MTF053 - Fluid Mechanics

2022-08-15 08.30 – 13.30

Approved aids:

- The formula sheet handed out with the exam (attached as an appendix)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Graph drawing calculator with cleared memory

Exam Outline:

- In total 6 problems each worth 10p

Grading:

number of points on exam (including bonus points)	24-35	36-47	48-60
grade	3	4	5

Problem 1 - Pipe Flow (10 p.)

Water from a treatment plant is pumped into a distribution system at a rate of 4.38 m^3/s, a pressure of 480 kPa, and a temperature of 20°C. The distribution pipe has an inner diameter of 750 mm and is made of cast iron.

(a) Estimate the pressure in the pipe at a station located 200 m downstream of the treatment plant if the pipe remains horizontal (5p.)

(b) After 20 years in operation, scale build-up is expected to cause the equivalent sand roughness of the pipe to increase by a factor of 10. How does this effect the water pressure at the station 200 m downstream of the treatment plant? (3p.)

(c) What does critical Reynolds number mean for a pipe flow? (1p.)

(d) What does fully developed pipe flow mean? (1p.)

Problem 2 - Lift & Drag (10 p.)

(a) A Boeing 757-200 has a maximum allowable total mass of 116000 kg, a wingspan of 38 m, and a total wing area of 185 m^2. At take off the flaps are deployed, which increases the lift coefficient and if configured optimally it is possible to get a lift coefficient of 3.5. Estimate the minimum take-off speed for a Boeing 757-200 if the take-off weight corresponds to the maximum allowable weight for the aircraft. (4p.)

(b) When the aircraft in the previous task reaches its operation altitude, it cruises at 900 km/h at an elevation of 10600 m where the air density is estimated to be 0.384 kg/m^3. The aircraft has two engines and a fuel capacity of 200000 L. The total fuel consumption at cruise conditions is 2 L/s. The density of the jet fuel is 804 kg/m^3. The intake diameter of each of the engines is 2.19 m and according to the specifications each engine can develop a thrust of 245 kN. Estimate the engine exhaust velocity. (4p.)

(c) The drag coefficient, C_D, for cylinder flow is drastically changed as the boundary layer becomes turbulent (before separating). Show schematically how C_D varies with the Reynolds number, Re_D, and indicate the locations for transition to turbulence and flow separation. (2p.)

Problem 3 - Flat-Plate Boundary-Layer Flow (10 p.)

Air at 20°C flows at 18 km/h over a flat surface that is 1.0 m long (flow direction) and 2.0 m wide (flow-normal direction). Boundary layer transition can be assumed to occur at $Re_x = 5.0 \times 10^5$.

(a) Determine the shear stress at the downstream end of the surface (4p.)

(b) Determine the average shear stress on the surface (3p.)

(c) Determine the drag force on the surface (1p.)

(d) Name two alternative ways to measure the boundary layer thickness than δ. How can these measures be interpreted physically? (1p.)

(e) For laminar flow over a flat plate, the velocity profile is self-similar - what does that mean? (1p.)

Problem 4 - Poiseuille Flow (10 p.)

SAE-30W oil at $20°C$ flows at a rate of $1.5\ L/s$ between two parallel plates spaced $5.0\ mm$ apart. The two plates are both $10.0\ m$ long (in the flow direction) and $2.0\ m$ wide (in the flow-normal direction).

(a) What is the required pressure difference between the inflow and the outflow? (5p.)

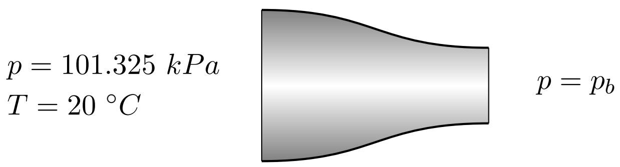
(b) What force is required to keep the upper surface from moving with the flow? (3p.)

(c) Determine the maximum flow velocity between the plates. (2p.)

Hint: the flow can be assumed to be two dimensional and steady state and gravity can be neglected.

Problem 5 - Nozzle Flow (10 p.)

