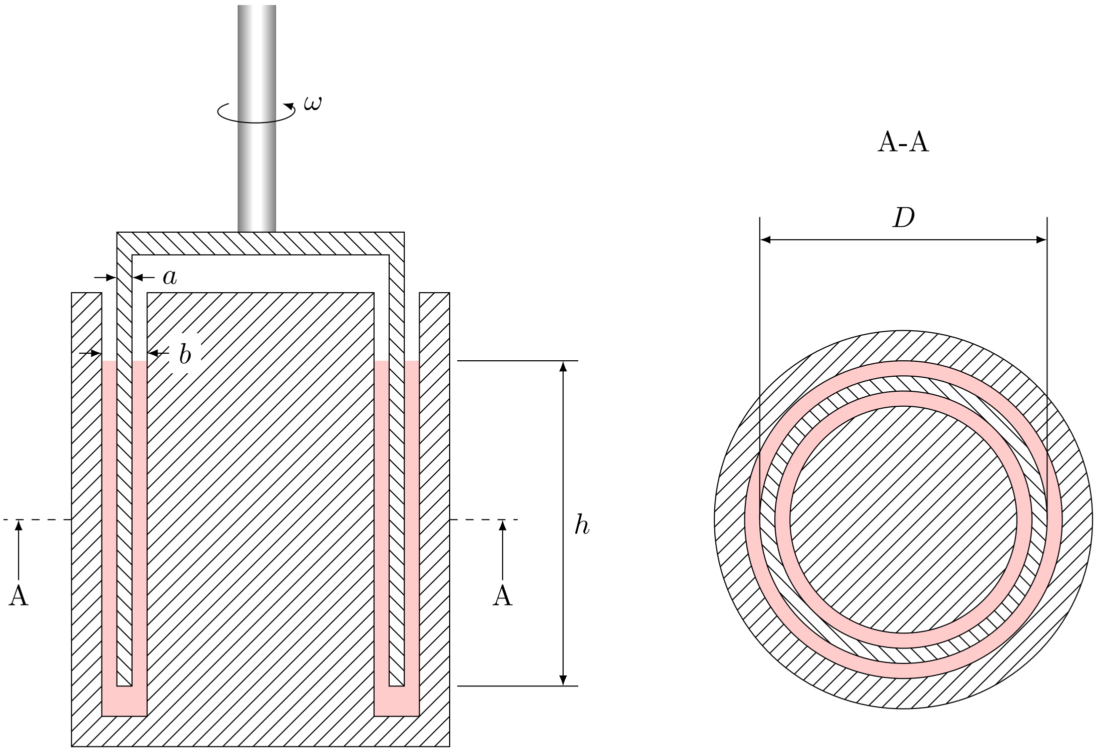
(a) The union water tower (see figure below) can be assumed to be a combination of a spherical water tank (diameter 12.0 m) and a cylinder (diameter 4.5 m, height 30.0m). Estimate the bending moment at the base of the water tower if the wind speed is 27 m/s. The velocity can be assumed to be constant, i.e. you don't have to account for the fact that in a real situation there would be a boundary layer. (6p)



(b) Explain the concepts favorable pressure gradient and adverse pressure gradient. What is the implication of pressure gradients on separation for an external flow (describe differences in separation tendency for flows with favorable pressure gradient, adverse pressure gradient, and no pressure gradient) (4p)

PROBLEM 5 - VISCOSITY (10 P.)

The construction illustrated below is used for measurement of the viscosity of oils. The device consists of a rotating cylinder and a stationary container. The gaps between the container walls and the cylinder are filled with oil and thus there is a thin oil layer on both the inside and outside of the cylinder. The upper part of the cylinder is not in contact with the oil. The material in the rotating cylinder can be assumed to be thin and thus the friction contribution from the end surface can be neglected.



