MTF053 - Fluid Mechanics

2021-10-29 08.30 – 13.30

Approved aids:

- The formula sheet handed out with the exam (attached as an appendix)
- *Beta - Mathematics Handbook for Science and Engineering*
- *Physics Handbook : for Science and Engineering*
- Graph drawing calculator with cleared memory

Exam Outline:

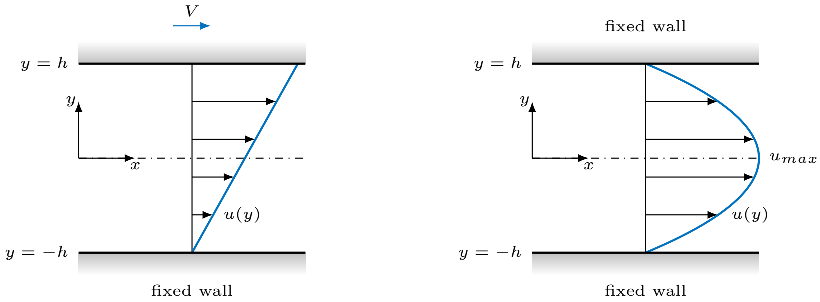
- In total 6 problems each worth 10p

Grading:

number of points on exam (including bonus points)	24-35	36-47	48-60
grade	3	4	5

PROBLEM 1 - COUETTE-POISEUILLE (10 P.)

A Couette flow is established in the fluid between two parallel plates if the plates move with different velocity as in the left figure below where the lower plate is fixed and the upper plate moves to the right with the constant velocity V. The right figure shows a Poiseuille flow generated as a fluid between two fixed parallel plates is exposed to a constant pressure gradient.



Now, let's combine the two elementary flows above, i.e. flow between two parallel plates of which the upper is moving with a constant velocity V and the fluid is exposed to a constant pressure gradient dp/dx.

(a) Derive an expression for the velocity profile $u(y)$ starting from the momentum equations on partial differential form (Eqn. 4.38). The vertical distance between the plates is $2h$ as in the figures above (4p)

(b) Find the velocity V as a function of the height (h), the fluid viscosity (μ), and the pressure gradient (dp/dx) such that the wall-shear stress at the upper wall is zero (2p)

(c) Find the vorticity at the center of the channel when the wall-shear stress at the upper wall is zero (2p)

(d) What is the physical interpretation of fluid viscosity? (1p)

(e) What does it mean that a fluid is Newtonian? (1p)

Problem 2 - Flow Deflection (10 p.)

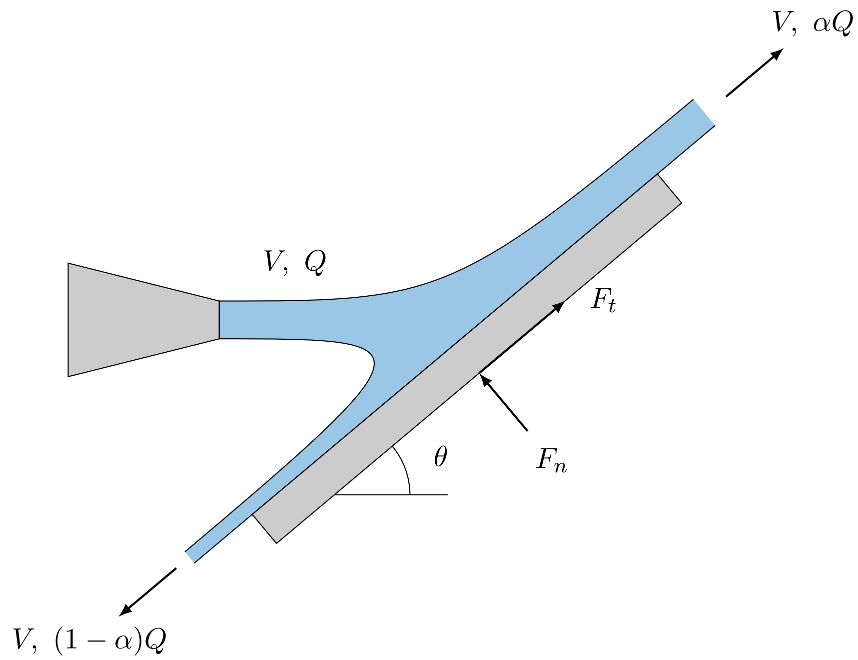
A jet strikes an inclined fixed plate and the jet flow is divided into two jets (as indicated in the picture below). The jet flow velocity is unchanged and the volume flow Q is separated such that the volume flow of the jet going upwards is $\alpha Q,\ \alpha \in [0,1]$ and consequently the volume flow of the jet going in the opposite direction is $(1-\alpha)Q$.

(a) Find α as a function of the deflection angle θ such that the tangential force F_t is zero (7p)

(b) Explain the physical meaning of each of the terms in Reynolds transport theorem (2p)

$$\frac{d}{dt}(B_{syst}) = \frac{d}{dt}\left(\int_{cv} \beta \rho d\mathcal{V}\right) + \int_{cs} \beta \rho (\mathbf{V}_r \cdot \mathbf{n}) dA$$

(c) Explain the physical meaning of the local acceleration term and the convective acceleration term (1p)



Problem 3 - Surface Roughness (10 p.)

