# MTF053, Fluid Mechanics - Home and exercise tasks 

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Please note that formulas, tables and graphs mentioned in this document and not included can be found at the formula sheet, denoted as "fs".

## Chapter 1

1.1 The belt in Fig. 1.1 moves at a steady velocity $V$ and skims the top of a tank of oil of viscosity $\mu$, as shown. Assuming a linear velocity profile in the oil, develop a simple formula for the required belt-drive power P as a function of $(h, L, V$, $b, \mu)$. What belt-drive power $P$, in watts, is required if the belt moves at $2.5 \mathrm{~m} / \mathrm{s}$ over SAE 30 W oil at $20^{\circ} \mathrm{C}$, with $L=2 \mathrm{~m}, b=60 \mathrm{~cm}$, and $h=3 \mathrm{~cm}$ ?


Fig. 1.1
1.2 A block of weight $W$ slides down an inclined plane while lubricated by a thin film of oil, as in Fig. 1.2. The film contact area is $A$ and its thickness is $h$. Assuming a linear velocity distribution in the film, derive an expression for the "terminal" (zero-acceleration) velocity $V$ of the block. Find the terminal velocity of the block if the block mass is $6 \mathrm{~kg}, A=35 \mathrm{~cm}^{2}, \theta=15^{\circ}$, and the film is 1 -mm-thick SAE 30 oil at $20^{\circ} \mathrm{C}$.
1.3 In Fig. 1.3, if the fluid is glycerin at $20^{\circ} \mathrm{C}$ and the width between plates is 6 mm , what shear stress (in Pa ) is required to move the upper plate at $5.5 \mathrm{~m} / \mathrm{s}$ ? What is the Reynolds number if $L$ is taken to be the distance between plates?
1.4 The Stokes-Oseen formula for drag force $F$ on a sphere of diameter $D$ in a fluid stream of low


Fig. 1.2


Fig. 1.3
velocity $V$, density $\rho$, and viscosity $\mu$ is

$$
F=3 \pi \mu D V+\frac{9 \pi}{16} \rho V^{2} D^{2}
$$

Is this formula dimensionally homogeneous?
1.5 The efficiency $\eta$ of a pump is defined as the (dimensionless) ratio of the power developed by the flow to the power required to drive the pump:

$$
\eta=\frac{Q \Delta p}{\text { input power }}
$$

where $Q$ is the volume rate of flow and $\Delta p$ is the pressure rise produced by the pump. Suppose that a certain pump develops a pressure rise of $35 \mathrm{lbf} / \mathrm{in}^{2}$ when its flow rate is $40 \mathrm{~L} / \mathrm{s}$. If the input power is 16 hp , what is the efficiency?
1.6 On a summer day in Narragansett, Rhode Island, the air temperature is $74^{\circ} \mathrm{F}$ and the barometric pressure is $14.5 \mathrm{lbf} / \mathrm{in}^{2}$. Estimate the air density in $\mathrm{kg} / \mathrm{m}^{3}$.
1.7 An aluminium cylinder weighing $30 \mathrm{~N}, 6 \mathrm{~cm}$ in diameter and 40 cm long, is falling concentrically through a long vertical sleeve of diameter 6.04 cm . The clearance is filled with SAE 50 oil at $20^{\circ} \mathrm{C}$. Estimate the terminal (zero acceleration) fall velocity. Neglect air drag and assume a linear velocity distribution in the oil. Hint: You are given diameters, not radii.
1.8 A disk of radius $R$ rotates at an angular velocity $V$ inside a disk-shaped container filled with oil of viscosity $\mu$, as shown in Fig. 1.8. Assuming a linear velocity profile and neglecting shear stress on the outer disk edges, derive a formula for the viscous torque on the disk.


