# Fluid Mechanics MTF053

Study Guide

Division of Fluid Dynamics
Department of Mechanics and Maritime Sciences
Chalmers University of Technology

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# 1 Introduction (Basic Concepts)

Answers to the questions can be found in the course book (*F. M. White Fluid Mechanics 8$^{th}$ ed.*) and in the complementary course material

MTF053_C01.pdf

MTF053_Formulas-Tables-and-Graphs.pdf

## Theory Questions

1.1 Explain how a fluid differs from a solid (see Fig. 1.1 below). How does a fluid element and solid element react under the presence of shear forces?

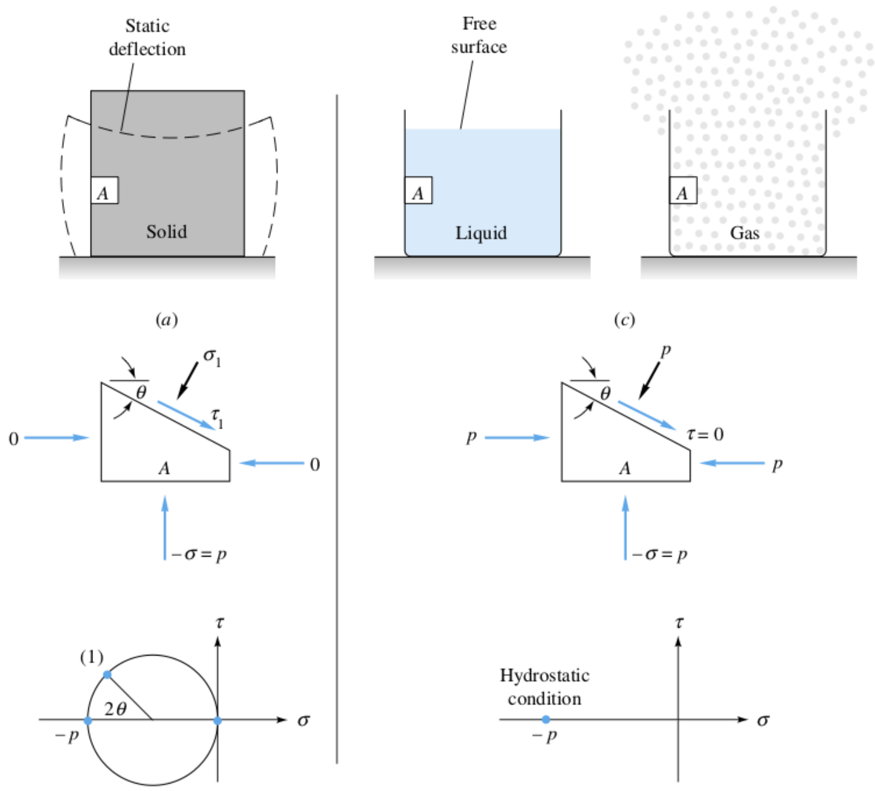


Figure 1.1: Fluid versus solid

1.2 The continuum concept is very central in fluid mechanics - explain this concept.

1.3 Primary and secondary dimensions:

(a) What does *primary dimension* mean?

(b) Which are the primary dimensions used in fluid mechanics?

(c) Give examples of secondary dimensions.

(d) What does a *dimensional homogenous equation* imply?

1.4 Explain the difference between Eulerian and Lagrangian frame of reference

1.5 Show that if the shear stress is proportional to the fluid element strain rate $\delta\theta/\delta t$, it is also proportional to the velocity gradient $du/dy$

1.6 What is the viscosity of a fluid?

1.7 For a viscous flow, what is the fluid velocity at a wall? What is this boundary condition called?

1.8 What does it mean that a fluid is Newtonian?

1.9 How is the Reynolds number defined? Explain in words what the Reynolds number is.

1.10 Explain the following concepts:

   (a) Steady-state flow
   (b) Unsteady flow
   (c) Incompressible flow
   (d) Inviscid flow
   (e) Turbulent flow

1.11 What is the vapor pressure of a fluid? Explain why cavitation may occur if the pressure becomes low enough in a fluid flow

1.12 Explain the difference between *streamline*, *pathline*, and *streakline*. Under what circumstances do these three line types coincide in a fluid flow?

