MTF053 - Fluid Mechanics

2022-10-28 08.30 – 13.30

Approved aids:

- The formula sheet handed out with the exam (attached as an appendix)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Graph drawing calculator with cleared memory

Exam Outline:

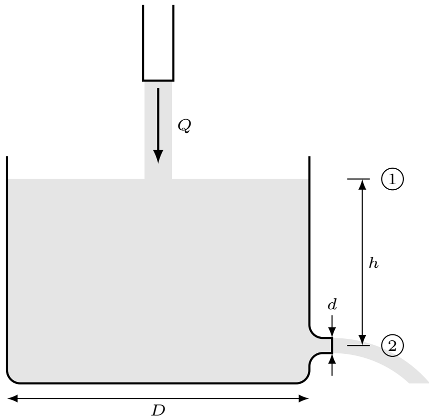
- In total 6 problems each worth 10p

Grading:

number of points on exam	24-35	36-47	48-60
grade	3	4	5

Problem 1 - Tank Flow (10 p.)

The water tank in the figure below is evacuated through the pipe at the bottom and continuously filled from the top with a flow rate such that the level is constant at $h = 0.20m$. The pipe diameter is $d = 0.01m$. The water holds the temperature $T = 20°C$. The pressure at the pipe exit and at the water surface at the top of the tank is the atmospheric pressure.



(a) Tank problems are often solved assuming that the fluid velocity at the surface is zero. Obviously, the fluid velocity cannot be zero, but it is often a good assumption. The assumption gets better as the ratio d/D decreases. Calculate the ratio d/D such that the error is 0.01%. (6p.)

If Q is the flow rate obtained with $V_1 \neq 0$ (the velocity of a fluid particle at the surface) and Q_0 the flow rate that you get if you assume that $V_1 = 0$, the error is calculated as follows:

$$error = \left| \frac{Q - Q_0}{Q} \right|$$

(b) For the calculated ratio d/D, calculate the flow rate by which the tank is filled from above and the velocity at which water is leaving the tank at the bottom. (1p.)

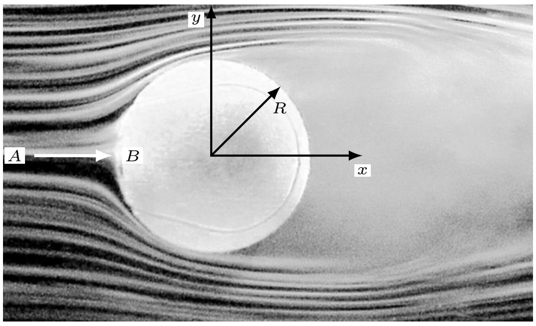
(c) Explain the difference between *streamline*, *pathline*, and *streakline*. Under what circumstances do these three line types coincide in a fluid flow? (1p.)

(d) When in use, the fluid in a fire extinguisher is flowing out from the tank through the hose. Describe the difference between

$$\left.\frac{dB}{dt}\right|_{system}, \text{ and } \frac{d}{dt}\left(\int_{cv} \beta \rho d\mathcal{V}\right)$$

if the extensive property B is fluid mass and if the control volume (CV) is fixed and aligned with the tank surface. (2p.)

Problem 2 - Tennis Ball (10 p.)



For the tennis ball in the figure, the velocity along the streamline $A \rightarrow B$ is given by

$$\mathbf{V} = u(x)\mathbf{e}_x = V_o\left(1 + \frac{R^3}{x^3}\right)\mathbf{e}_x$$

where R is the radius of the tennis ball and x is the coordinate from the center of the ball positive in the flow direction.

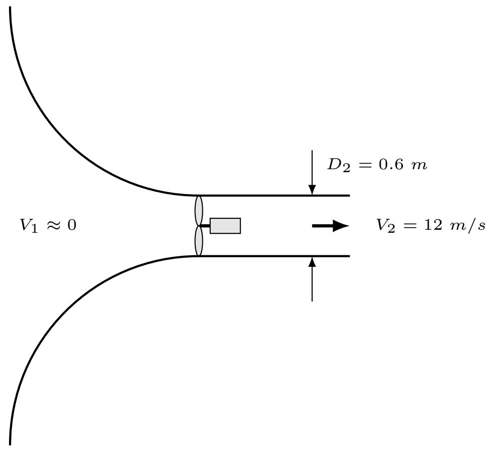
(a) Assuming the flow around the ball to be steady-state, derive an expression describing the acceleration experienced by a fluid particle along the streamline $A \rightarrow B$ (5p.)