Fig. 1.8
1.9 For low-speed (laminar) steady flow through a circular pipe, as shown in Fig. 1.9, the velocity $u$ varies with radius and takes the form

$$
u=B \frac{\Delta p}{\mu}\left(r_{0}^{2}-r^{2}\right)
$$

where $\mu$ is the fluid viscosity and $\Delta p$ is the pressure drop from entrance to exit. What are the dimensions of the constant $B$ ?
1.10 An airplane flies at $555 \mathrm{mi} / \mathrm{h}$. At what altitude in the standard atmosphere will the airplane's Mach number be exactly 0.8 ?
1.11 Early mountaineers boiled water to estimate their altitude. If they reach the top and find that


Fig. 1.9
water boils at $84^{\circ} \mathrm{C}$, approximately how high is the mountain?

## Chapter 2

2.1 In Fig. 2.1 all fluids are at $20^{\circ} \mathrm{C}$. Determine the pressure difference ( Pa ) between points $A$ and $B$.


Fig. 2.1
2.2 The cylindrical tank in Fig. 2.2 is being filled with water at $20^{\circ} \mathrm{C}$ by a pump developing an exit pressure of 175 kPa . At the instant shown, the air pressure is 110 kPa and $H=35 \mathrm{~cm}$. The pump stops when it can no longer raise the water pressure. For isothermal air compression, estimate $H$ at that time.


Fig. 2.2
2.3 Water flows downward in a 1.524 m long pipe at $45^{\circ}$, as shown in Fig. 2.3. The pressure drop $p_{1}-p_{2}$ is partly due to gravity and partly due to friction. The mercury manometer reads a 15.24 cm height difference. What is the total pressure drop $p_{1}-p_{2}$ in Pa ? What is the pressure drop due to friction only between 1 and 2 in Pa? Does the manometer reading correspond only to friction drop? Why?
Note that the input data is slightly modified from the book.


Fig. 2.3
2.4 A spar buoy is a buoyant rod weighted to float and protrude vertically, as in Fig. 2.4. It can be used for measurements or markers. Suppose that the buoy is maple wood $(\mathrm{SG}=0.6), 5 \mathrm{~cm}$ by 5 cm by 3.5 m , floating in seawater $(\mathrm{SG}=1.025)$. How many pounds of steel ( $\mathrm{SG}=7.85$ ) should be added to the bottom end so that $h=0.5 \mathrm{~m}$ ?

Note that the steel sphere in Fig P2.113 is approximated by a square block mounted below the wood.


Fig. 2.4
2.5 In Fig. 2.5, if pressure gage $A$ reads 140 kPa absolute, find the pressure in the closed air space $B$. The manometer fluid is Meriamred oil, $\mathrm{SG}=$ 0.827 .


Fig. 2.5
2.6 In Fig. 2.6, pressure gage $A$ reads 1.5 kPa (gage). The fluids are at $20^{\circ} \mathrm{C}$. Determine the elevations $z$, in meters, of the liquid levels in the open piezometer tubes $B$ and $C$.


Fig. 2.6
2.7 The can in Fig. 2.7 floats in the position shown. What is its weight in N ?


Fig. 2.7
2.8 An intrepid treasure-salvage group has discovered a steel box, containing gold doubloons and
other valuables, resting in 25 m of seawater. They estimate the weight of the box and treasure (in air) at $30,000 \mathrm{~N}$. Their plan is to attach the box to a sturdy balloon, inflated with air to 3 atm pressure. The empty balloon weighs 1000 N . The box is 0.6 m wide, 1.5 m long, and 0.5 m in high. What is the proper diameter of the balloon to ensure an upward lift force on the box that is 20 percent more than required?

## Chapter 3

3.1 For steady low-Reynolds-number (laminar) flow through a long tube (see Prob. 1.9, Fig. 1.9), the axial velocity distribution is given by $u=C\left(R^{2}-r^{2}\right)$, where $R$ is the tube radius and $r \leq R$. Integrate $u(r)$ to find the total volume flow Q through the tube.
3.2 The open tank in Fig. 3.3 contains water at $20^{\circ} \mathrm{C}$ and is being filled through section 1. Assume incompressible flow. First derive an analytic expression for the water-level change $\mathrm{d} h / \mathrm{d} t$ in terms of arbitrary volume flows $\left(Q_{1}, Q_{2}, Q_{3}\right)$ and tank diameter $d$. Then, if the water level $h$ is constant, determine the exit velocity $V_{2}$ for the given data $V_{1}=3 \mathrm{~m} / \mathrm{s}$ and $Q_{3}=0.01 \mathrm{~m}^{3} / \mathrm{s}$.