1.13 The energy per unit mass $e$ can be expressed as follows:

$$e = \hat{u} + \frac{1}{2}V^2 + gz$$

The three components on the right-hand side represents different forms of energy what does each of the terms represent physically?

1.14 How does the fluid viscosity vary with temperature in liquids and gases, respectively.

## Recommended Problems

## 2 Pressure Distribution in a Fluid

Answers to the questions can be found in the course book (*F. M. White Fluid Mechanics 8$^{th}$ ed.*) and in the complementary course material

MTF053_C02.pdf

MTF053_Formulas-Tables-and-Graphs.pdf

### Theory Questions

2.1 If you would hold your thumb tightly over the upper end of straw filled with water and held vertically, would the water not pour out - explain why. Is there a limit for how heigh a water pilar can be without pouring out? If so, how heigh can it be?

2.2 Show that the pressure difference in a fluid can be calculated as $\Delta p = -\rho g \Delta z$ starting from Newton's second law ($F = ma$), the fluid density can be assumed to be constant.

2.3 Show that the force balancing gravity in a fluid at rest is caused by the pressure gradient.

2.4 Show that the normal of a constant-pressure surface must be aligned with the gravity vector in a fluid at rest.

2.5 How does the hydrodynamic pressure distribution differ in liquids and gases?

2.6 How can we make use of Pascal's law when analysing manometer tubes?

2.7 Describe the implication of buoyancy for immersed bodies and floating bodies, respectively.

2.8 How is the buoyancy of a body immersed in a fluid calculated? Show how this relation is obtained.

### Recommended Problems

# 3 Integral Relations for a Control Volume

Answers to the questions can be found in the course book (*F. M. White Fluid Mechanics 8$^{th}$ ed.*) and in the complementary course material

MTF053_C03.pdf

MTF053_Formulas-Tables-and-Graphs.pdf

## Theory Questions

3.1 Rewrite Newton's second law using the momentum of a system. What is the name of this relation?

3.2 Define the angular momentum of a system

3.3 Volume flow and massflow:

  (a) Show how the volume flow $Q$ and massflow $\dot{m}$ over a control volume surface can be calculated in a general way

  (b) How are volume flow $Q$ and massflow $\dot{m}$ related if the density is constant

  (c) How is the volume-averaged mean velocity over a surface defined for a fluid with constant density?

3.4 Give examples of when it is appropriate to use fixed control volume, moving control volume, and deformable control volume, respectively.

3.5 Reynolds transport theorem:

  (a) In Reynolds transport theorem, $B$ and $\beta$ denotes extensive and intensive properties respectively. Explain the difference between $B$ and $\beta$.

  (b) If an intensive property, $\beta$, is known, how is the corresponding extensive property, $B$, calculated?

  (c) Give two examples of intensive and extensive properties

  (d) Explain the physical meaning of each of the terms in Reynolds transport theorem:

$$\frac{d}{dt}\left(B_{syst}\right) = \frac{d}{dt}\left(\int_{cv} \beta \rho d\mathcal{V}\right) + \int_{cs} \beta \rho \left(\mathbf{V}_r \cdot \mathbf{n}\right) dA \qquad (3.16)$$

  (e) How can the generic form of Reynolds transport theorem (above) be be simplified for a fix control volume?

  (f) What does it mean that inlets and outlets are one-dimensional?

  (g) How can we simplify Reynolds transport theorem for one-dimensional inlets and outlets?

3.6 The continuity equation:

  (a) Derive the continuity equation on integral form for a fixed control volume using Reynolds transport theorem

(b) Explain the physical meaning of each of the terms in the continuity equation on integral form

(c) How can we simplify the continuity equation on integral form under the following circumstances (assuming that the control volume is fixed)?

$$\int_{cv} \frac{\partial \rho}{\partial t} d\mathcal{V} + \int_{cs} \rho \left(\mathbf{V} \cdot \mathbf{n}\right) dA \tag{3.21}$$

i. inlets and outlets can be assumed to be one-dimensional

ii. steady-state flow

iii. incompressible unsteady flow

3.7 The momentum equation:

(a) Derive the momentum equation on integral form starting from Reynolds transport theorem

(b) Explain the physical meaning of each of the terms in the momentum equation on integral form

(c) How can we simplify the momentum equation on integral form under the following circumstances?