In order to estimate the surface roughness of a badly corroded pipe, pressure is measured at two positions in the pipe as water with a temperature of $20°C$ flows through the pipe at a flow rate $Q = 20\ m^3/h$. The inner diameter of the pipe is 5.0 cm and the pipe slopes downward at an angle of 8°.

station	pressure [kPa]	z-coordinate [m]
1	420	12
2	250	3

(a) Estimate the average surface roughness ε (6p)

(b) Estimate the percent change in head loss if the pipe were smooth (same flow rate) (2p)

(c) What does critical Reynolds number mean for a pipe flow? (1p)

(d) Why does the Moody chart not give reliable values in the Reynolds number range $2000 < Re < 4000$? (1p)

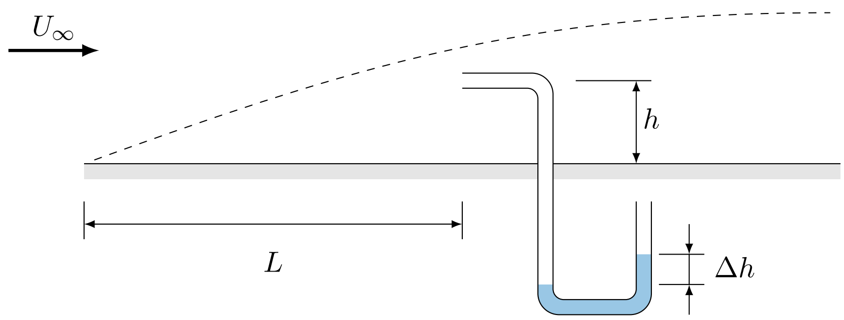
Problem 4 - Boundary Layer Flow (10 p.)

A stagnation tube is mounted on a flat plate as shown in the figure below. The entrance of the tube is located at the axial distance $L = 0.5\ m$ from the leading edge of the plate. The vertical distance from the flat plate surface to the center of the orifice of the stagnation tube is $h = 2.0\ mm$. The freestream velocity is $U_\infty = 15\ m/s$. The fluid is air at $20°C$ and atmospheric pressure. The stagnation tube is attached to water-manometer.

(a) Calculate Δh if the boundary layer is laminar (8p)

(b) Explain the closure problem related to the Reynolds-averaged flow equations (1p)

(c) How does the turbulence viscosity ν_t compare to the kinematic viscosity ν in the viscous sublayer and in the fully turbulent region, respectively? (1p)



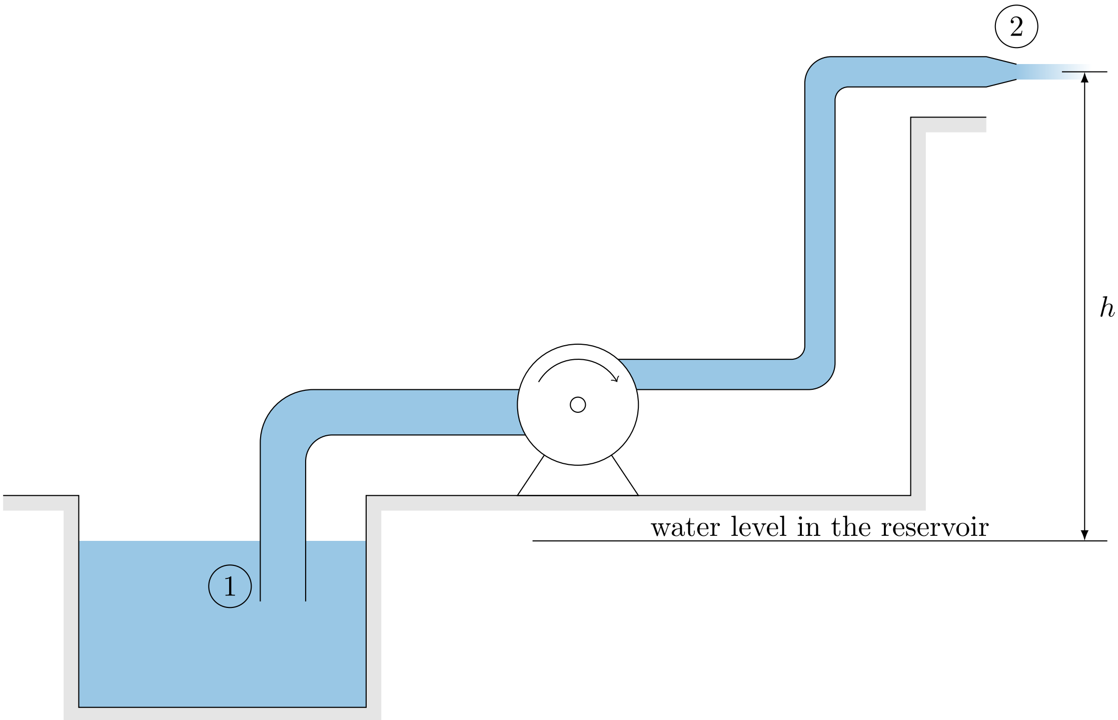
Problem 5 - Pump (10 p.)

The construction schematically represented in the figure below is used to pump up water from a reservoir. The pipe between the pump and the reservoir has the diameter $D_1 = 30.0\ cm$. At the height $h = 10.0\ m$ above the water level in the reservoir, a nozzle with the exit diameter $D_2 = 15.0 cm$ is attached to the pipe. The water leaves the nozzle with an average velocity of $V_{exit} = 5.0\ m/s$. The friction losses in the pipe system can be approximated as V_{exit}^2/g and the flow can be assumed to be turbulent in all pipes.