Air flows through a converging nozzle. The entrance of the nozzle (the left-hand side) is open to the atmosphere, where the temperature is $20\ °C$, and the pressure is $101.325\ kPa$. The nozzle exit diameter is $10.0\ mm$, and the back pressure exerted at the exit of the nozzle is p_b.



Determine the mass flow rate through the nozzle if the back pressure is

(a) $p_b = 50.0\ kPa$ (5p.)

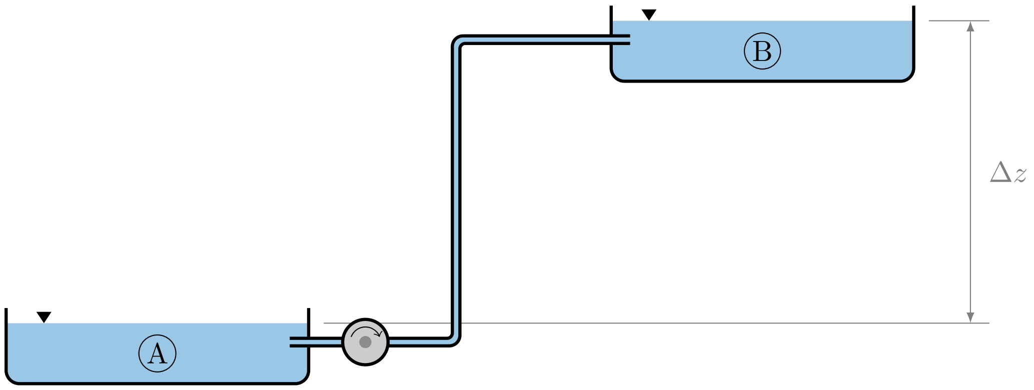
(b) $p_b = 70.0\ kPa$ (5p.)

PROBLEM 6 - WATERPUMP (10 P.)

A pump delivers water with the temperature $20°C$ from a lower reservoir (A) to an upper reservoir (B) at a rate of 6.3 L/s. Both reservoirs are open to atmosphere. The pump lifts the water the vertical distance $\Delta z = 20.0\ m$. The reservoirs are large, which means that vertical movement of the reservoir water surface can be neglected. The pipe system head loss (friction losses in the system), h_L, varies with the volume flow rate, Q, according to the following empirical expression

$$h_L = 10^5 Q^2$$

where the dimension of h_L is $[m]$ and the dimension of Q is $[m^3/s]$



(a) Calculate the electric power needed to drive the pump if the pump electric efficiency is $\eta = 0.8$ (6p.)

Hint: pump power, P_P $[W]$, is related to pump head, h_P $[m]$, as

$$P_P = \rho g Q h_P$$

where ρ is the fluid density, g is the gravity constant, and Q is the volume flow rate in m^3/s

(b) Often, but not in this specific problem, something called kinetic energy correction factors are introduced in energy equation problems. Why is that done? Show that the kinetic energy correction factor is $\alpha = 2.0$ for laminar, incompressible pipe flow. (2p.)

(c) The Bernoulli equation is a simplified form of the energy equation.

$$\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho} + \frac{1}{2}V_2^2 + gz_2 = const$$

In what ways are the Bernoulli equation above more limited than the energy equation on the form given below?

$$\left(\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1\right) = \left(\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2\right) + \frac{\hat{u}_2 - \hat{u}_1 - q}{g}$$

(2p.)

P1

WATER FROM A TREATMENT PLANT IS PUMPED INTO A DISTRIBUTION SYSTEM AT A RATE OF $Q = 4.38\ m^3/s$, PRESSURE $P = 480\ kPa$, AND TEMPERATURE $T = 20°C$

INNER DIAMETER OF PIPE $D = 0.75\ m$

(PIPE MADE OF CAST IRON)

a) ESTIMATE THE PRESSURE 200 m DOWNSTREAM OF THE TREATMENT PLANT

CAST IRON $\Rightarrow \varepsilon = 0.26\ mm$

$$V_{av} = Q/A = \frac{4Q}{\pi D^2}$$

$$Re_D = \frac{\rho V_{av} D}{\nu}$$

WATER @ 20°C $\Rightarrow \rho = 998\ kg/m^3$

$\mu = 10^{-3}\ kg/m\,s$ $\quad (\nu = \mu/\rho)$

$\Rightarrow Re_D \simeq 7.42 \cdot 10^6$ (TURBULENT)