Fig. 3.2
3.3 An incompressible fluid flows past an impermeable flat plate, as in Fig. 3.3, with a uniform inlet profile $u=U_{0}$ and a cubic polynomial exit profile

$$
u \approx U_{0}\left(\frac{3 \eta-\eta^{3}}{2}\right) \text { where } \eta=\frac{y}{\delta}
$$

Compute the volume flow $Q$ across the top surface of the control volume.


Fig. 3.3
3.4 An incompressible fluid in Fig. 3.4 is being squeezed outward between two large circular disks by the uniform downward motion $V_{0}$ of the upper disk. Assuming one-dimensional radial outflow, use the control volume shown to derive an expression for $V(r)$.


Fig. 3.4
3.5 Water fills a cylindrical tank to depth $h$. The tank has diameter $D$. The water flows out at average velocity $V_{0}$ from a hole in the bottom of area $A_{0}$. Use the Reynolds transport theorem to find an expression for the instantaneous depth change $\mathrm{d} h / \mathrm{d} t$.
3.6 Incompressible steady flow in the inlet between parallel plates in Fig. 3.16 is uniform, $u=U_{0}=$ $8 \mathrm{~cm} / \mathrm{s}$, while downstream the flow develops into the parabolic laminar profile $u=a z\left(z_{0}-z\right)$, where a is $a$ constant. If $z_{0}=3 \mathrm{~cm}$ and the fluid is SAE 30 oil at $20^{\circ} \mathrm{C}$, what is the value of $u_{\text {max }}$ in $\mathrm{cm} / \mathrm{s}$ ?
3.7 A spherical tank, of diameter 35 cm , is leaking air through a 5 mm -diameter hole in its side. The air exits the hole at $360 \mathrm{~m} / \mathrm{s}$ and a density of 2.5 $\mathrm{kg} / \mathrm{m}^{3}$. Assuming uniform mixing, a) find a formula for the rate of change of average density in the tank and b) calculate a numerical value for


Fig. 3.6
$(\mathrm{d} \rho / \mathrm{d} t)$ in the tank for the given data. Hint, use conservation of mass.
3.8 Gasoline enters section 1 in Fig. 3.8 at $0.5 \mathrm{~m}^{3} / \mathrm{s}$. It leaves section 2 at an average velocity of $12 \mathrm{~m} / \mathrm{s}$. What is the average velocity at section 3 ? Is it in or out?


Fig. 3.8
3.9 The pipe flow in Fig. 3.9 fills a cylindrical surge tank as shown. At time $t=0$, the water depth in the tank is 30 cm . Estimate the time required to fill the remainder of the tank.


Fig. 3.9
3.10 A thin layer of liquid, draining from an inclined plane, as in Fig. 3.10, will have a laminar velocity profile $u \approx U_{0}\left(2 y / h-y^{2} / h^{2}\right)$, where $U_{0}$ is the surface velocity. If the plane has width $b$ into the paper, determine the volume rate of flow in the film. Suppose that $h=13 \mathrm{~mm}$ and the flow rate per meter of channel width is $5 \mathrm{~L} /(\mathrm{m} \cdot \mathrm{min})$. Estimate $U_{0}$ in $\mathrm{mm} / \mathrm{s}$.


Fig. 3.10
3.11 In some wind tunnels the test section is perforated to suck out fluid and provide a thin viscous boundary layer. The test section wall in Fig. 3.11 contains 1200 holes of 5 mm diameter each per square meter of wall area. The suction velocity through each hole is $V_{s}=8 \mathrm{~m} / \mathrm{s}$, and the test-section entrance velocity is $V_{1}=35 \mathrm{~m} / \mathrm{s}$. Assuming incompressible steady flow of air at $20^{\circ} \mathrm{C}$, compute a) $V_{0}$, b) $V_{2}$, and c) $V_{f}$, in $\mathrm{m} / \mathrm{s}$.