(a) Derive an expression for the moment required to turn the cylinder based on the geometrical dimensions given in the figure above. (6p)

(b) Calculate h in the figure above such that the viscosity meter can be used to measure the viscosity of oils of the type SAE 10W-30 in the temperature range 20 $°C$ - 80 $°C$ if the turning moment must not exceed 1.5×10^{-2} Nm and if all other parameters are defined according to the table below (3p)

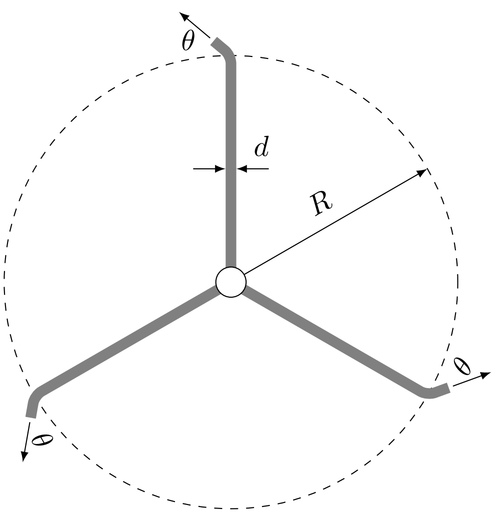
ω	one revolution per second
a	1.5 mm
b	3.5 mm
D	50 mm

(c) What is the viscosity of a fluid? (0.5p)

(d) What does it mean that a fluid is Newtonian? (0.5p)

Problem 6 - Sprinkler (10 p.)

The figure below sows a three-armed lawn sprinkler from above. Water enters the sprinkler from below (normal to the paper) at a flow rate of $Q = 8.0 \times 10^{-4}\ m^3/s$. The radius of the sprinkler is $R = 0.175\ m$ and the inner diameter of the sprinkler arms is $d = 7.0\ mm$. Friction between rotating and stationary parts of the sprinkler can be neglected.



(a) Derive an expression for calculation of the rotational velocity of the sprinkler as function of the angle θ (and other relevant parameters) (6p)

(b) Calculate the rotational velocity if the angle θ is 40° (1p)

(c) For what angle θ will we get the maximum rotation velocity? (1p)

(d) Make a schematic representation of the non-dimensional velocity u^+ as a function of the non-dimensional wall distance y^+ for a turbulent boundary layer. The velocity profile can be divided into different regions. Name these regions. (2p)

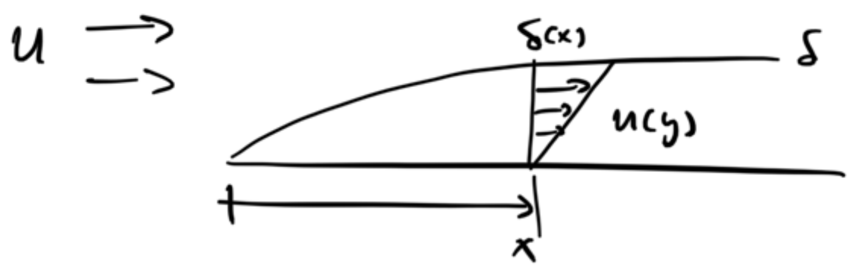
P1

Given :

Velocity profile $u(y) = U\frac{y}{\delta}$

where U is the free stream velocity and δ is the boundary-layer thickness

Note! $\delta = \delta(x)$



$$u(y) = U\frac{y}{\delta} \qquad 0 \le y \le \delta$$
$$u = U \qquad y > \delta$$

Derive $\tau_w(x)$ using the momentum thickness relation given by

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$u(y) = U\frac{y}{\delta} \Rightarrow \theta = \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy$$

$$\theta = \int_0^\delta \frac{y}{\delta} - \frac{y^2}{\delta^2}\, dy = \left[\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right]_0^\delta =$$

$$= \frac{\delta^2}{2\delta} - \frac{\delta^3}{3\delta^2} = \frac{\delta}{2} - \frac{\delta}{3} = \frac{\delta}{6}$$

$$\therefore \boxed{\theta = \frac{\delta}{6}, \quad \delta = \delta(x)}$$

Eqn. 7.5 : $\tau_w = \rho U^2 \frac{d\theta}{dx}$

$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \left\{\frac{\partial u}{\partial y} = \frac{U}{\delta}\right\} = \mu\frac{U}{\delta}$ $\Rightarrow$

$$\Rightarrow \mu\frac{U}{\delta} = \rho U^2 \frac{d}{dx}\left(\frac{\delta(x)}{6}\right)$$

$$\mu\frac{U}{\delta} = \rho U^2 \frac{1}{6}\frac{d\delta}{dx} \Rightarrow \frac{6\mu}{\rho U} dx = \delta\, d\delta$$

$$\frac{1}{2}\delta^2 = \frac{6\mu x}{\rho U}$$

$$\left(\Rightarrow \frac{\delta}{x} = \sqrt{12}\left(\frac{\mu}{\rho U x}\right)^{1/2}\right)$$

$$\tau_w(x) = \frac{\mu U}{\sqrt{12}\, x \left(\frac{\mu}{\rho U x}\right)^{1/2}}$$