(a) Calculate the efficiency of the pump ($\eta = power_{out}/power_{in}$) if 20 kW delivered to the pump (8p)

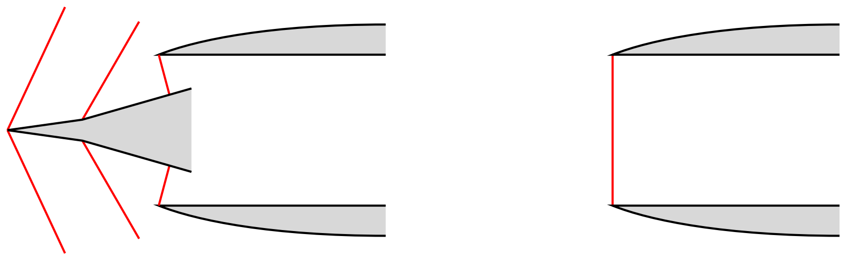
(b) Why is the kinetic energy correction factor larger for laminar flows than for turbulent flows ($\alpha_{lam} = 2.0$, $\alpha_{turb} \approx 1.0$)? (1p)

(c) Give three examples of sources of local losses in a pipe system (1p)



Problem 6 - Engine Inlet (10 p.)

Engine inlets designed for supersonic operation often feature inlet cones for gradual deceleration of the flow by setting up a system of oblique shocks. In the schematic figure below, two engine inlets are compared. The engine inlet to the left has an inlet cone were the flow angle is changed in two discrete steps, which will produce two oblique shocks. In each of the two steps, the flow is bent 8 degrees. After passing the two oblique shocks the flow passes a normal shock when reaching the engine nacelle. In the example to the right, the flow is decelerated by a single normal shock at the engine inlet face.
In reality, the engine inlets are circular but for simplicity let's assume that it is possible to analyse the flow in two dimensions.



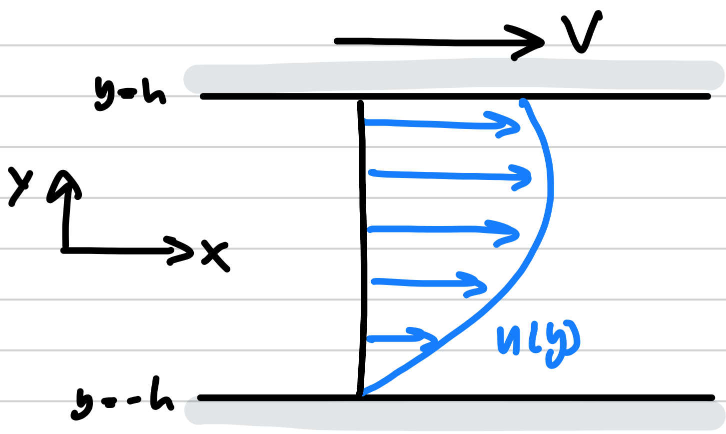
(a) Considering that the oblique shock formed at the tip of the cone needs to deflect the flow an angle of 8 degrees, make an estimate of the lowest Mach number for which the engine inlet will function as intended (2p)

(b) Calculate the Mach number of the flow entering the engine in the two cases if the freestream Mach number is 3.0 (6p)

(c) Explain why the engine inlet design with the oblique shock system (left figure) would be more efficient than the an engine inlet design with a single normal shock at the inlet plane (right figure) (2p)

P1

DERIVE AN EXPRESSION FOR THE VELOCITY DISTRIBUTION IN A COUETTE–POISEUILLE FLOW



a) FIND $u(y)$

ASSUMPTIONS: STEADY STATE, INCOMPRESSIBLE, FULLY DEVELOPED, ONLY FLOW IN X-DIRECTION

CONTINUITY EQUATION (Eqn 4.4)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$v = 0$, $w = 0$ (ONLY FLOW IN X-DIRECTION)

$$\Rightarrow \frac{\partial u}{\partial x} = 0$$

MOMENTUM EQUATION (Eqn 4.38)

$$\rho\left(\underbrace{\cancel{\frac{\partial u}{\partial t}}}_{\text{steady state}} + \underbrace{\cancel{u\frac{\partial u}{\partial x}}}_{\text{continuity}} + \cancel{v}\frac{\partial u}{\partial y} + \cancel{w}\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \underbrace{\cancel{\rho g_x}}_{\text{not important}} + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right)$$

$$\Rightarrow \quad \mu \frac{\partial^2 u}{\partial y^2} = \frac{\partial p}{\partial x}$$

Y - COMPONENT:

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = - \frac{\partial p}{\partial y} + \rho g_y +$$

$$+ \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)$$

$$\Rightarrow \quad \frac{\partial p}{\partial y} = 0$$

Z - COMPONENT $\Rightarrow \quad \frac{\partial p}{\partial z} = 0$

$$\Rightarrow \quad p = p(x) \quad \Rightarrow$$

$$\boxed{\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu} \frac{dp}{dx}}$$

INTEGRATE TWICE:

$$\frac{\partial u}{\partial y} = \frac{1}{\mu} \frac{dp}{dx} y + C_1$$

$$u(y) = \frac{1}{2\mu} \frac{dp}{dx} y^2 + C_1 y + C_2$$

BOUNDARY CONDITIONS (NO SLIP)

$$u(-h) = 0 \quad ; \quad u(h) = V$$

$$0 = \frac{1}{2\mu}\frac{dp}{dx}h^2 - C_1 h + C_2$$

$$+ \quad V = \frac{1}{2\mu}\frac{dp}{dx}h^2 + C_1 h + C_2$$

$$V = \frac{1}{\mu}\frac{dp}{dx}h^2 + 2C_2$$

$$\Rightarrow C_2 = -\frac{1}{2\mu}\frac{dp}{dx}h^2 + \frac{V}{2}$$

$$0 = \frac{1}{2\mu}\frac{dp}{dx}h^2 - C_1 h + C_2$$

$$- \quad V = \frac{1}{2\mu}\frac{dp}{dx}h^2 + C_1 h + C_2$$

$$-V = -2C_1 h$$

$$\Rightarrow C_1 = \frac{V}{2h}$$

$$\boxed{u(y) = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - h^2) + \frac{V}{2}\left(1 - \frac{y}{h}\right)}$$

b) FIND THE VELOCITY $V = V(h, \mu, dp/dx)$ SUCH THAT THE WALL SHEAR STRESS AT THE UPPER WALL IS ZERO.