$$\frac{d}{dt}\left(m\mathbf{V}\right)_{syst} = \sum \mathbf{F} = \frac{d}{dt}\left(\int_{cv} \mathbf{V}\rho d\mathcal{V}\right) + \int_{cs} \mathbf{V}\rho \left(\mathbf{V}_r \cdot \mathbf{n}\right) dA \tag{3.35}$$

i. fixed control volume

ii. fixed control volume and one-dimensional inlets and outlets

iii. fixed control volume, one-dimensional inlets and outlets, and steady-state flow

3.8 What is gauge pressure?

3.9 The Bernoulli equation:

(a) Derive the Bernoulli equation for steady-state, incompressible flow along a streamline

$$\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho} + \frac{1}{2}V_2^2 + gz_2 = const \tag{3.53}$$

(b) What assumptions are made in the derivation of the Bernoulli equation?

3.10 The energy equation:

(a) What does $Q$ and $W$ in the energy equation represent?

(b) The energy per unit mass, $e$, and the work are both divided into parts, what parts are the terms divided into?

(c) The Bernoulli equation is a simplified form of the energy equation.

$$\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + gz_1 = \frac{p_2}{\rho} + \frac{1}{2}V_2^2 + gz_2 = const \tag{3.53}$$

In what ways are the Bernoulli equation above more limited than the energy equation on the form given below?

$$\left(\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1\right) = \left(\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2\right) + \frac{\hat{u}_2 - \hat{u}_1 - q}{g} \tag{3.72}$$

(d) Why is the kinetic energy korrection factor introduced?

(e) Show that the kinetic energy correction factor is $\alpha = 2.0$ for laminar, incompressible pipe flow.

3.11 Explain how to measure velocity using a Prandtl tube (Pitot-static tube) and derive the relation needed to estimate the velocity

3.12 Explain how a venturi meter works and derive the relation needed to estimate the velocity

## Recommended Problems

# 4 Differential Relations for Fluid Flow

Answers to the questions can be found in the course book (*F. M. White Fluid Mechanics 8$^{th}$ ed.*) and in the complementary course material

MTF053_C04.pdf

MTF053_Formulas-Tables-and-Graphs.pdf

## Theory Questions

4.1 What is the difference between the control volume in an integral formulation and a differential formulation of the governing equations?

4.2 Rewrite the acceleration using the chain rule and group the terms into local acceleration terms and convective acceleration terms

4.3 Explain the physical meaning of the local acceleration term and the convective acceleration term

4.4 The continuity equation on differential form:

(a) Derive the continuity equation on differential form starting from the integral form for a fixed control volume

$$\int_{cv} \frac{\partial \rho}{\partial t} d\mathcal{V} + \sum_i (\rho_i A_i V_i)_{out} - \sum_i (\rho_i A_i V_i)_{in} = 0 \tag{3.22}$$

and let the size of the control volume reduce to infinitesimal size

(b) How can we simplify the continuity equation on differential form under the following circumstances?

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0 \tag{4.4}$$

i. steady-state flow

ii. incompressible flow

4.5 The momentum equation on differential form:

(a) Derive the momentum equation on differential form starting from the integral form

$$\sum \mathbf{F} = \frac{d}{dt}\left(\int_{cv} \mathbf{V} \rho d\mathcal{V}\right) + \sum (\dot{m}_i \mathbf{V}_i)_{out} - \sum (\dot{m}_i \mathbf{V}_i)_{in} \tag{3.40}$$

and let the size of the control volume reduce to infinitesimal size

(b) A fluid element is subjected to both body forces and surface forces. Give an example of a body force and name the two surface forces.

(c) Make a sketch of a control volume used for the derivation of the governing equations on differential form and indicate the stresses acting on the surfaces of the cube in one selected direction. Write down the resulting surface force in the selected direction.

(d) What generates the stresses denoted $\tau_{ij}$. What is the difference between $\tau_{ij}$ and $\sigma_{ij}$.

4.6 The $x$-component of the Navier-Sokes equations reads:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = g_x - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \tag{4.38}$$

What is the physical meaning of each of the terms in the equation?

4.7 Under what circumstances can the general formulation of the momentum equation be reduced to the Navier-Stokes equation?

4.8 If the integral angular momentum equation is applied to a control volume and the control volume is reduced to infinitesimal size, one will see a special feature of the stress tensor $\tau_{ij}$. What feature is that?