Fig. 3.11
3.12 A liquid jet of velocity $v_{j}$ and area $A_{j}$ strikes a single $180^{\circ}$ bucket on a turbine wheel rotating at angular velocity $\Omega$, as in Fig. 3.12. Derive an expression for the power $P$ delivered to this wheel at this instant as a function of the system parameters. *At what angular velocity is the maximum power delivered? *How would your analysis differ if there were many, many buckets on the wheel, so that the jet was continually striking at least one bucket?
*Removed from the exercises task
3.13 Assume that the outflow velocity profile is given by $u(y) \approx U_{0} \sin (\pi y /(2 \delta))$ for the flow over the flat plate, shown in Fig. 3.13. The fluid is water at $20^{\circ} \mathrm{C}, U_{0}=3 \mathrm{~m} / \mathrm{s}, \delta=2 \mathrm{~mm}$, and $L=0.45 \mathrm{~cm}$. Estimate the drag force per unit width of the plate.


Fig. 3.12


Fig. 3.13
3.14 The water jet in Fig. 3.14 strikes normal to a fixed plate. Neglect gravity and friction, and compute the force $F$ in newtons required to hold the plate fixed.


Fig. 3.14
3.15 Consider incompressible flow in the entrance of a circular tube, as in Fig. 3.15. The inlet flow is uniform, $u_{1}=U_{0}$. The flow at section 2 is developed pipe flow. Find the wall drag force $F$ as a function of $\left(p_{1}, p_{2}, \rho, U_{0}, R\right)$ if the flow at section 2 is
a) Laminar: $u_{2}=u_{\max }\left(1-\frac{r^{2}}{R^{2}}\right)$
b) Turbulent: $u_{2}=u_{\max }\left(1-\frac{r}{R}\right)^{1 / 7}$
3.16 In Fig. 3.16 the jet strikes a vane that moves to the right at constant velocity $V_{c}$ on a frictionless cart. Compute a) the force $F_{x}$ required to restrain the cart and b) the power $P$ delivered to the cart. Also find the cart velocity for which c)


Fig. 3.15
the force $F_{x}$ is a maximum and d) the power $P$ is a maximum.


Fig. 3.16
3.17 A necked-down section in a pipe flow, called a venturi, develops a low throat pressure that can aspirate fluid upward from a reservoir, as in Fig. 3.17. Using Bernoulli's equation with no losses, derive an expression for the velocity $V_{1}$ that is just sufficient to bring reservoir fluid into the throat.


Fig. 3.17
3.18 A venturi meter, shown in Fig. 3.18, is a carefully designed constriction whose pressure difference is a measure of the flow rate in a pipe. Using Bernoulli's equation for steady incompress-
ible flow with no losses, show that the flow rate $Q$ is related to the manometer reading $h$ by

$$
Q=\frac{A_{2}}{\sqrt{1-\left(D_{2} / D_{1}\right)^{4}}} \sqrt{\frac{2 g h\left(\rho_{\mathrm{M}}-\rho\right)}{\rho}}
$$

where $\rho_{\mathrm{M}}$ is the density of the manometer fluid.


Fig. 3.18
3.19 In Fig. 3.19 both fluids are at $20^{\circ} \mathrm{C}$. If $V_{1}=$ $0.5 \mathrm{~m} / \mathrm{s}$ and losses are neglected, what should the manometer reading $h \mathrm{~m}$ be?


Fig. 3.19


Fig. 3.20
and $b$ ) the rotation rate ( $\mathrm{r} / \mathrm{min}$ ) if there is no retarding torque.


Fig. 3.21
3.22 The three-arm lawn sprinkler of Fig. 3.22 receives $20^{\circ} \mathrm{C}$ water through the center at 2.7 $\mathrm{m}^{3} / \mathrm{h}$. If collar friction is negligible, what is the steady rotation rate in $\mathrm{r} / \mathrm{min}$ for a) $\theta=0^{\circ}$ and b) $\theta=40^{\circ}$ ?