$$\tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=h}$$

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{dp}{dx}y + \frac{V}{2h}$$

$$\tau_w = \frac{dp}{dx}h + \frac{V\mu}{2h}$$

$$\tau_w = 0 \Rightarrow \frac{V\mu}{2h} = -\frac{dp}{dx}h \Rightarrow$$

$$\boxed{V = \frac{-2h^2}{\mu}\frac{dp}{dx}}$$

c) CALCULATE THE VORTICITY AT THE CENTER $(y=0)$ FOR THE CONDITIONS IN TASK b.

$$\zeta = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)e_x + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)e_y +$$

$$+ \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)e_z \Rightarrow$$

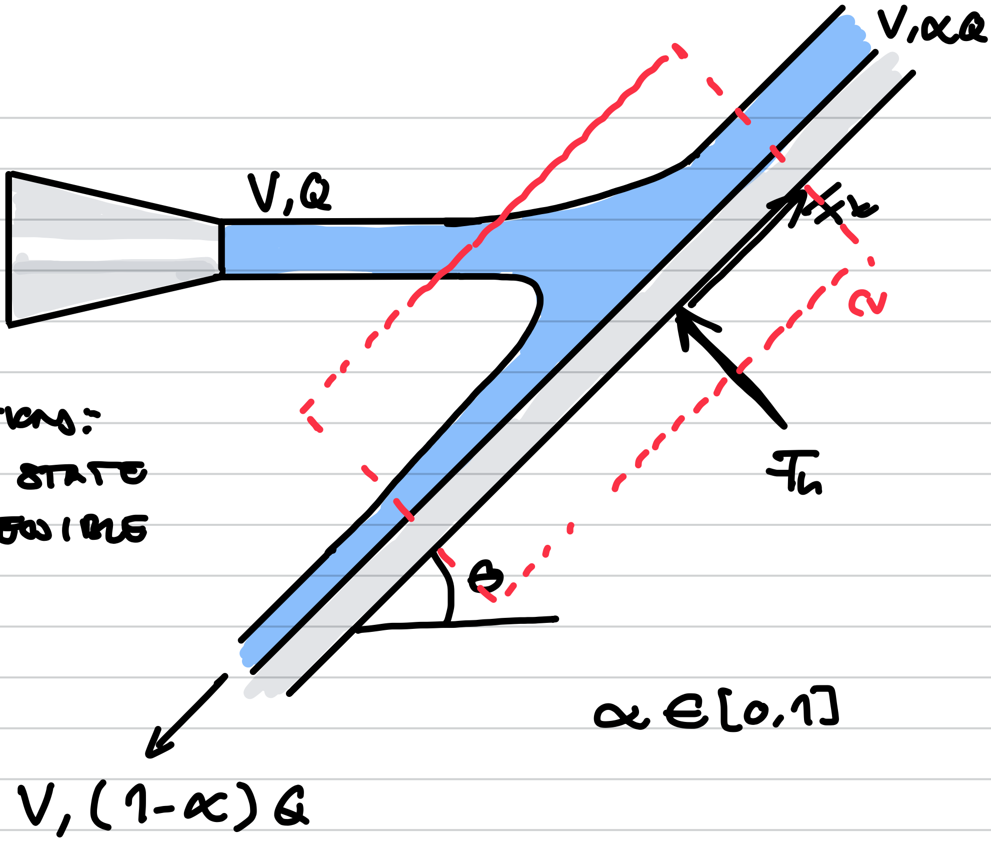
$$\Rightarrow \zeta = \left(0,\ 0,\ -\frac{\partial u}{\partial y}\right)$$

$$\left.\begin{aligned} \frac{\partial u}{\partial y} &= \frac{1}{\mu}\frac{dp}{dx}\, y + \frac{V}{2h} \\ V &= \frac{-2h^2}{\mu}\frac{dp}{dx} \\ y &= 0 \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \left.\frac{\partial u}{\partial y}\right|_{y=0} = -\frac{h}{\mu}\frac{dp}{dx}$$

$$\Rightarrow \boxed{\zeta = \left(0,\ 0,\ \frac{h}{\mu}\frac{dp}{dx}\right)}$$

P2



ASSUMPTIONS:
STEADY STATE
INCOMPRESSIBLE

A JET IS DEFLECTED BY AN INCLINED PLATE

THE JET IS DIVIDED INTO TWO JETS

FIND $\alpha = \alpha(\theta)$ SUCH THAT $F_t = 0$

CONSERVATION OF LINEAR MOMENTUM

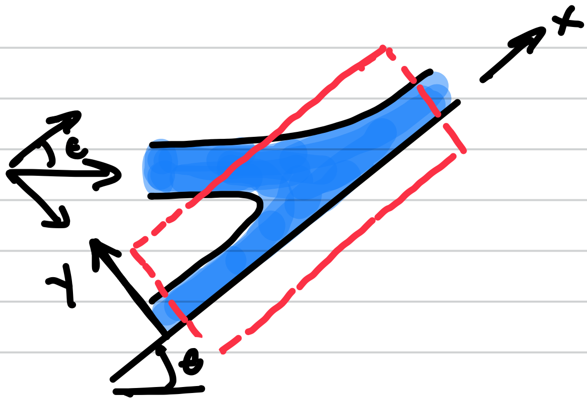
(Eqn 3.40)

$$\Sigma \mathbf{F} = \underbrace{\frac{d}{dt}\left(\int_{CV} \mathbf{V} \rho \, dV\right)}_{=0} + \sum_i (\dot{m}_i \mathbf{V}_i)_{out} - \sum_i (\dot{m}_i \mathbf{V}_i)_{in}$$

$$\dot{m} = \rho Q$$

SET UP BALANCE IN A COORDINATE SYSTEM

ALIGNED WITH THE PLATE.