4.9 What is the physical meaning of the terms in the simplified energy equation?

$$\rho C_v \frac{dT}{dt} = k\nabla^2 T + \Phi \tag{4.53}$$

4.10 Simplify the following system of equations for incompressible flow. Use the relation between the stress tensor and velocity derivatives applicable for Newtonian fluids.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \tag{4.56}$$

$$\rho \frac{d\mathbf{V}}{dt} = \rho \mathbf{g} - \nabla p + \nabla \cdot \tau_{ij} \tag{4.57}$$

$$\rho \frac{d\hat{u}}{dt} + p\left(\nabla \cdot \mathbf{V}\right) = \nabla \cdot (k\nabla T) + \Phi \tag{4.58}$$

What unknown properties can be calculated and how many equations do we have?

4.11 What boundary conditions are often used for velocity and temperature at a solid wall?

4.12 2D incompressible duct flow:

(a) Simplify the continuity equation and Navier-Stokes equations without pressure gradient driven by a moving upper wall, i.e., Couette flows.

(b) Simplify the continuity equation and Navier-Stokes equations for pressure-driven flow, i.e., Poiseuille flows.

(c) Show that the Couette flow is rotational by analyzing a small fluid element.

4.13 How is vorticity defined?

## Recommended Problems

## 5 Dimensional Analysis and Similarity

Answers to the questions can be found in the course book (*F. M. White Fluid Mechanics 8$^{th}$ ed.*) and in the complementary course material

MTF053_C05.pdf

MTF053_Dimensional-Analysis-and-Similarity.pdf

MTF053_Formulas-Tables-and-Graphs.pdf

### Theory Questions

5.1 If you are going to do an experimental investigation of a problem including a number of important physical variables, why is it beneficial to divide the variables into non-dimensional groups?

5.2 Explain Buckinghams Π-theorem.

5.3 Rewrite the continuity equation and the $x$-component of the momentum equation in non-dimensional form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{4.12}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = g_x - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \tag{4.38}$$

5.4 The flow resistance $F_D$ of a body immersed in a flow can be divided into pressure forces and friction forces (MTF053_Dimensional-Analysis-and-Similarity.pdf).

(a) Show using Reynolds similarity that the pressure forces can be written as

$$F_{D_p} = C_{D_p}(Re)A_p\frac{\rho U^2}{2}$$

where the drag coefficient $C_{D_p}$ is a function of the Reynolds number only and $A_p$ is the projected area of the body.

(b) Show using Reynolds similarity that the pressure forces can be written as

$$F_{D_f} = C_{D_f}(Re)A_p\frac{\rho U^2}{2}$$

where the drag coefficient $C_{D_f}$ is a function of the Reynolds number only and $A_p$ is the projected area of the body.

5.5 Explain the concepts *geometric similarity*, *kinematic similarity*, and *dynamic similarity*

### Recommended Problems

# 6 Viscous Flow In Ducts

Answers to the questions can be found in the course book (*F. M. White Fluid Mechanics 8$^{th}$ ed.*) and in the complementary course material

MTF053_C06.pdf

MTF053_Turbulence.pdf

MTF053_Formulas-Tables-and-Graphs.pdf

## Theory Questions

6.1 How do we usually define the Reynolds number for pipe flows?

6.2 What does critical Reynolds number mean for a pipe flow?

6.3 What does entrance length mean? How is the flow velocity profile changed during the entrance length?

6.4 What does fully developed pipe flow mean?

6.5 Give three examples of sources of local losses in a pipe system

6.6 Start from Bernoullis extended equation and the momentum equation for a pipe with the length $L$ and diameter $D = 2R$ and show that

$$\Delta p_f = 2\tau_w \frac{L}{R}$$

6.7 The Darcy friction factor is defined as

$$f = \frac{8\tau_w}{\rho V^2} \tag{6.11}$$

Rewrite the friction loss from the previous question using the Darcy friction factor

6.8 For laminar pipe flows, show that:

$$f = \frac{64}{Re_D} \tag{6.13}$$

6.9 What parameters effects the magnitude of $f$?

6.10 For fully developed laminar pipe flow, the velocity profile can be expressed as

$$u = u_{max}\left(1 - \frac{r^2}{R^2}\right)$$

Show that the average velocity in fully developed laminar pipe flow is half the maximum velocity.