Fig. 3.22
3.23 The pipe bend of Fig. 3.23 has $D_{1}=27 \mathrm{~cm}$ and $D_{2}=13 \mathrm{~cm}$. When water at $20^{\circ} \mathrm{C}$ flows through the pipe at $1 \mathrm{~m}^{3} / \mathrm{min}, p_{1}=194 \mathrm{kPa}$ (gage). Compute the torque required at point $B$ to hold the bend stationary.
3.24 There is a steady isothermal flow of water at $20^{\circ} \mathrm{C}$ through the device in Fig. 3.24. Heattransfer, gravity, and temperature effects are


Fig. 3.23
negligible. Known data are $D_{1}=9 \mathrm{~cm}, Q_{1}=$ $220 \mathrm{~m}^{3} / \mathrm{h}, p_{1}=150 \mathrm{kPa}, D_{2}=7 \mathrm{~cm}, Q_{2}=$ $100 \mathrm{~m}^{3} / \mathrm{h}, p_{2}=225 \mathrm{kPa}, D_{3}=4 \mathrm{~cm}$, and $p_{3}=265 \mathrm{kPa}$. Compute the rate of shaft work done for this device and its direction.


Fig. 3.24
3.25 A power plant on a river, as in Fig. 3.25, must eliminate 55 MW of waste heat to the river. The river conditions upstream are $Q_{i}=2.5 \mathrm{~m}^{3} / \mathrm{s}$ and $T_{i}=18^{\circ} \mathrm{C}$. The river is 45 m wide and 2.7 m deep. If heat losses to the atmosphere and ground are negligible, estimate the downstream river conditions $\left(Q_{0}, T_{0}\right)$. Hint: The width and depth of the river is regarded constant.
3.26 When the pump in Fig. 3.26 draws $220 \mathrm{~m}^{3} / \mathrm{h}$ of water at $20^{\circ} \mathrm{C}$ from the reservoir, the total friction head loss is 5 m . The flow discharges through a nozzle to the atmosphere. Estimate the pump power in kW delivered to the water.
$3.27^{*}$ The large turbine in Fig. 3.27 diverts the river flow under a dam as shown. System friction losses are $h_{f}=3.5 V^{2} /(2 g)$, where $V$ is the average velocity in the supply pipe. For what river flow rate in $\mathrm{m}^{3} / \mathrm{s}$ will the power extracted be 25 MW? Which of the two possible solutions has a better "conversion efficiency"?


Fig. 3.25


Fig. 3.26


Fig. 3.27

## Chapter 4

4.1 Flow through the converging nozzle in Figure 4.1 can be approximated by the one-dimensional velocity distribution

$$
u \approx V_{0}\left(1+\frac{2 x}{L}\right) \quad v \approx 0 \quad w \approx 0
$$

a) Find a general expression for the fluid acceleration in the nozzle. b) For the specific case $V_{0}=3 \mathrm{~m} / \mathrm{s}$ and $L=0.15 \mathrm{~m}$, compute the acceleration, in g's, at the entrance and at the exit.


Fig. 4.1
4.2 Calculate for the nozzle in 4.1 a) the acceleration at $x=L \mathrm{~b}$ ) the time it takes a fluid particle to move from $x=0$ to $x=L$.
4.3 A two-dimensional velocity field is given by

$$
\mathbf{V}=\left(x^{2}-y^{2}+x\right) \mathbf{i}-(2 x y+y) \mathbf{j}
$$

in arbitrary units. At $(x, y)=(1,2)$, compute a) the accelerations $a_{x}$ and $a_{y}$, b) the velocity component in the direction $\theta=10^{\circ}$, c) the direction of maximum velocity, and d) the direction of maximum acceleration.
4.4 Derive the equation for cylindrical coordinates (Eq. D.2) by considering the flux of an incompressible fluid in and out of the elemental control volume in Fig. 4.1.
4.5 An incompressible flow in polar coordinates is given by

$$
\begin{aligned}
& v_{r}=K \cos \theta\left(1-\frac{b}{r^{2}}\right) \\
& v_{\theta}=K \sin \theta\left(1+\frac{b}{r^{2}}\right)
\end{aligned}
$$