(b) At what axial position will the magnitude of the acceleration be the biggest? (2p.)

(c) The diameter of a tennis ball is 2.7 inches (68.58 mm). When a professional tennis player serves, the velocity of the ball can reach velocities of up to 200.0 km/h. What is the maximum magnitude of acceleration that a fluid particle approaching the stagnation point at the front of the tennis ball will experience at this velocity? (2p.)
note: it is a ridiculously high value

(d) Finally a question related to another sport. Why do dimpled golf balls have lower pressure drag than golf balls with smooth surfaces? (1p)

Problem 3 - Not a Big Fan (10 p.)



An axial-flow fan installed in an air ventilation system is driven by an electric motor marked 0.4 kW. The flow velocity in the air intake ahead of the fan can be assumed to be zero and after the fan, the fluid velocity is 12.0 m/s. The duct in which the fan is installed has the diameter $D = 0.6m$.

(a) Based on the data provided above and assuming that the flow is adiabatic and that the pressure is the atmospheric pressure both upstream and downstream of the fan, calculate the efficiency for the fan installation. (6p.)

hints:

- A fan can be represented as shaft work added to the flow in the same way as shaft work can represent a pump
- Efficiency (η) is defined as the energy that has been transferred to the flow by the fan divided by the energy consumed by the electric motor
- Head (dimension $[m]$) can be converted to flow power (dimension $[W]$) by multiplying with mass flow ($\dot{m}$ $[kg/s]$) times gravity constant (g $[m/s^2]$)

(b) What does Q and W in the energy equation represent? (1p.)

(c) What is the physical meaning of the terms in the energy equation on the form given below? (1p.)

$$\rho C_v \frac{dT}{dt} = k\nabla^2 T + \Phi$$

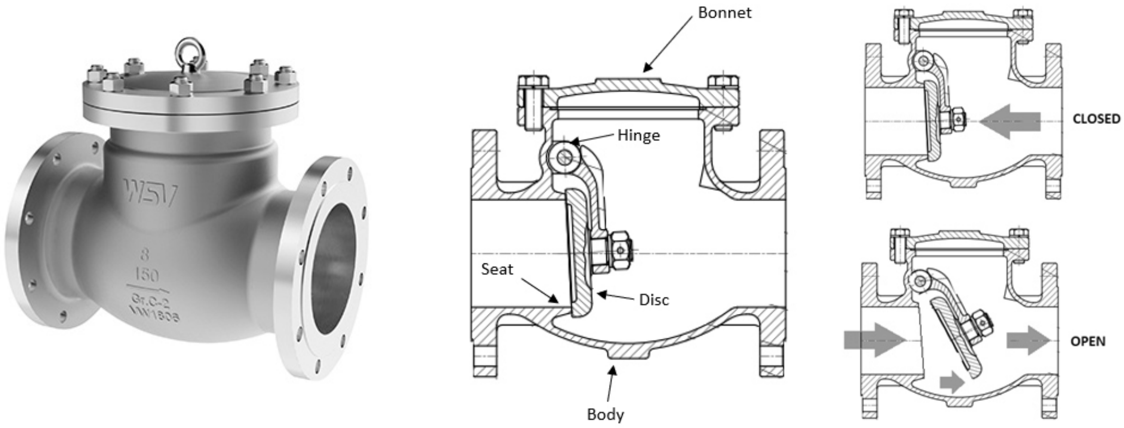
(d) The Bernoulli equation on the form given below is derived for steady-state, incompressible, frictionless flow along a streamline

$$\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho} + \frac{1}{2}V_2^2 + gz_2 = const$$

In what ways are the Bernoulli equation more limited than the energy equation on the form given below? (2p.)

$$\left(\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1\right) = \left(\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2\right) + \frac{\hat{u}_2 - \hat{u}_1 - q}{g}$$

Problem 4 - Valve Test (10 p.)