THE ENERGY EQN (3.73)

$$\left(\frac{p}{\rho g} + \cancel{\frac{V^2}{2g}} + \cancel{z}\right)_1 = \left(\frac{p}{\rho g} + \cancel{\frac{V^2}{2g}} + \cancel{z}\right)_2 + \cancel{h_t} - \cancel{h_p} + h_f$$

FULLY DEVELOPED $\Rightarrow V_1 = V_2 = V_{av}$

$z_1 = z_2$

$h_t = 0$ (no turbine)

$h_p = 0$ (no pump)

$$\Rightarrow \quad \frac{p_1 - p_2}{\rho g} = h_f$$

(6.10) $$h_f = f \frac{L}{D} \frac{V_{av}^2}{2g}$$

WE NEED A FRICTION FACTOR..

COLEBROOK (6.48)

$$\frac{1}{\sqrt{f}} = -2.0 \log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re_D\sqrt{f}}\right)$$

SOLVE BY ITERATING (ALT. USE MOODY CHART OR THE EXPLICIT HAALAND EQN.)

$$\Rightarrow f = 0.0155$$

$$P_1 - P_2 = \rho f \frac{L}{D} \frac{V_{av}^2}{2}$$

$$\Rightarrow \quad \underline{P_2 = 277.2 \text{ kPa}}$$

b)

AFTER 20 YEARS OF USE THE RELATIVE ROUGHNESS IS EXPECTED TO HAVE INCREASED BY A FACTOR OF 10

CALCULATE THE DOWNSTREAM PRESSURE FOR THESE CONDITIONS.

$$\varepsilon = 10 \cdot 0.26 = 2.6 \text{ mm}$$

FINDING f USING COLEBROOK'S FORMULA GIVES

$$f \approx 0.027$$

$$P_1 - P_2 = \rho f \frac{L}{D} \frac{V_{av}^2}{2} \Rightarrow \underline{P_2 = 123.2 \text{ kPa}}$$

P2

BOEING 757-200

MASS 116000 kg (m)

WINGSPAN 38 m

TOTAL WING AREA 185 m^2 (A_p)

WITH FLAPS DEPLOYED $C_L = 3.5$

a) CALCULATE THE MINIMUM TAKE-OFF SPEED

$$(7.66) \qquad C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A_p}$$

$F_L = mg$ (THE LIFT FORCE MUST BALANCE THE WEIGHT)

$$\Rightarrow \quad V = \sqrt{\frac{F_L}{\frac{1}{2}\rho C_L A_p}}$$

WITH $\rho = 1.2$ (AIR @ 20°C) WE GET

$$V = 54.1 \text{ m/s}$$

b) CRUISE AT 900 km/h @ 10600 m

$\rho = 0.384 \text{ kg/m}^3$

TWO-ENGINE CONFIGURATION

FUEL CAPACITY : 200000 L

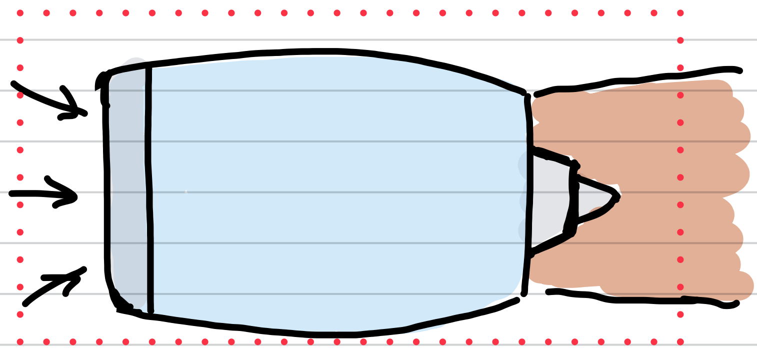
FUEL CONSUMPTION : 2 L/s (Q_f)

JET FUEL DENSITY : $\rho_f = 804 \text{ kg/m}^3$

ENGINE INTAKE DIAMETER : 2.19 m

ENGINE THRUST : 245 kN

TASK: ESTIMATE ENGINE EXHAUST VELOCITY.