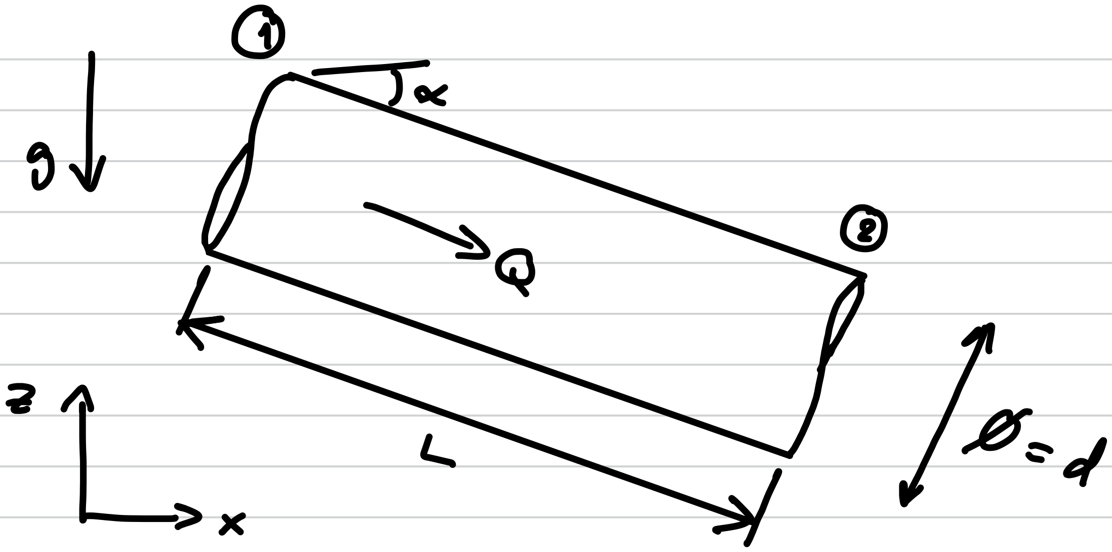
$$F_t = \rho \alpha Q V - \rho (1-\alpha) Q V - \rho Q V \cos\theta$$

$$F_t = 0 \Rightarrow \rho Q V \left[2\alpha - 1 - \cos\theta \right] = 0$$

$$\Rightarrow 2\alpha = \cos\theta + 1 \Rightarrow$$

$$\boxed{\alpha = \frac{\cos\theta + 1}{2}}$$

P3



GIVEN:

WATER @ 20°C ⇒ $\rho = 998\ kg/m^3$

$\eta = 0{,}001\ kg/ms$

$Q = 20.0\ m^3/h$

$d = 0{,}05\,m$

$\alpha = 8°$

$P_1 = 420\ kPa$

$Z_1 = 12.0\ m$

$P_2 = 250\ kPa$

$Z_2 = 3.0\ m$

a) ESTIMATE THE AVERAGE SURFACE ROUGHNESS ϵ

$$Q = 20\,m^3/h = \frac{20}{3600} = 0.00556\,m^3/s$$

ASSUMING INCOMPRESSIBLE, STEADY-STATE FLOW

THE AVERAGE VELOCITY IS :

$$V = \frac{Q}{A} = \frac{4Q}{\pi d^2} = 2.83 \text{ m/s}$$

PIPE LENGTH : $L = \dfrac{\Delta z}{\sin \alpha} = 64.7 \text{ m}$

NOW, USING THE STEADY-STATE FLOW ENERGY EQUATION (EQ. 3.73)

(NO PUMPS OR TURBINES)

$$\left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_1 = \left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right)_2 + h_f$$

INCOMPRESSIBLE & STEADY STATE $\Rightarrow V_1 = V_2 = V \Rightarrow$

$$\left(\frac{p}{\rho g} + z\right)_1 = \left(\frac{p}{\rho g} + z\right)_2 + h_f$$

$\Rightarrow h_f = 26.36 \text{ m}$

FROM THE DEFINITION OF THE FRICTION FACTOR WE HAVE THAT

$$h_f = f \frac{L}{d} \frac{V^2}{2g} \Rightarrow f = 0.05$$

THE REYNOLDS NUMBER IS

$$Re_p = \frac{\rho V d}{\mu} \approx 1{,}9 \cdot 10^5$$

FROM THE MOODY CHART WE CAN NOW GET AN ESTIMATE OF ε/d

$$\varepsilon/d \simeq 0{,}021 \Rightarrow \varepsilon = 1{,}05 \text{ mm}$$

b) ESTIMATE THE CHANGE IN HEAD LOSS IF THE PIPE WAS SMOOTH (SAME FLOW RATE)