A large check valve designed to be installed in a water supply system is to be tested in model scale. The full-scale valve has an inlet diameter of $D_p = 0.6\ m$ and the water system is supposed to be able to handle water with the temperature $T = 20°C$ at a flow rate of $Q = 0.85\ m^3/s$. The reason for doing the test is to get estimates of forces on hinges and installation bolts. Geometric similarity and Reynolds number similarity is assumed to be sufficient to obtain dynamic similarity.

The following constraints are given by the test facility:

valve inlet diameter
$70\ mm \leq D_{inlet} \leq 140\ mm$
flow rate
$Q \leq 0.20\ m^3/s$
certified fluids
Water SAE 30W oil SAE 10W oil SAE 10W30 oil Ethylene glycol Ethanol

(a) Is it possible to design a model-scale valve for testing in the facility with the given constraints and obtain dynamic similarity? (6p)
(*your answer must be justified by calculations*)

(b) Explain the concepts *geometric similarity* and *dynamic similarity* (2p.)

(c) How does the fluid viscosity vary with temperature in liquids and gases, respectively? (2p.)

Problem 5 - Pipe Flow (10 p.)

(a) A sprinkler system used in case of fire is fed by a pipe made from galvanized iron that can be assumed to be in new condition. When the sprinkler system is activated the flow rate through the pipe should be 0.2 m^3/s. The total length of the pipe is 35 m and the head loss must not exceed 50 m. Calculate the smallest pipe diameter that fulfills the constraints. (6p.)

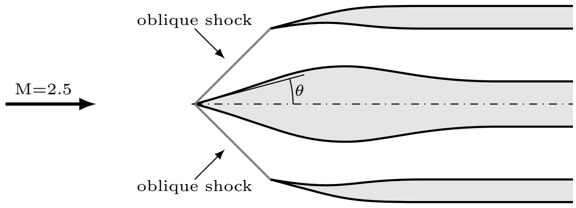
(b) Why does the Moody chart not give reliable values in the Reynolds number range $2000 < Re < 4000$? (1p.)

(c) Show that, for a laminar pipe flow, the friction factor f can be calculated as

$$f = \frac{64}{ReD}$$

(3p.)

Problem 6 - Engine Intake for Supersonic Flight (10 p.)



The center cone of the engine intake in the figure below has an angle θ that corresponds to 90% of maximum possible flow deflection at $M = 2.5$. This means that if the engine is operated at a freestream Mach number of 2.5, an oblique shock will be attached to the leading edge of the cone.

(a) Calculate the cone angle at the leading edge (θ) (2p)

(b) Calculate the Mach number downstream of the oblique shock attached to the leading edge if the freestream Mach number is 2.5 (2p.)

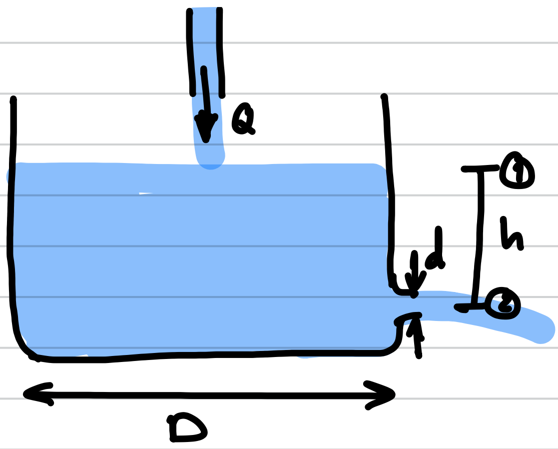
(c) Calculate the pressure and temperature downstream of the oblique shock if the upstream temperature and pressure is $T_1 = 20°C$ and $p_1 = 1.0\ bar$, respectively. (2p.)

(d) Show schematically, what the flow in the in the vicinity of the engine intake would look like if the angle of the center cone was larger than the maximum deflection angle for the freestream Mach number. (2p.)

(e) What is required for a process to be isentropic? (1p.)

(f) Which of the properties h_o, T_o, a_o, p_o, and ρ_o are constants in a flow if the flow is adiabatic and isentropic, respectively? (1p.)