CONTROL VOLUME

STEADY STATE:

$$F = \sum_i (\dot{m}_i V_i)_{out} - \sum_i (\dot{m}_i V_i)_{in} \quad (3.40)$$

$$\dot{m}_M = \frac{\pi}{4} D_m^2 \cdot \rho = 361.6 \text{ kg/s}$$

$$\dot{m}_{fuel} = \frac{Q_f}{2} \cdot \rho_f = 0{,}804 \text{ kg/s}$$

(TWO ENGINES)

$$\dot{m}_{out} = \dot{m}_M + \dot{m}_{fuel}$$

$$F = \dot{m}_{out} V_j - \dot{m}_M V_m$$

$$\Rightarrow V_j = 925 \text{ m/s}$$ RELATIVE TO THE ENGINE

(ABSOLUTE VELOCITY $V_j - V_{in} = 675$ m/s)

P3

AIR @ 20°C FLOWS AT 18 km/h OVER A FLAT SURFACE. SURFACE LENGTH $L = 1.0\,m$

SURFACE WIDTH $b = 2.0\,m$

TRANSITION CAN BE ASSUMED TO OCCUR AT

$Re_x = 5.0 \cdot 10^5$

a) CALCULATE THE SHEAR STRESS AT THE DOWNSTREAM END.

AIR @ 20°C $\Rightarrow \rho = 1.2\,kg/m^3$, $\mu = 1.8 \cdot 10^{-5}\,kg/ms$

$$Re_L = \frac{\rho U L}{\mu} = 3.3 \cdot 10^5 < 5.0 \cdot 10^5$$

$\Rightarrow$ LAMINAR

$$(7.25) \qquad \tau_w = \frac{0{,}332\,\rho U^2}{Re_L^{1/2}} = \underline{17{,}3\,mPa}$$

b) CALCULATE THE AVERAGE SHEAR STRESS

(7.27)

$$\bar{\tau}_w = \frac{b \int_0^L \tau_w(x)\,dx}{L b} = C_D \frac{1}{2}\rho U^2 = \left[\frac{1.328}{Re_L^{1/2}}\right]\frac{1}{2}\rho U^2$$

$$\Rightarrow \bar{\tau}_w = \underline{34.5\,mPa}$$

c) CALCULATE THE TOTAL DRAG FORCE

$$D = \overline{\tau_w} \cdot bL = \underline{69\ mN}$$

P4

SAE-30W OIL FLOWS AT A FLOW RATE OF $Q = 1.5\ L/s$ BETWEEN TWO PARALLEL PLATES SPACED 5.0 mm (h) APART

SAE-30W @ 20°C

$L = 10.0\ m$

$\rho = 891\ kg/m^3$

$b = 2.0\ m$

$\mu = 0,29\ kg/ms$

CONTINUITY:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$v = w = 0 \Rightarrow \frac{\partial u}{\partial x} = 0$$

MOMENTUM-X:

$$\rho\left(\underbrace{\cancel{\frac{\partial u}{\partial t}}}_{\text{steady state}} + u\underbrace{\cancel{\frac{\partial u}{\partial x}}}_{\text{continuity}} + \underbrace{\cancel{v\frac{\partial u}{\partial y}}}_{v=0} + \underbrace{\cancel{w\frac{\partial u}{\partial z}}}_{w=0}\right) = -\frac{\partial p}{\partial x} + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right)$$

$$\Rightarrow \frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2}$$

MOMENTUM-Y :

$$\rho\left(\cancel{\frac{\partial v}{\partial t}} + u\cancel{\frac{\partial v}{\partial x}} + v\cancel{\frac{\partial v}{\partial y}} + w\cancel{\frac{\partial v}{\partial z}}\right) = -\frac{\partial p}{\partial y} +$$

$$+ \mu\left(\cancel{\frac{\partial^2 v}{\partial x^2}} + \cancel{\frac{\partial^2 v}{\partial y^2}} + \cancel{\frac{\partial^2 w}{\partial z^2}}\right)$$

$$\Rightarrow \frac{\partial p}{\partial y} = 0$$

IN THE SAME WAY , MOMENTUM-Z $\Rightarrow \frac{\partial p}{\partial z} = 0$

$$\Rightarrow p = p(x)$$

$$\Rightarrow \frac{dp}{dx} = \mu \frac{\partial^2 u}{\partial y^2}$$

INTEGRATE :