6.11 Compare the velocity profiles for fully developed laminar and turbulent flow, which of the flows gives the highest wall shear stress for a given mass flow?

6.12 For fully developed turbulent pipe flow, the mean velocity profile can be approximated using the 1/7-rule

$$u = u_{max}\left(\frac{r}{R}\right)^{1/7}$$

Why should this not be used directly for the calculation of wall shear stress?

6.13 Give three characteristics of turbulent flow

6.14 Turbulent flow is dissipative. What does that mean?

6.15 What defines the largest and smallest length scales in a turbulent flow?

6.16 Explain the concept of Reynolds decomposition

6.17 Why do one often want to use time-averaged equations when studying turbulent flow while that is not the case for laminar flows?

6.18 Explain the closure problem related to the Reynolds-averaged flow equations

6.19 In the Reynolds decomposition, the velocity components and pressure are divided into an average part and a fluctuating part as for example

$$u = \bar{u} + u' \tag{6.16}$$

Define the time average and show that the time average of the fluctuating component is identically equal to zero.

6.20 How is the intensity of the fluctuating velocity component specified?

6.21 Derive the continuity equation for the time-averaged velocity field for incompressible turbulent flow starting from the general continuity equation on differential form

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} \tag{4.4}$$

6.22 Derive the $x$-component of the Navier-Stokes equation for turbulent flow. Explain the physical meaning of each of the terms in the equation.

6.23 Show that the term $-\rho\overline{u'v'}$ can be interpreted as a shear stress

6.24 How can the turbulent shear stress be related to the mean flow using the turbulence viscosity $\mu_t$? What is this assumption called?

6.25 Define the friction velocity $u^*$

6.26 Derive the average velocity distribution in the viscous sublayer starting from

$$\overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{v}}{\partial y} = \frac{1}{\rho}\frac{\partial \tau}{\partial y}$$

6.27 How does the turbulence viscosity $\mu_t$ compare to the fluid viscosity $\mu$ in the viscous sublayer and in the fully turbulent region, respectively?

6.28 How does the total shear stress vary with distance from the wall in the viscous sublayer and in the fully turbulent region, respectively?

6.29 How are the velocity profiles characterized mathematically in the viscous sublayer and in the fully turbulent region, respectively?

6.30 Make a schematic representation of the non-dimensional velocity $u^+$ as a function of the non-dimensional wall distance $y^+$ for a turbulent boundary layer. The velocity profile can be divided into different regions. Name these regions.

6.31 What relation for the local average velocity is used for the derivation of the friction factor $f$ for turbulent pipe flow?

6.32 The Moody chart is an engineering tool that can be used for estimation of pressure losses in a pipe flow

(a) Why does the Moody chart not give reliable values in the Reynolds number range $2000 < Re < 4000$?

(b) How does $f$ vary with the Reynolds number in the fully turbulent regime?

(c) What is the effect of surface roughness on the friction factor?

(d) What can we say about the Moody chart when it comes to accuracy?

6.33 How is the hydraulic diameter defined and how can it be used for calculation of the friction factor $f$ for laminar and turbulent flow in non-circular ducts?

6.34 Define the loss coefficient $K$

## Recommended Problems

# 7 Flow Past Immersed Bodies

Answers to the questions can be found in the course book (*F. M. White Fluid Mechanics 8$^{th}$ ed.*) and in the complementary course material

MTF053_C07.pdf

MTF053_Equation-for-Boundary-Layer-Flows.pdf

MTF053_Turbulence.pdf

MTF053_Formulas-Tables-and-Graphs.pdf

## Theory Questions

7.1 Make a schematic sketch of the velocity profile in a flat-plate boundary layer showing the extent of the viscous region at $Re_L = 10$ and $Re_L = 10^7$, respectively. How does the extent of the viscous region $\delta$ relate to the characteristic length, $L$, for the two Reynolds numbers? Indicate what parts of the boundary layer that is laminar and turbulent respectively as well as the transition region.

7.2 Make a schematic sketch of the flow over a cylinder at $Re_D = 10^5$. Indicate the stagnation point, separation points and the wake region.