P1

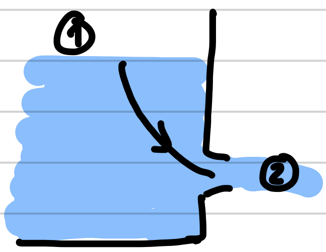


$d = 0.01\,m$
$h = 0.2\,m$

A WATER TANK WITH AN OPEN DISCHARGE IS FILLED CONTINUOUSLY FROM THE TOP SUCH THAT THE LEVEL OF WATER IS CONSTANT AT HEIGHT h OVER THE DISCHARGE ORIFICE. THE WATER TEMPERATURE IS 20°C

a) TANK PROBLEMS ARE OFTEN SOLVED ASSUMING THAT THE FLUID VELOCITY AT THE SURFACE IS ZERO (OFTEN A GOOD ASSUMPTION)
OBTAIN THE RATIO d/D SUCH THAT THE ERROR IS 0.01%

1) $V_1 = 0$



BERNOULLI (3.57)

$$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{1}{2}\frac{V_1^2}{g}} + z_1 = \cancel{\frac{P_2}{\rho g}} + \frac{1}{2}\frac{V_2^2}{g} + z_2$$

$$\Rightarrow V_2 = \sqrt{2g(z_1 - z_2)} = \sqrt{2gh}$$

$$Q_0 = \frac{\pi d^2}{4}\sqrt{2gh}$$

2) $V_1 \neq 0$

Continuity $\Rightarrow$ $\cancel{\rho} V_1 \frac{\pi D^2}{4} = \cancel{\rho} V_2 \frac{\pi d^2}{4}$ (3.22)

$\Rightarrow V_1 = \left(\frac{d}{D}\right)^2 V_2$ (1)

BERNOULLI (3.54)

$$\frac{\cancel{P_1}}{\cancel{\rho} g} + \frac{V_1^2}{2g} + z_1 = \frac{\cancel{P_2}}{\cancel{\rho} g} + \frac{V_2^2}{2g} + z_2$$

WITH V_1 FROM (1) $\Rightarrow$

$$\frac{V_2^2}{2g}\left(\frac{d}{D}\right)^4 + z_1 = \frac{V_2^2}{2g} + z_2$$

$$V_2^2\left(1-\left(\frac{d}{D}\right)^4\right) = 2gh$$

$$V_2 = \sqrt{2gh/\left(1-\left(\frac{d}{D}\right)^4\right)}$$

$$Q = \frac{\pi d^2}{4}\sqrt{2gh/\left(1-\left(\frac{d}{D}\right)^4\right)}$$

$$\varepsilon = \left| \frac{Q - Q_0}{Q} \right|$$

$$\Rightarrow \varepsilon = \left(1 - \sqrt{\left(1 - \left(\frac{d}{D}\right)^4\right)}\right)$$

$$\left(1 - \left(\frac{d}{D}\right)^4\right) = (1 - \varepsilon)^2$$

$$\Rightarrow \frac{d}{D} = \left(1 - (1 - \varepsilon)^2\right)^{1/4}$$

$$\varepsilon = 0.0001 \Rightarrow \underline{\frac{d}{D} = 0.12}$$

b) WITH d/D CALCULATED IN a),

CALCULATE THE FLOW RATE BY WHICH THE

TANK IS FILLED FROM ABOVE AND THE

EXIT VELOCITY (V_c)

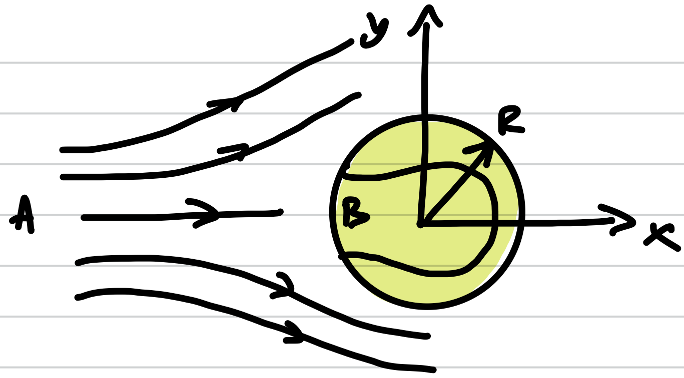
CONTINUITY $\Rightarrow$ $Q_{in} = Q_{out}$.