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{dp}{dx} y + C_1$$

$$u(y) = \frac{1}{2\mu}\frac{dp}{dx} y^2 + C_1 y + C_2$$

BOUNDARY CONDITIONS:

$$u(0) = 0 \Rightarrow C_2 = 0$$

$$u(h) = 0 \Rightarrow \frac{1}{2\mu}\frac{dp}{dx} h^2 + C_1 h = 0$$

$$\Rightarrow C_1 = -\frac{h}{2\mu}\frac{dp}{dx}$$

$$u(y) = \frac{h^2}{2\mu}\frac{dp}{dx}\left(\left(\frac{y}{h}\right)^2 - \left(\frac{y}{h}\right)\right)$$

$$Q = b\int_0^h u(y)\,dy =$$

$$= b\int_0^h \frac{h^2}{2\mu}\frac{dp}{dx}\left(\left(\frac{y}{h}\right)^2 - \left(\frac{y}{h}\right)\right)dy =$$

$$= \frac{bh^2}{2\mu}\frac{dp}{dx}\left[\frac{y^3}{3h^2} - \frac{y^2}{2h}\right]_0^h =$$

$$= \frac{bh^2}{2\mu}\frac{dp}{dx}\left(\frac{h}{3} - \frac{h}{2}\right) = \frac{-bh^3}{12\mu}\frac{dp}{dx}$$

$$Q = -\frac{bh^3}{12\mu}\frac{dp}{dx} \Rightarrow \frac{dp}{dx} = \underline{\underline{-20.9\ kPa/m}}$$

b) WHAT FORCE IS REQUIRED TO KEEP THE UPPER SURFACE FROM MOVING WITH THE FLOW?

$$\tau_w = \mu\left.\frac{\partial u}{\partial y}\right|_{y=h}$$

$$u(y) = \frac{h^2}{2\mu}\frac{dp}{dx}\left[\left(\frac{y}{h}\right)^2 - \left(\frac{y}{h}\right)\right]$$

$$\frac{\partial u}{\partial y} = \frac{h^2}{2\mu}\frac{dp}{dx}\left(\frac{2y}{h^2} - \frac{1}{h}\right)$$

$$\tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=h} = \frac{h^2}{2}\frac{dp}{dx}\left(\frac{2}{h} - \frac{1}{h}\right)$$

$$= \frac{h}{2}\frac{dp}{dx}$$

$$F = \tau_w \cdot b \cdot L = \frac{hbL}{2}\frac{dp}{dx} = \underline{1.04\ kN}$$

C) CALCULATE THE MAXIMUM VELOCITY

$$\frac{\partial u}{\partial y} = \frac{h^2}{2\mu}\frac{dp}{dx}\left(\frac{2y}{h^2} - \frac{1}{h}\right) = 0 \Rightarrow$$

$$\Rightarrow \frac{2y}{h^2} = \frac{1}{h} \Rightarrow y = \frac{h}{2}$$

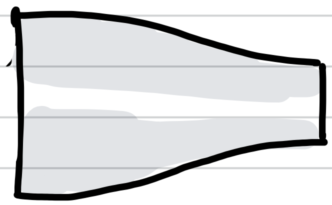
(AS EXPECTED)

$$u(h/2) = \frac{h^2}{2\mu}\frac{dp}{dx}\left(\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)\right)$$

$$= -\frac{h^2}{8\mu}\frac{dp}{dx} = \underline{0.225\ m/s}$$

P_5

P_0, T_0



$P = P_b$

$P_0 = 101.325$ kPa

$T_0 = 293$ K

EXIT DIAMETER 10 mm

DETERMINE THE MASS FLOW RATE IF THE BACK PRESSURE IS

a) $P_b = 50.0$ kPa

b) $P_b = 70.0$ kPa

THE CRITICAL PRESSURE IS CALCULATED AS

$$(9.32) \qquad \frac{p^*}{P_0} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)}$$