Does this field satisfy continuity? For consistency, what should the dimensions of constants $K$ and $b$ be? Sketch the surface where $v_{r}=0$ and interpret.
4.6 An excellent approximation for the twodimensional incompressible laminar boundary layer on the flat surface in Fig. 4.6 is

$$
u \approx U\left(2 \frac{y}{\delta}-2 \frac{y^{3}}{\delta^{3}}+\frac{y^{4}}{\delta^{4}}\right) \quad \text { for } y \leq \delta
$$

where $\delta=C x^{1 / 2}, C=$ const.
a) Assuming a no-slip condition at the wall, find an expression for the velocity component $v(x, y)$ for $y \leq \delta$. b) Then find the maximum value of $v$ at the station $x=1 \mathrm{~m}$, for the particular case of airflow, when $U=3 \mathrm{~m} / \mathrm{s}$ and $\delta=1.1 \mathrm{~cm}$.


Fig. 4.6
4.7 How is the continuity equation written for
a) stationary, compressible flow in yz?
b) instationary, incompressible flow in $x z$ ?
c) instationary, compressible flow in y?
d) stationary, compressible flow in plane polar coordinates?
4.8 For an incompressible plane flow in polar coordinates, we are given

$$
v_{r}=r^{3} \cos \theta+r^{2} \sin \theta
$$

Find the appropriate form of circumferential velocity for which continuity is satisfied.
4.9 Air flows under steady, approximately onedimensional conditions through the conical nozzle in Fig. 4.9. If the speed of sound is approximately $340 \mathrm{~m} / \mathrm{s}$, what is the minimum nozzlediameter ratio $D_{e} / D_{0}$ for which we can safely neglect compressibility effects if $\left.V_{0}=\mathrm{a}\right) 10 \mathrm{~m} / \mathrm{s}$ and b) $30 \mathrm{~m} / \mathrm{s}$ ?
4.10 A frictionless, incompressible steady flow field is given by

$$
\mathbf{V}=2 x y \mathbf{i}-y^{2} \mathbf{j}
$$

in arbitrary units. Let the density be $\rho_{0}=$ constant and neglect gravity. Find an expression for the pressure gradient in the x direction.


Fig. 4.9
4.11 As mentioned in Sec. 4.10, the velocity profile for laminar flow between two plates, as in Fig. 4.11, is

$$
u=\frac{4 u_{\max } y(h-y)}{h^{2}} \quad v=w=0
$$

If the wall temperature is $T_{w}$ at both walls, use the incompressible flow energy equation (4.75) to solve for the temperature distribution $T(y)$ between the walls for steady flow.


Fig. 4.11
4.12 Oil, of density $\rho$ and viscosity $\mu$, drains steadily down the side of a vertical plate, as in Fig. 4.12. After a development region near the top of the plate, the oil film will become independent of $z$ and of constant thickness $\delta$. Assume that $w=w(x)$ only and that the atmosphere offers no shear resistance to the surface of the film. a) Solve the Navier-Stokes equation for $\mathrm{w}(\mathrm{x})$, and sketch its approximate shape. b) Suppose that film thickness $\delta$ and the slope of the velocity profile at the wall $[\partial w / \partial x]_{\text {wall }}$ are measured with a laser-Doppler anemometer (Chap. 6). Find an expression for oil viscosity $\mu$ as a function of $\left(\rho, \delta, g,[\partial w / \partial x]_{\text {wall }}\right)$
4.13 The flow pattern in bearing lubrication can be illustrated by Fig. 4.13, where a viscous oil $(\rho, \mu)$

Plate


Fig. 4.12
is forced into the gap $h(x)$ between a fixed slipper block and a wall moving at velocity $U$. If the gap is thin, $h \ll L$, it can be shown that the pressure and velocity distributions are of the form $p=$ $p(x), u=u(y), v=w=0$. Neglecting gravity, reduce the Navier-Stokes equations (4.38) to a single differential equation for $u(y)$. What are the proper boundary conditions? Integrate and show that

$$
u=\frac{1}{2 \mu} \frac{\mathrm{~d} p}{\mathrm{~d} x}\left(y^{2}-y h\right)+U\left(1-\frac{y}{h}\right)
$$

where $h=h(x)$ may be an arbitrary, slowly varying gap width.