$$Q_{in} = \frac{\pi d^2}{4} \sqrt{2gh / (1 - (d/D)^4)}$$

$$\Rightarrow Q_{in} = 1.56 \cdot 10^{-4} \ m^3/s \quad (0.16 \ L/s)$$

$$V_2 = \sqrt{2gh / (1 - (d/D)^4)} = 1.98 \ m/s$$

P_2



VELOCITY ALONG THE STREAMLINE A → B

$$\mathbf{V} = u(x)\mathbf{e}_x = V_0\left(1 + \frac{R^3}{x^3}\right)\mathbf{e}_x$$

a) ASSUMING STEADY-STATE FLOW, DERIVE AN EXPRESSION DESCRIBING THE ACCELERATION EXPERIENCED BY A FLUID PARTICLE ALONG THE STREAMLINE A → B

$$\mathbf{a} = \frac{D\mathbf{V}}{Dt} = \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V} \qquad (4.2)$$

$$\mathbf{V} = (u, 0, 0) \Rightarrow$$

$$\mathbf{a} = \underbrace{\cancel{\frac{\partial u}{\partial t}}}_{\text{STEADY STATE}} + u\frac{\partial u}{\partial x}$$

$$u = V_0\left(1 + \frac{R^3}{x^3}\right) \Rightarrow \frac{\partial u}{\partial x} = -\frac{3V_0R^3}{x^4}$$

$$a = u\frac{\partial u}{\partial x}e_x = -3V_0^2R^3\left(\frac{1}{x^4}+\frac{R^3}{x^7}\right)e_x$$

b) MAXIMUM ACCELERATION:

$$\frac{\partial a}{\partial x} = -3V_0^2R^3\left(\frac{-4}{x^5}-\frac{7R^3}{x^8}\right)e_x$$

$$3V_0^2R^3\left(\frac{4}{x^5}+\frac{7R^3}{x^8}\right)=0 \Rightarrow$$

$$4x^3 = -7R^3 \Rightarrow x = \left(\frac{-7R^3}{4}\right)^{1/3}$$

c)

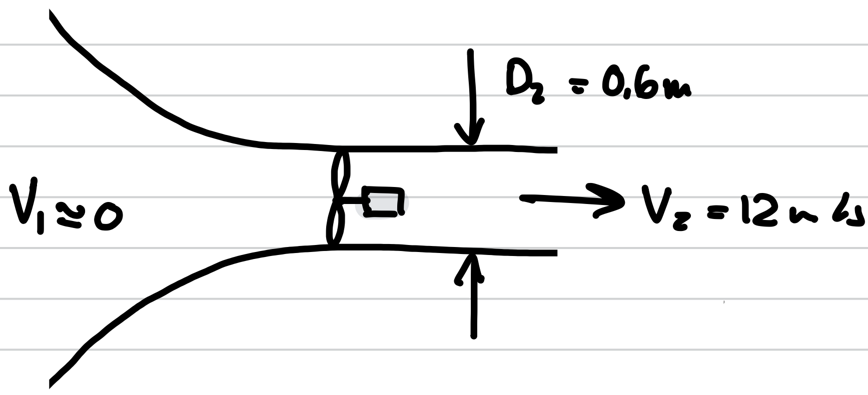
$R = 68,58$ mm $(2.7'')$

$V_0 = 200$ km/h $(55,56$ m/s$)$

$$x = \left(\frac{-7R^3}{4}\right)^{1/3} = -0.0413 \text{ m}$$

$$a = -3V_0^2R^3\left(\frac{1}{x^4}+\frac{R^3}{x^7}\right) = -5,49\cdot10^4 \text{ m/s}^2$$

P3



Given:

Axial flow fan (0.4 kW)

Inlet velocity ≈ 0 (large inlet)

Downstream velocity ~12 m/s

$D = 0.6\,m$

a) Assumptions: Adiabatic (no heat addition)