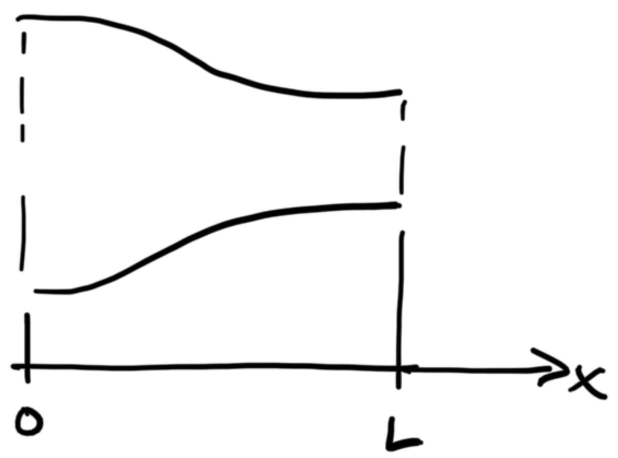
$$\Rightarrow \tau_w(x) = \frac{\mu^{1/2} U^{3/2} \rho^{1/2}}{\sqrt{12}\, x^{1/2}}$$

Blasius laminar profile $\Rightarrow$

$$\tau_{w_B}(x) = \frac{0.332\, \rho^{1/2} \mu^{1/2} U^{3/2}}{x^{1/2}}$$

$$\frac{\tau_w(x)}{\tau_{w_B}(x)} = \frac{(1/\sqrt{12})}{0.332} = 0.87$$

P2



GIVEN: LINEAR VELOCITY DISTRIBUTION

$$\left.\begin{aligned} u(0) &= U_0 \\ u(L) &= 3U_0 \end{aligned}\right\} \Rightarrow u(x) = U_0\left(1 + \frac{2}{L}x\right)$$

GET FLOW ACCELERATION THROUGH THE CONVERGENT NOZZLE.

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial w}{\partial z}$$

STEADY-STATE : $\frac{\partial u}{\partial t} = 0$

ONE-DIMENSIONAL FLOW $\Rightarrow$ $v=0$, $w=0$

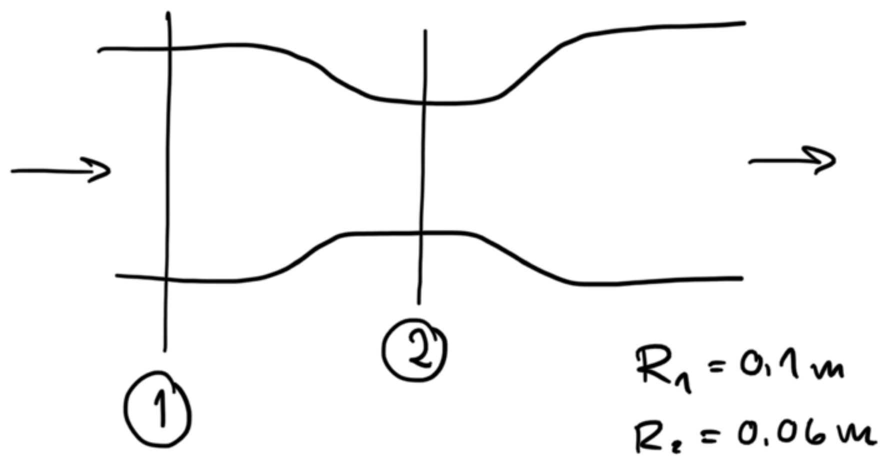
$$\Rightarrow \frac{Du}{Dt} = u\frac{\partial u}{\partial x}$$

$$\frac{\partial u}{\partial x} = \frac{2u_0}{L} \Rightarrow \frac{Du}{Dt} = \frac{2u_0^2}{L}\left(1 + \frac{2}{L}x\right)$$

$$\left.\frac{Du}{Dt}\right|_{x=0} = \frac{2u_0^2}{L}$$

$$\left.\frac{Du}{Dt}\right|_{x=L} = \frac{6u_0^2}{L}$$

P3)



Assume incompressible, steady-state flow.

continuity: $u_1 A_1 = u_2 A_2 = Q \quad (1)$

energy eqn:

$$\frac{P_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{u_2^2}{2g} + z_2 + h_L$$

$$\left.\begin{matrix} z_1 = z_2 \\ h_L \simeq 0. \end{matrix}\right\} \Rightarrow P_1 + \frac{1}{2}\rho u_1^2 = P_2 + \frac{1}{2}\rho u_2^2 \quad (2)$$