Fig. 4.13
4.14 Consider a viscous film of liquid draining uniformly down the side of a vertical rod of radius a, as in Fig. 4.14. At some distance down the rod the film will approach a terminal or fully developed draining flow of constant outer radius $b$,
with $v_{z}=v_{z}(r), v_{\theta}=v_{r}=0$. Assume that the atmosphere offers no shear resistance to the film motion. Derive a differential equation for $v_{z}$, state the proper boundary conditions, and solve for the film velocity distribution. How does the film radius $b$ relate to the total film volume flow rate $Q$ ?


Fig. 4.14

## Chapter 5

5.1 A prototype automobile is designed for cold weather in Denver, Colorado ( $10^{\circ} \mathrm{C}, 83 \mathrm{kPa}$ ). Its drag force is to be tested on a one-seventh-scale model in a wind tunnel at $70 \mathrm{~m} / \mathrm{s}, 20^{\circ} \mathrm{C}$, and 1 atm . If the model and prototype are to satisfy dynamic similarity, what prototype velocity, in $\mathrm{m} / \mathrm{s}$, needs to be matched?
5.2 When tested in water at $20^{\circ} \mathrm{C}$ flowing at $2 \mathrm{~m} / \mathrm{s}$, an 8 cm diameter sphere has a measured drag of 5 N . What will be the velocity and drag force on a 1.5 m diameter weather balloon moored in sea-level standard air under dynamically similar conditions?
5.3 To find the drag force of a full size zeppelin in air at $20^{\circ} \mathrm{C}$ with a velocity of $U=6 \mathrm{~m} / \mathrm{s}$ a small scale model is used to save money. The model is $1 / 30$ of the full size and is investigated in water to reach dynamically similar condition. a) At what velocity must the water flow for this to be true? b) What is the drag of the full size zeppelin if the model measures 2700 N ? c) What is the required power to drive the zeppelin?
5.4 The wall shear stress $\tau_{w}$ in a boundary layer is assumed to be a function of stream velocity $U$, boundary layer thickness $\delta$, local turbulence velocity $u^{\prime}$, density $\rho$, and local pressure gradient $\mathrm{d} p / \mathrm{d} x$. Using $(\rho, U, \delta)$ as repeating variables, rewrite this relationship as a dimensionless function.
5.5 A simply supported beam of diameter $D$, length $L$, and modulus of elasticity $E$ is subjected to a fluid crossflow of velocity $V$, density $\rho$, and viscosity $\mu$. Its center-deflection $\delta$ is assumed to be a function of all these variables. a) Rewrite this proposed function in dimensionless form.
Note that the book has a task b) that can be disregarded.
5.6 A smooth steel ( $\rho=7800 \mathrm{~kg} / \mathrm{m}^{2}$ ) sphere with diameter 0.025 m is immersed in a stream of gasoline at $20^{\circ} \mathrm{C}$ moving at $1.5 \mathrm{~m} / \mathrm{s}$. Ignoring its acceleration phase, what will its terminal (constant) fall velocity be?

## Chapter 6

6.1 In flow past a body or wall, early transition to turbulence can be induced by placing a trip wire on the wall across the flow, as in Fig. 6.1. If the trip wire in Fig. 6.1 is placed where the local velocity is $U$, it will trigger turbulence if $U d / \nu=850$, where $d$ is the wire diameter. If the sphere diameter, $D$, is 20 cm and transition is observed at $R e_{D}=90000$, what is the diameter of the trip wire in mm?


Fig. 6.1
6.2 Water at $20^{\circ} \mathrm{C}$ is to be siphoned through a tube 1 m long and 2 mm in diameter, as in Fig. 6.2. Is there any height, $H$, for which the flow might not be laminar? What is the flow rate if $H=50$ cm ? Neglect the tube curvature.