Insignificant pressure increase over fan $P_1 \sim P_2 = P_{atm}$

Calculate efficiency

Energy equation (3.70)

$$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + \cancel{z_2} + \cancel{h_{turbine}}$$

$$- \underset{(fan)}{h_{pump}} + h_f$$

HEAD LOSS h_f IS INCLUDED IN THE FAN INSTALLATION

$$\Rightarrow \frac{V_2^2}{2g} = h_{fan} = 7.34\,m$$

$$P_{fan} = h_{fan}\,\dot{m}g \quad \text{(FROM HINT)}$$

$$\dot{m} = \rho Q = \rho A V = \rho \frac{\pi D_2^2}{4} V_2$$

VENTILATION SYSTEM $\Rightarrow$ ASSUME AIR @ 20°C

$\Rightarrow \rho = 1.2\ kg/m^3$

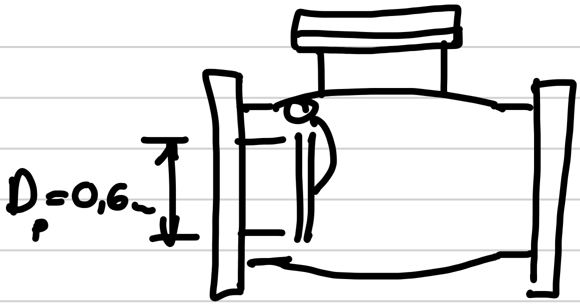
$$\Rightarrow \dot{m} = 4.07\ kg/s$$

$$P_{fan} = h_{fan}\,\dot{m}g = 293\,W$$

$$\eta = \frac{P_f}{P_m} = \frac{293}{400} = 0.73$$

FAN INSTALLATION EFFICIENCY : 73%

P4



$Q_p = 0,85\,m^3/s$

WATER SUPPLY SYSTEM $\Rightarrow$ WATER @ 20°

$\rho = 998\ kg/m^3\ ,\ \mu = 10^{-3}\ kg/ms$

IS IT POSSIBLE TO DESIGN A MODEL SCALE VALVE THAT FITS THE EXPERIMENTAL TEST FACILITY AND FULFILL GEOMETRIC SIMILARITY AND REYNOLDS NUMBER SIMILARITY?

REYNOLDS NUMBER BASED ON DIAMETER

$$\left.\begin{aligned} Re_D &= \frac{\rho V D}{\mu} \\ Q &= V\frac{\pi D^2}{4} \end{aligned}\right\} \Rightarrow Re_D\ (\text{PROTOTYPE}) = 1.8 \cdot 10^6$$

$$Re_{D,P} = Re_{D,M} \Rightarrow \frac{\rho_M V_M D_M}{\mu_M} = Re_{D_P}$$

WE ARE GIVEN 6 LIQUIDS THAT CAN BE USED FOR THE TEST

ASSUME THAT WE CAN USE WATER.

$$\frac{\rho_M V_M D_M}{\mu_M} = \frac{\rho_P V_P D_P}{\mu_P}$$

$\rho_M = \rho_P$, $\mu_M = \mu_P \Rightarrow V_M D_M = V_P D_P$

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2} \Rightarrow$$

$$\frac{Q_M}{D_M} = \frac{Q_P}{D_P}$$

ACCORDING TO THE SPECIFICATIONS

$0{,}07\,m \leq D_M \leq 0{,}14\,m$

$D_M = 0{,}07\,m \Rightarrow Q_M = 0.1\,m^3/s$

$D_M = 0.14\,m \Rightarrow Q_M = 0{,}2\,m^3/s$

SO, WITH WATER @ 20°C, WE CAN CHOSE ANY DIAMETER WITHIN THE SPECIFIED RANGE WITHOUT GETTING ABOVE THE MAXIMUM $Q = 0.20\,m^3/s$

P5

GALVANIZED IRON PIPE (NEW CONDITION)

FLOW RATE $Q = 0.2\ m^3/s$

LENGTH $L = 35\,m$

HEAD LOSS $h_f \leq 50\,m$

a) CALCULATE THE SMALLEST PIPE DIAMETER THAT FULFILLS THE CONSTRAINTS.