7.3 The boundary layer equations are derived from the continuity equation and the Navier-Stokes equation on non-dimensional form (MTF053_Equation-for-Boundary-Layer-Flows.pdf)

(a) Define the non-dimensional properties $x^*$, $u^*$, and $p^*$ and explain the included properties.

(b) Give the order of magnitude of $x^*$, $u^*$, and $p^*$ that holds for larger part of the boundary layer. The estimation of magnitude is made using the following terms $\partial u^*/\partial x^*$, $\partial u^*/\partial y^*$, $\partial^2 u^*/\partial {x^*}^2$, and $\nu^*$. Use relevant figures to justify the estimation of the derivatives.

(c) What assumption is made to be able to derive the boundary layer equations?

(d) Show that the static pressure is independent of the distance from the wall in a laminar two-dimensional boundary layer. Use the non-dimensional form of the $y$-component of the Navier-Stokes equations

$$u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = -\frac{\partial p^*}{\partial y^*} + \frac{1}{Re}\left(\frac{\partial^2 v^*}{\partial {x^*}^2} + \frac{\partial^2 v^*}{\partial {y^*}^2}\right)$$

The following size estimates can be assumed to hold (does not need to be proved): $u^* \sim 1$, $v^* \sim \delta^*$, and $Re \sim {\delta^*}^{-2}$

*note: here $\delta^*$ is the non-dimensional boundary layer thickness, not the displacement thickness*

(e) How does the laminar and turbulent boundary layer equations differ? How does this effect the possibility to solve the equations?

7.4 Show using the Bernoulli equation (with small differences in elevation) that the pressure gradient for the flow over a flat plate depends on the density, the derivative of the freestream velocity, and the freestream velocity. Justify the use of the Bernoulli equation for this purpose.

7.5 Describe in words the steps taken in Blasius solution of the boundary layer equations for laminar flow over a flat plate.

7.6 For laminar flow over a flat plate, the velocity profile is self-similar - what does that mean?

7.7 For a laminar boundary layer over a flat plate, the local wall shear stress coefficient $c_f$ can be obtained as

$$c_f = \frac{0.664}{\sqrt{Re_x}} \tag{7.25}$$

The wall shear stress coefficient, $c_f$, is related to the wall shear stress, $\tau_w$, and the free stream velocity, $U$, as

$$c_f = \frac{2\tau_w}{\rho U^2} \tag{7.10}$$

Show how to obtain the total friction force (the flat plate drag), $D$, for one side of a flat plate with the length $L$. Derive an expression for the drag force on non-dimensional form using the drag coefficient $C_D$.

7.8 Name two alternative ways to measure the boundary layer thickness than $\delta$. How can these measures be interpreted physically?

7.9 Derive von Kármán's momentum integral relation

$$\tau_w = \rho U^2 \frac{d\theta}{dx} \tag{7.5}$$

starting from

$$D(x) = \rho b U^2 \theta \tag{7.3}$$

where

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

Describe how to derive an approximate solution for the boundary layer thickness and the wall shear stress for laminar flow over a flat plate using the von Kármán momentum integral relation.

7.10 In what way is the transition location effected by (assume other properties to be constant)

(a) increased freestream velocity $U$ for a given $Re_{x,tr}$

(b) surface roughness $\varepsilon$

(c) freestream turbulence

(d) positive pressure gradient

7.11 Show how the velocity profile as well as its first and second derivative and the pressure gradient change in a boundary layer when the flow separates

7.12 The drag coefficient, $C_D$, for cylinder flow is drastically changed as the boundary layer becomes turbulent (before separating). Show schematically how $C_D$ varies with the Reynolds number, $Re_D$, and indicate the location transition to turbulence and separation.

7.13 The drag coefficient, $C_D$, can be divided into two components, which two components? What phenomena are associated with each of the two components?

7.14 Make a schematic representation of the pressure distribution around a cylinder for inviscid flow (potential flow), viscous flow with laminar and turbulent separation respectively. Explain why the pressure varies the way it does. Which of the three cases will give the lowest and highest pressure drag?