(1) in (2) $\Rightarrow$

$$P_1 + \frac{1}{2}\rho \frac{Q^2}{A_1^2} = P_2 + \frac{1}{2}\rho \frac{Q^2}{A_2^2}$$

$$\frac{2}{\rho}(P_1 - P_2) = Q^2\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)$$

$$\frac{2}{\rho}(P_1 - P_2) = Q^2\left(\frac{A_1^2 - A_2^2}{A_1^2 A_2^2}\right)$$

$$Q^2 = \left(\frac{A_1^2 A_2^2}{A_1^2 - A_2^2}\right)\frac{2}{\rho}(P_1 - P_2)$$

$$\boxed{Q = A_1 A_2 \sqrt{\frac{2}{\rho}\,\frac{P_1 - P_2}{A_1^2 - A_2^2}}}$$

$Q_{min} = 0.005\ m^3/s$

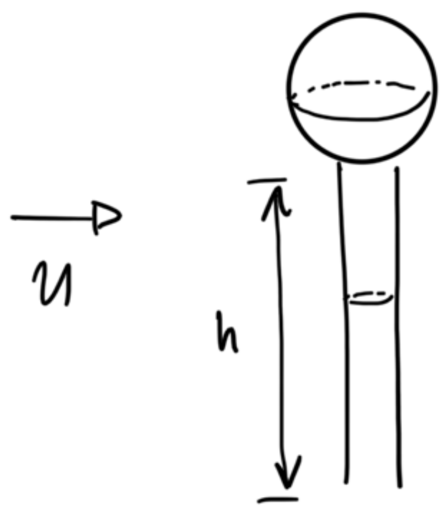
$Q_{max} = 0.050\ m^3/s$

$$Q^2 = \left(\frac{A_1^2 A_2^2}{A_1^2 - A_2^2}\right)\frac{2}{\rho}\Delta p$$

$$\Delta p = \frac{\rho}{2}\left(\frac{A_1^2 - A_2^2}{A_1^2 A_2^2}\right)Q^2 \Rightarrow$$

$$\begin{cases} \Delta P_{min} = 1.11 \cdot 10^3\ Pa \\ \Delta P_{max} = 1.11 \cdot 10^5\ Pa \end{cases}$$

P4



$D_s = 12.0\,m$

$D_c = 4.5\,m$

$h = 30.0\,m$

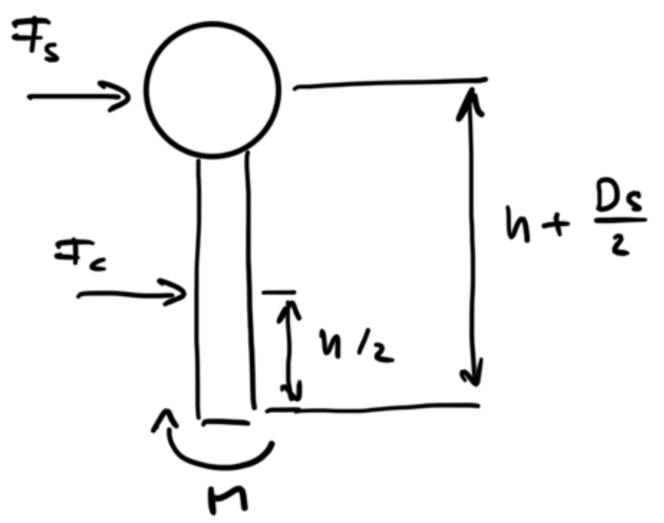
$U = 27.0\,m/s$

air @ 20°C:

$\rho = 1.2\ kg/m^3$

$\mu = 1.81 \cdot 10^{-5}\ kg/m{\cdot}s$

CALCULATE THE BENDING MOMENT AT THE BASE OF THE WATER TOWER.



$$M = F_s \cdot \left(h + \frac{D_s}{2}\right) + F_c \frac{h}{2} \qquad (1)$$

PROJECTED AREA:

CYLINDER: $D_c \cdot h$

SPHERE: $\frac{\pi D_s^2}{4}$

FORCES:

$$F_s = \frac{1}{2} \rho U^2 \frac{\pi D_s^2}{4} C_{D_s} \qquad (2)$$

$$F_c = \frac{1}{2} \rho U^2 h D_c C_{D_c} \qquad (3)$$

DRAG COEFFICIENTS:

$$Re_s = \frac{\rho U D_s}{\mu} \approx 2.15 \cdot 10^7 \Rightarrow$$

$$\Rightarrow C_{D_s} \approx 0.25 \qquad (4)$$

$$Re_c = \frac{\rho U D_c}{\mu} \approx 8.05 \cdot 10^6 \Rightarrow$$

$$\Rightarrow C_{D_c} \approx 0.44 \qquad (5)$$

(4) in (2) $\Rightarrow$

$$F_s \approx 1.26 \cdot 10^4\ N \qquad (6)$$

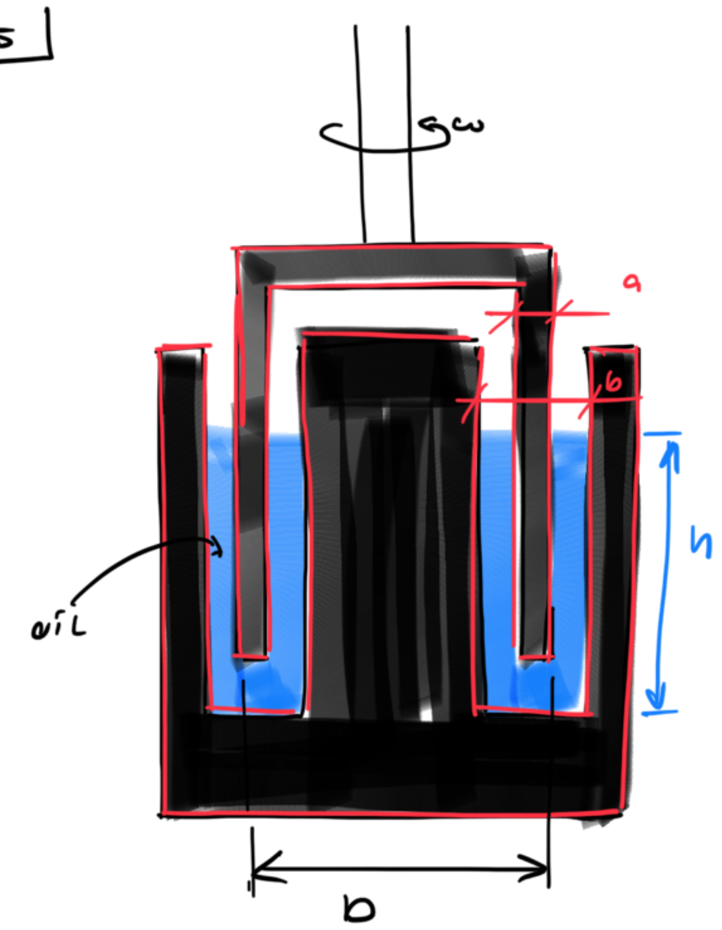
(5) in (3) $\Rightarrow$

$$F_c = 2.58 \cdot 10^4\ N \qquad (7)$$

(6) & (7) in (1) $\Rightarrow$

$$\boxed{M \approx 8.39 \cdot 10^5\ Nm}$$

P5



FIND AN EXPRESSION FOR THE MOMENT REQUIRED TO TURN THE CYLINDER

THE ROTATING CYLINDER IS THIN AND THE CONTRIBUTION FROM THE END SURFACE CAN BE NEGLECTED. THE INNER AND OUTER SURFACES OF THE ROTATING CYLINDER WILL CONTRIBUTE TO THE RESISTING MOMENT.

OUTSIDE:

$$\tau_o = \mu \frac{\partial u}{\partial y} = \mu \frac{\omega D}{2} \frac{2}{b-a} = \mu \frac{\omega D}{b-a} \quad (1)$$

$$F_o = \tau_o \cdot 2\pi \frac{D}{2} h = \mu \pi h \frac{\omega D^2}{b-a} \quad (2)$$

$$M_o = F_o \frac{D}{2} = \frac{1}{2} \mu \pi h \frac{\omega D^3}{b-a} \quad (3)$$

INSIDE:

$$\tau_i = \mu \frac{\partial u}{\partial y} = \mu \frac{\omega (D-2a)}{2} \frac{2}{b-a} = \mu \frac{\omega (D-2a)}{b-a} \quad (4)$$

$$F_i = \tau_i \, 2\pi \frac{(D-2a)}{2} h = \mu \pi h \frac{\omega (D-2a)^2}{b-a} \quad (5)$$

$$M_i = F_i \frac{(D-2a)}{2} = \frac{1}{2} \mu \pi h \frac{\omega (D-2a)^3}{b-a} \quad (6)$$