WITH THE FLOW RATE KEPT THE SAME, THE HEAD LOSS IS A LINEAR FUNCTION OF THE FRICTION FACTOR

$$h_f = f \frac{L}{d} \frac{V^2}{2g} = f \cdot \text{const}$$

FOR THE ROUGH PIPE, $f = 0{,}05$

FOR A SMOOTH PIPE AT THE SAME REYNOLDS NUMBER, THE MOODY CHART GIVES

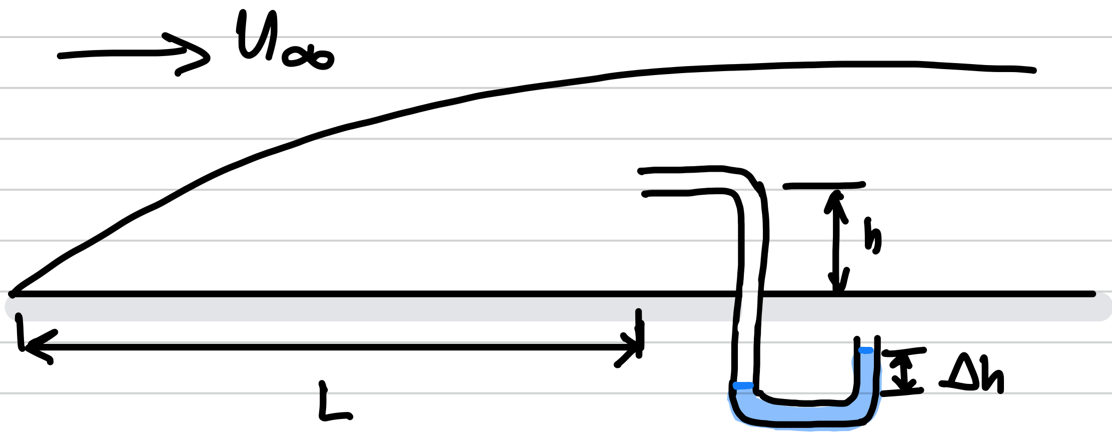
$$f = 0{,}017$$

THUS THE CHANGE IN HEAD LOSS IS:

$$\frac{h_{f\,(rough)} - h_{f\,(smooth)}}{h_{f\,(rough)}} =$$

$$= \frac{f_{rough} - f_{smooth}}{f_{rough}} \simeq 66\%$$

P_4



GIVEN:

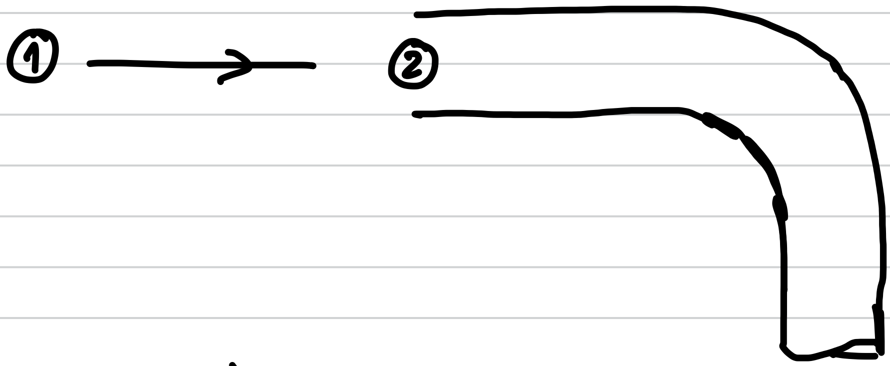
$L = 0.5\ m$
$h = 0.002\ m$
$U_\infty = 15\ m/s$

FLUID: AIR @ 20°C $\Rightarrow$ $\rho = 1.2\ kg/m^3$
$\mu = 1.8 \cdot 10^{-5}\ kg/ms$

MANOMETER FLUID: WATER @ 20°C $\Rightarrow$
$\rho_w = 998\ kg/m^3$, $\mu_w = 10^{-3}\ kg/ms$

a) CALCULATE Δh IF THE BOUNDARY LAYER IS LAMINAR.

SET UP THE BERNOULLI EQUATION FOR THE DECELERATION IN FRONT OF THE STAGNATION TUBE.



(EQN 3.59)

$$P_1 + \rho\frac{V_1^2}{2} + \rho g z_1 = P_2 + \rho\frac{V_2^2}{2} + \rho g z_2$$