TABLE 6.1 $\Rightarrow$ $\varepsilon = 0.15\ mm$

ASSUME TURBULENT FLOW.

$$\left.\begin{aligned} h_f &= f\frac{L}{D}\frac{V^2}{2g} \\ V &= \frac{4Q}{\pi D^2} \end{aligned}\right\} \Rightarrow \boxed{D^5 = f\,\frac{8Q^2 L}{g\pi^2 h_f}} \qquad (1)$$

$$\boxed{Re_D = \frac{\rho V D}{\mu} = \frac{4\rho Q}{\pi \mu D}} \qquad (2)$$

COLEBROOK (6.48)

$$\boxed{\frac{1}{\sqrt{f}} = -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re_D\sqrt{f}}\right)} \qquad (3)$$

SOLUTION PROCESS:

1) GUESS f (FRICTION FACTOR)

2) CALCULATE D FROM (1)

3) CALCULATE Re_D FROM (2)

4) SOLVE (3) TO GET AN UPDATED VALUE OF f

REPEAT 2–4 UNTIL f DOES NOT CHANGE SIGNIFICANTLY IN STEP 4

STARTING GUESS: $f = 0.03$

ITERATE $\Rightarrow$ $f = 0.0203$

$D = 0.136\ m$

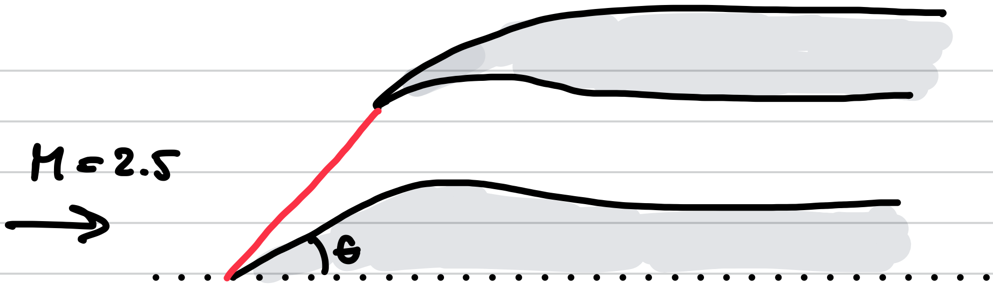
$V = 13.73\ m/s$

$Re_D \approx 1.87 \cdot 10^6 \gg 2300 \Rightarrow$ TURBULENT

SO...