7.15 Why do dimpled golf balls have lower pressure drag than golf balls with smooth surfaces?

## Recommended Problems

# 9 Compressible Flow

Answers to the questions can be found in the course book (*F. M. White Fluid Mechanics 8$^{th}$ ed.*) and in the complementary course material

MTF053_C09.pdf

MTF053_Compressible-Flow-Hugoniot-Equation.pdf

MTF053_Formulas-Tables-and-Graphs.pdf

## Theory Questions

9.1 How is the Mach number defined?

9.2 Show by estimation of the density variation in a fluid flow that the criteria for incompressible flow is $M \ll 1$
*hint: see Eqn 4.13 - 4.17 in F. M. White Fluid Mechanics 8$^{th}$ ed.*

9.3 What is the upper Mach number often used as the limit for incompressible flow?

9.4 Name the different Mach number regimes in compressible flow and specify the corresponding Mach number ranges

9.5 What is an adiabatic process?

9.6 What is required for a process to be isentropic?

9.7 Write down the relation between internal energy and temperature and between enthalpy and temperature for

(a) an ideal gas with constant specific heats (calorically perfect gas)

(b) an ideal gas with specific heats that are functions of temperature (thermally perfect gas)

9.8 Derive an expression for the speed of sound for a generic fluid. In the derivation, an assumption regarding the pressure derivative is made – what is the assumption?

9.9 The speed of sound in a generic fluid is given by

$$a = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s} \tag{9.15}$$

From the relation above, derive an expression for the speed of sound in a calorically perfect gas (perfect gas with constant specific heats) as a function of temperature. Use the isentropic relation

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} \tag{9.9}$$

9.10 The adiabatic energy equation

(a) Write down the energy equation for steady-state adiabatic compressible flow without potential energy changes and no viscous work. The equation should be written out using enthalpy.

(b) Rewrite the energy equation from the previous question but now expressed in terms of temperature

(c) Now, derive the following relation:

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2 \tag{9.26}$$

9.11 Which of the properties $h_o$, $T_o$, $a_o$, $p_o$, and $\rho_o$ are constants in a flow if the flow is adiabatic and isentropic, respectively?

9.12 What is a critical property (such as for example the critical temperature $T^*$)?

9.13 Using the continuity equation and energy equation on differential form together with the definition of speed of sound, the following relation can be derived

$$\frac{dV}{V} = \frac{dA}{A}\frac{1}{M^2 - 1} = -\frac{dp}{\rho V^2} \tag{9.40}$$

Show, using the relation given above, how the velocity and pressure changes in a flow through a divergent or convergent duct for initially subsonic flow or initially supersonic flow

9.14 Explain the concept of choking

9.15 Normal shocks:

(a) Explain what a normal shock is. What happens with the velocity, pressure and total temperature over a normal shock? How is the critical area, $A^*$ effected?

(b) The normal shock equation system has two solutions. How do we know which solution that is the correct one?

(c) How is the Hugoniot relation derived?

(d) Derive a relation for the pressure ratio over a normal shock

(e) How does pressure ($p$), temperature ($T$), density ($\rho$), Mach number ($M$), total pressure ($p_o$), and total temperature ($T_o$) change over a normal shock?

9.16 Explain, using a schematic figure, how pressure and velocity varies in a nozzle (the two types of nozzles specified below) as the downstream pressures (back pressures) changes. The upstream end of the nozzle is connected to a large reservoir (tank) with the pressure $p_o$ and temperature $T_o$. What happens with the massflow when the downstream pressure changes?

(a) convergent nozzle

(b) convergent-divergent nozzle

9.17 Make a schematic sketch showing how pressure waves expands around an object moving at

(a) subsonic speed

(b) sonic speed

(c) supersonic speed

Show the location of object at the time $t$ and the time $t + \Delta t$. In your sketch, show features such as Mach waves and the Mach angle

9.18 Oblique shocks:

(a) Show schematically how the velocity (normal velocity component, tangential velocity component, and the total velocity) changes over an oblique shock. Indicate the shock angle, $\beta$, and the deflection angle, $\theta$.

(b) Show schematically how the oblique shock formed ahead of a wedge traveling at supersonic speed if

i. $\theta < \theta_{max}$

ii. $\theta > \theta_{max}$

(c) Can the relations for total quantities for a normal shock be used for an oblique shock? Explain why/why not.

9.19 Prandtl-Meyer expansions:

(a) What is a Prandtl-Meyer expansion. Show with a figure.

(b) How does pressure ($p$), temperature ($T$), density ($\rho$), Mach number ($M$), total pressure ($p_o$), and total temperature ($T_o$) change over an expansion region?

## Recommended Problems