$z_1 = z_2$

$V_2 = 0$

$$\Rightarrow \quad \rho\frac{V_1^2}{2} = P_2 - P_1$$

$$V_1^2 = 2\,\frac{P_2 - P_1}{\rho} = 2\,\frac{\Delta P}{\rho} \qquad (1)$$

THE MANOMETER READING WILL BE PROPORTIONAL TO ΔP

$$\Delta p = \rho_w g \Delta h \qquad (2)$$

(2) IN (1) $\Rightarrow$ $$V_1^2 = \frac{2\rho_w g \Delta h}{\rho} \qquad (3)$$

TO CALCULATE Δh, WE NEED A VALUE FOR V_1

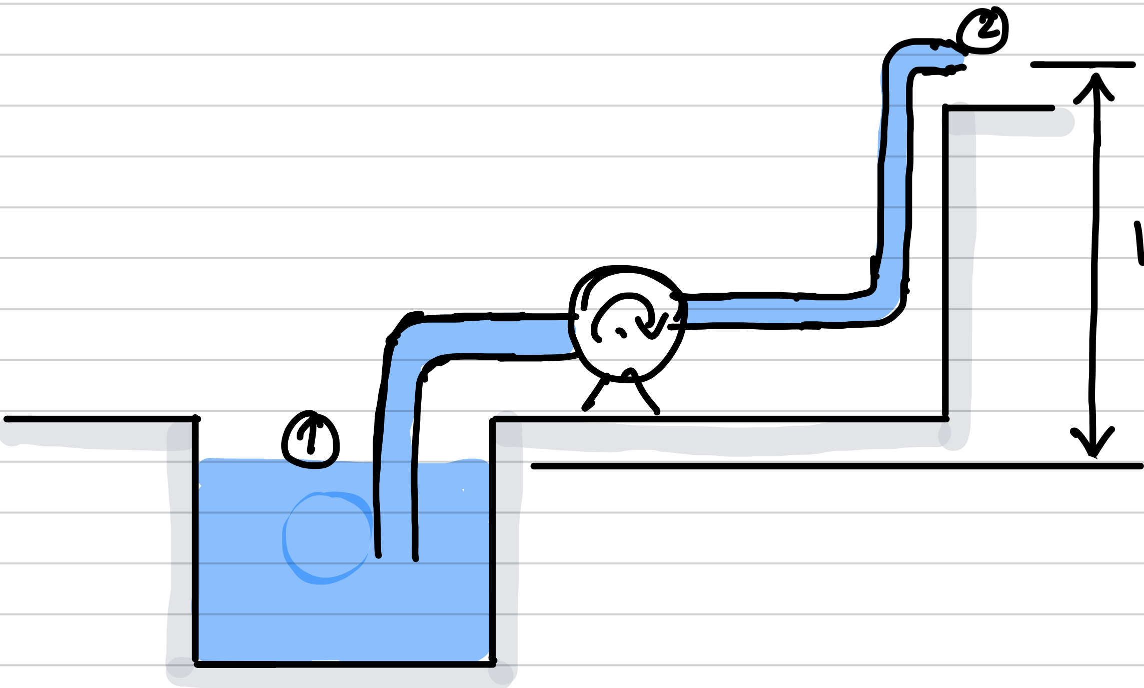
SINCE THE BOUNDARY LAYER IS LAMINAR, WE CAN USE THE BLASIUS VELOCITY PROFILE.

$$\eta = y\sqrt{\frac{u_\infty}{\nu x}} = h\sqrt{\frac{u_\infty}{\nu L}} \approx 2.83$$

$$\Rightarrow \frac{V_1}{u_\infty} \approx 0.8126 \qquad (4)$$

(4) IN (8) $\Rightarrow$ $\Delta h = 9.1\ m$

P5



GIVEN:

$D_1 = 0.3\,m$ $\qquad V_2 = 5\,m/s$

$D_2 = 0.15\,m$

$h = 10.0\,m$

FRICTION LOSSES : $\frac{V_2^2}{g}$

TURBULENT FLOW IN ALL PIPES

a) CALCULATE THE EFFICIENCY IF 20 kW IS DELIVERED TO THE PUMP

ENERGY EQUATION (EQ 3.78)

$$\left(\frac{p}{\rho g} + \frac{\alpha V^2}{2g} + z\right)_1 = \left(\frac{p}{\rho g} + \frac{\alpha V^2}{2g} + z\right)_2 + h_t - h_p + h_f$$

TURBULENT FLOW $\Rightarrow \alpha_1 = \alpha_2 \approx 1.0$

$P_1 = P_2 = P_{atm}$

$V_1 \approx 0$ (LARGE TANK)

$z_2 - z_1 = h$

$h_t = 0$

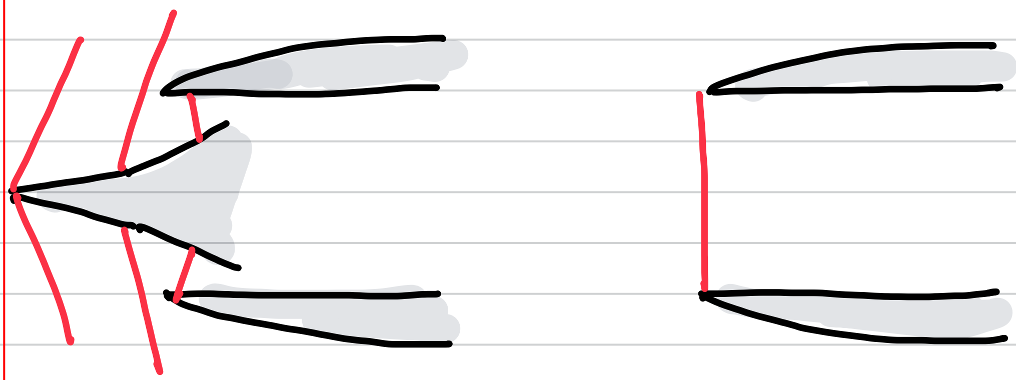
$h_f = \dfrac{V_2^2}{g}$

$$\Rightarrow h_p = h + \frac{V_2^2}{2g} + \frac{V_2^2}{g} = h + \frac{3V_2^2}{2g} = 13.8\,m$$

$$P_p = \rho g h_p Q = 11957\ W$$

$$\eta = \frac{11957}{20000} = 0{,}60$$

P6

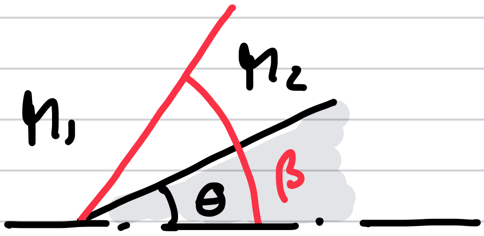


a) FIND THE LOWEST MACH NUMBER FOR WHICH THE CONE INLET WILL FUNCTION AS INTENDED.

THE ANSWER WOULD BE THE LOWEST MACH NUMBER FOR WHICH A 8° DEFLECTION IS POSSIBLE AT THE SECOND FLOW DEFLECTION

SO, THE θ-β-M-RELATION GIVES THAT THE LOWEST MACH NUMBER FOR WHICH AN 8° DEFLECTION IS POSSIBLE IS $M = 1.4$

THUS THE MACH NUMBER AHEAD OF THE SECOND SHOCK MUST BE 1.4



To find M_1 such that $M_2 = 1.4$ an iterative approach is required..