$$\boxed{D = 0.136\ m}$$

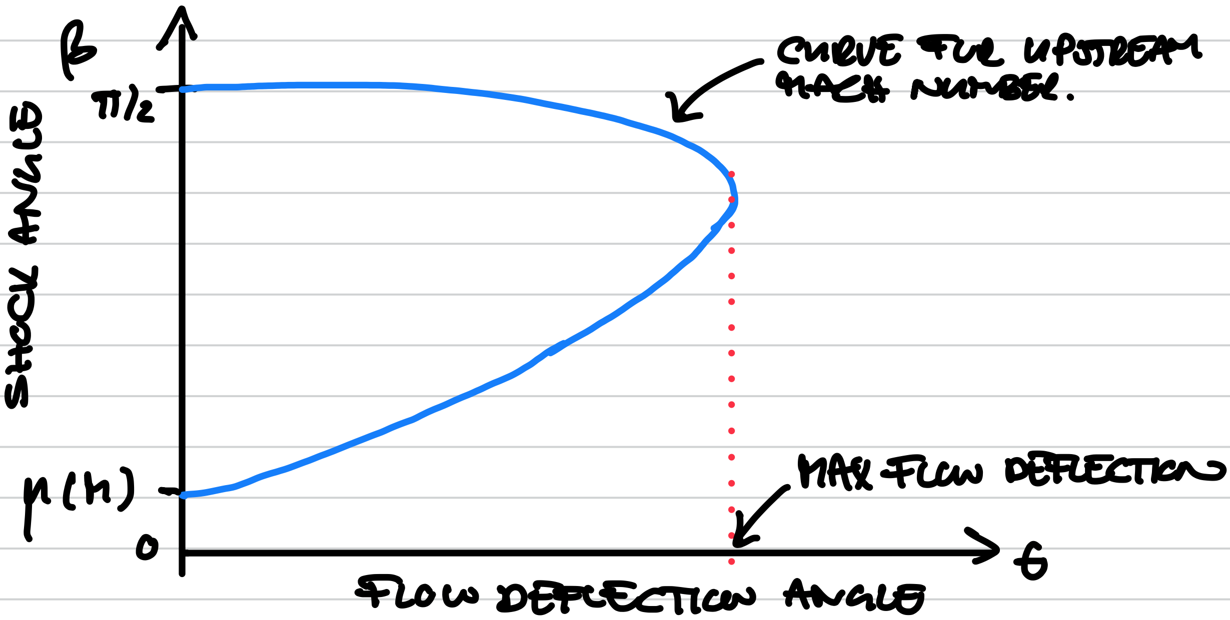
P6



θ CORRESPONDS TO 90% OF THE MAXIMUM FLOW DEFLECTION @ M=2.5

a) FIND A VALUE FOR THE FLOW DEFLECTION θ

THE MAXIMUM FLOW DEFLECTION FOR A GIVEN MACH NUMBER IS GIVEN BY THE θ-β-M RELATION



=> MAX FLOW DEFLECTION FOR $M_1 = 2.5$

$\theta_{max} = 29.8°$

$$\theta = 0.9\,\theta_{max} = 26.8^\circ$$

b) CALCULATE THE MACH NUMBER DOWNSTREAM OF THE SHOCK

$$\theta-\beta-M \Rightarrow (M_1 = 2.5, \theta = 26.8^\circ) \Rightarrow \beta = 53.6^\circ$$

$$\left.\begin{aligned} (9.82) &\Rightarrow M_{n_1} = M_1 \sin(\beta) \\ (9.57) &\Rightarrow M_{n_2}^2 = \frac{(\gamma-1)M_{n_1}^2 + 2}{2\gamma M_{n_1}^2 - (\gamma-1)} \\ (9.82) &\Rightarrow M_{n_2} = M_2 \sin(\beta-\theta) \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \boxed{M_2 = 1.3}$$

c) CALCULATE THE PRESSURE AND TEMPERATURE DOWNSTREAM OF THE SHOCK IF THE UPSTREAM CONDITIONS ARE

$p_1 = 1.0\text{ bar}$ & $T_1 = 20^\circ C$ $(293\,K)$

$$(9.55)\quad \frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n_1}^2 - 1\right)$$

$$(9.58)\quad \frac{T_2}{T_1} = \left(2 + (\gamma-1)M_{n_1}^2\right)\frac{2\gamma M_{n_1}^2 - (\gamma-1)}{(\gamma+1)^2 M_{n_1}^2}$$

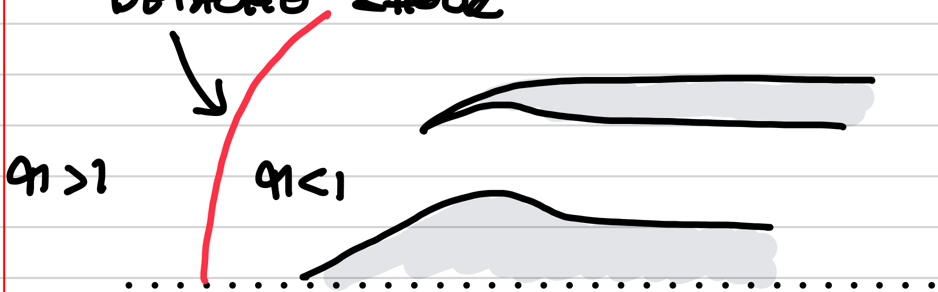
$$\Rightarrow \boxed{\begin{aligned} P_2 &= 4.6 \text{ bar} \\ T_2 &= 498 \text{ K} \end{aligned}}$$

$$(498\text{ K} = 225°\text{C})$$

d) SHOW SCHEMATICALLY WHAT THE FLOW IN THE VICINITY OF THE NOZZLE WOULD LOOK LIKE IF $\theta > \theta_{max}$

NO OBLIQUE SHOCK SOLUTION POSSIBLE $\Rightarrow$

DETACHE SHOCK



THE EXACT LOCATION AND STRENGTH OF THE DETACHED SHOCK DEPENDS ON THE UPSTREAM MACH NUMBER.