TURNING MOMENT:

$$M = M_i + M_o = \frac{\mu \pi h \omega}{2(b-a)} \left[D^3 + (D-2a)^3 \right] \quad (7)$$

b)

$\omega = 2\pi / s$

$a = 1.5$ mm

$b = 3.5$ mm

$D = 50.0$ mm

SAE 10W30 oil @ $20°C \leq T \leq 80°C$

$M \leq 1.5 \cdot 10^{-2}$ Nm

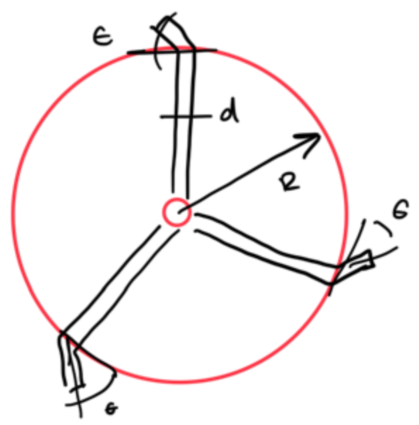
$T = 20°C \Rightarrow \mu = 1.7 \cdot 10^{-1}$ kg/ms

INCREASING THE TEMPERATURE WILL REDUCE THE VISCOSITY OF THE OIL AND THUS THE LOWEST TEMPERATURE IN THE GIVEN RANGE WILL BE THE ONE THAT SETS THE OIL LEVEL (h)

($T = 80°C \Rightarrow \mu = 1.57 \cdot 10^{-2}$ kg/ms)

(7) $\Rightarrow$ **h = 78 mm**

P6



$Q = 8.0 \cdot 10^{-4}\ m^3/s$

$R = 0{,}175\ m$

$d = 7.0\ mm$

FRICTION CAN BE NEGLECTED

DERIVE AN EXPRESSION THAT RELATES THE ROTATIONAL VELOCITY (Ω) TO THE ANGLE θ.

STARTING POINT : THE ANGULAR MOMENTUM EQUATION ON INTEGRAL FORM FOR A FINITE NUMBER OF INLETS AND OUTLETS.

$$\sum M = \frac{d}{dt}\int_{cv} (r \times u)\rho\, dV + \sum_{cs} r_{out} \times \dot m_{out} u_{out} - \sum_{cs} r_{in} \times \dot m_{in} u_{in}$$

STEADY STATE: $\frac{d}{dt}\int_{cv} (r \times u)\rho\, dV = 0$

NO TANGENTIAL VELOCITY COMPONENT AT THE INLET

$$\sum_{cs} r_{in} \times \dot m_{in} u_{in} = 0$$

$\Rightarrow$

$$\sum M = \sum_{cs} r_{out} \times \dot m_{out} u_{out}$$

$$u = \frac{Q}{3A}\cos\theta - \Omega R$$

$$A = \frac{\pi d^2}{4}$$

Ω : ROTATING VELOCITY OF THE SPRINKLER.

$$\sum M = \dot m \begin{bmatrix} 0 \\ R \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ \frac{Q}{3A}\sin\theta \\ \frac{Q}{3A}\cos\theta - \Omega R \end{bmatrix}$$

$$\Rightarrow \sum M = \dot m R\left(\frac{Q}{3A}\cos\theta - \Omega R\right)$$

NO FRICTION $\Rightarrow \sum M = 0 \Rightarrow$

$$\frac{Q}{3A}\cos\theta - \Omega R = 0$$

$$\Rightarrow \Omega = \frac{Q}{3AR}\cos\theta$$

$$\boxed{\Omega = \frac{4Q\cos\theta}{3\pi d^2 R}}$$

$$\Omega = \frac{4\,Q\cos\theta}{6\pi^2 d^2 R}\ \text{rev/s}$$

$$\Omega = \frac{40\,Q\cos\theta}{\pi^2 d^2 R}\ \text{rpm}$$

$$\Omega_{max} = \Omega(\theta = 0) = \frac{40Q}{\pi^2 d^2 R}\ \text{rpm}$$

$$\Omega(\theta = 40^\circ) = 290\ \text{rpm}\ \ (30\ \text{rad/s})$$