#1. Guess M_1

#2. Find β using the θ-β-M relation

#3. Calculate M_{n1} (eqn 9.82)

$$M_{n1} = M_1 \sin\beta$$

#4. Calculate M_{n2} using the normal-shock relation (eqn 9.57)

$$M_{n2}^2 = \frac{(\gamma - 1)M_{n1}^2 + 2}{2\gamma M_{n1}^2 + 2}$$

#5. Calculate M_2 (eqn 9.82)

$$M_{n2} = M_2 \sin(\beta - \theta)$$

Repeat #2 – #5 until $M_2 = 1.4$

$\Rightarrow M_1 = 1.68$

NOTE!

THE WAY THE PROBLEM IS GIVEN, THE SOLUTION ABOVE IS CORRECT. HOWEVER, THIS WAS NOT THE INTENTION. THE TASK WAS SUPPOSED TO BE TO FIND THE MINIMUM MACH NUMBER FOR WHICH A FLOW DEFLECTION OF $8°$ IS POSSIBLE AT THE WHEN THE FLOW MEETS THE CONE $\Rightarrow$

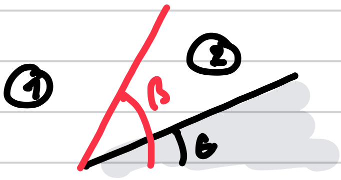
$$M_1 = 1.4$$

THE CORRECT SOLUTION IS A BIT TO MUCH FOR THE 2.0p AWARDED...

b) CALCULATE THE MACH NUMBER OF THE FLOW ENTERING THE ENGINE FOR THE TWO CASES IF THE FREESTREAM MACH NUMBER IS 3.0.

1) <u>TWO OBLIQUE SHOCKS FOLLOWED BY</u> A <u>NORMAL SHOCK.</u>

OBLIQUE SHOCK 1:



$M_1 = 3.0$

$\theta - \beta - M$ RELATION WITH $M_1 = 3.0$

AND $\theta = 8^\circ \Rightarrow \beta = 25.6^\circ$

EQN 9.82

$M_{n_1} = M_1 \sin\beta$

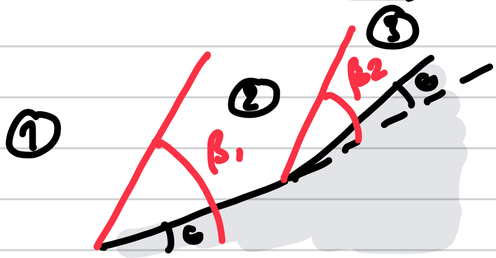
EQN 9.57

$$M_{n_2}^2 = \frac{(\gamma - 1)M_{n_1}^2 + 2}{2\gamma M_{n_1}^2 - (\gamma - 1)}$$

EQN 9.82

$M_{n_2} = M_2 \sin(\beta - \theta) \Rightarrow \underline{M_2 = 2.60}$

OBLIQUE SHOCK 2:



SAME APPROACH AS FOR THE FIRST SHOCK

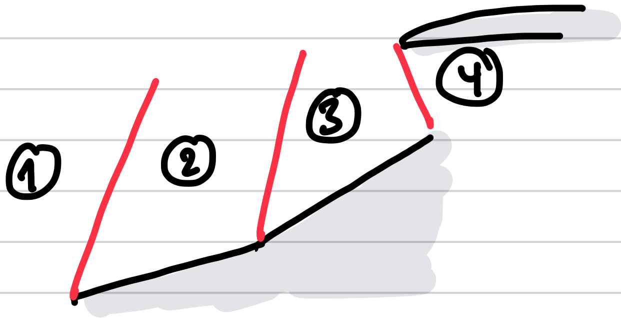
WITH UPSTREAM MACH NUMBER $M_2 = 2.60$

$\theta - \beta - M \Rightarrow \beta_2 = 28.9^\circ$

EQN 9.82 + EQN 9.57 $\Rightarrow M_3 = 2.26$

NORMAL SHOCK

NORMAL SHOCK WITH THE UPSTREAM MACH NUMBER $M_3 = 2.26$

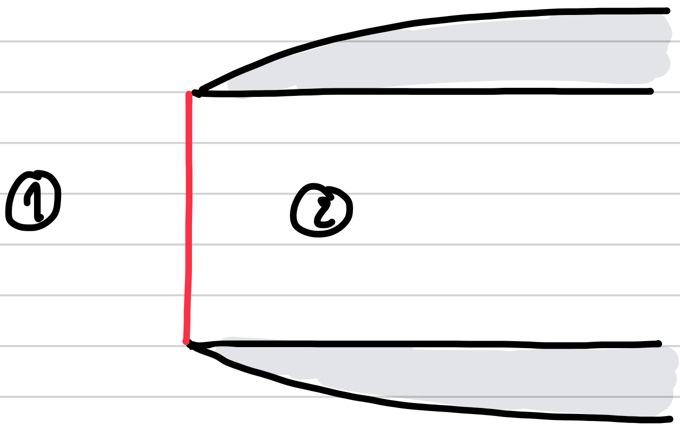


(EQN 9.57)

$$M_4^2 = \frac{(\gamma-1)M_3^2 + 2}{2\gamma M_3^2 - (\gamma - 1)} \Rightarrow \underline{M_4 = 0.54}$$

2)

NORMAL SHOCK INTAKE



EQN (9.57)

$$M_2^2 = \frac{(\gamma-1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma - 1)} \Rightarrow \underline{M_2 = 0.48}$$