$\Rightarrow p^* = 53.5$ kPa

a) $P_b = 50$ kPa $< p^* \Rightarrow$ CHOKED

$$\dot{m} = \frac{P_0 A^*}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

$M = 1$ AT THE NOZZLE EXIT $\Rightarrow$

$$A^* = A_e = \frac{\pi D_e^2}{4}$$

$$\Rightarrow \dot{m} = \underline{0.0188 \text{ kg/s}}$$

b)

$$p_b = 70 \text{kPa} > p^*$$

$\Rightarrow$ SUBSONIC, NOT CHOKED

$$\frac{p_0}{p_b} = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\gamma/(\gamma-1)} \Rightarrow M_e \qquad (9.28)$$

$$\frac{T_0}{T_e} = \left(1 + \frac{\gamma - 1}{2} M_e^2\right) \Rightarrow T_e \qquad (9.26)$$

$$\rho_e = \frac{p_e}{R T_e}$$

$$u_e = M_e a_e = M_e \sqrt{\gamma R T_e}$$

$$\dot{m}_e = \rho_e u_e A_e = \underline{0{,}0176 \text{ kg/s}}$$

P6

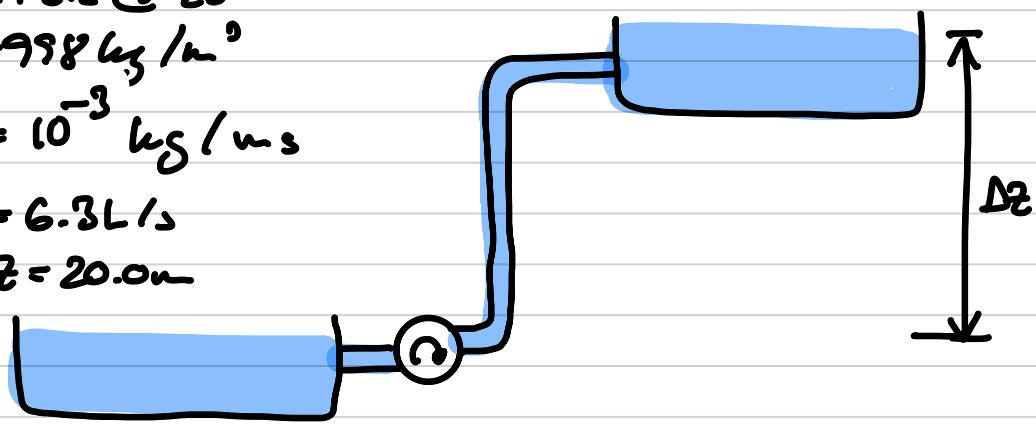
WATER @ 20°

$\rho = 998\ kg/m^3$

$\mu = 10^{-3}\ kg/ms$

$Q = 6.3\ L/s$

$\Delta z = 20.0\ m$



ENERGY EQUATION (8.73)

$$\left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_1 = \left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_2 + h_t - h_p + h_L$$

$V_1 = V_2$

$P_1 = P_2$

$z_2 - z_1 = \Delta z$

$h_t = 0$

$h_L = 10^5 Q^2$ (GIVEN) $= 3.97\ m$

a) CALCULATE THE ELECTRIC POWER NEEDED TO DRIVE THE PUMP IF THE PUMP EFFICIENCY $\eta = 0.8$

$$h_p = \Delta z + h_L = \Delta z + 10^5 Q^2 = 23.97\ m$$

POWER

$$P_p = \rho g Q h_p = \rho g Q(\Delta z + 10^5 Q^2)$$

$$= 1478 \text{ W}$$

$$P_{in} = \frac{P_p}{\eta} = \underline{1848 \text{ W}}$$