Fig. 6.2
6.3 Two tanks of water at $20^{\circ} \mathrm{C}$ are connected by a capillary tube 4 mm in diameter and 3.5 m long. The surface of tank 1 is 30 cm higher than the surface of tank 2. a) Estimate the flow rate in $\mathrm{m}^{3} / \mathrm{h}$. Is the flow laminar? b) For what tube diameter will $R e_{d}$ be 500 ?
6.4 For the configuration shown in Fig. 6.4, the fluid is ethyl alcohol at $20^{\circ} \mathrm{C}$, and the tanks are very wide. Find the flow rate which occurs in $\mathrm{m}^{3} / \mathrm{h}$. Is the flow laminar?


Fig. 6.4
6.5 The pipe flow in Fig. 6.5 is driven by pressurized air in the tank. What gage pressure $p_{1}$ is needed to provide a $20^{\circ} \mathrm{C}$ water flow rate $Q=60 \mathrm{~m}^{3} / \mathrm{h}$ ?
6.6 Water at $20^{\circ} \mathrm{C}$ is to be pumped through 600 m of pipe from reservoir 1 to 2 at a rate of $0.1 \mathrm{~m}^{3} / \mathrm{s}$, as shown in Fig. 6.6. If the pipe is cast iron of diameter 15 cm and the pump is 75 percent efficient, what horsepower pump is needed?
6.7 You wish to water your garden with 30 m of 15 mm -diameter hose whose roughness is 0.3 mm . What will be the delivery, in $\mathrm{L} / \mathrm{s}$, if the gage pressure at the faucet is 400 kPa ? If there is no nozzle (just an open hose exit), what is


Fig. 6.5


Fig. 6.6
the maximum horizontal distance the exit jet will carry?
6.8 The reservoirs in Fig. 6.8 contain water at $20^{\circ} \mathrm{C}$. If the pipe is smooth with $L=4500 \mathrm{~m}$ and $d=$ 4 cm , what will the flow rate in $\mathrm{m}^{3} / \mathrm{h}$ be for $\Delta z=100 \mathrm{~m} ?$


Fig. 6.8
6.9 SAE 10 oil at $20^{\circ} \mathrm{C}$ flows at an average velocity of $2 \mathrm{~m} / \mathrm{s}$ between two smooth parallel horizontal
plates 3 cm apart. Estimate a) the centerline velocity, b) the head loss per meter, and c) the pressure drop per meter.
6.10 The system in Fig. 6.10 consists of 1200 m of 5 cm cast iron pipe, two $45^{\circ}$ and four $90^{\circ}$ flanged longradius elbows, a fully open flanged globe valve, and a sharp exit into a reservoir. If the elevation at point 1 is 400 m , what gage pressure is required at point 1 to deliver $0.005 \mathrm{~m}^{3} / \mathrm{s}$ of water at $20^{\circ} \mathrm{C}$ into the reservoir?


Fig. 6.10
6.11 For the pitot-static pressure arrangement of Fig. 6.11, the manometer fluid is (colored) water at $20^{\circ} \mathrm{C}$. Estimate a) the centerline velocity, b) the pipe volume flow, and c) the (smooth) wall shear stress.


Fig. 6.11
6.12 An engineer who took college fluid mechanics on a pass-fail basis has placed the static pressure hole far upstream of the stagnation probe, as in Fig. 6.12, thus contaminating the pitot measurement ridiculously with pipe friction losses. If the pipe flow is air at $20^{\circ} \mathrm{C}$ and 1 atm and the manometer fluid is Meriam red oil $(\mathrm{SG}=0.827$ or $\rho=827 \mathrm{~kg} / \mathrm{m}^{3}$ ), estimate the air center-
line velocity for the given manometer reading of 16 cm . Assume a smooth-walled tube.


Fig. 6.12
6.13 The following turbulent flow velocity data $u(y)$, for air at $24^{\circ} \mathrm{C}$ and 1 atm near a smooth flat wall were taken in the University of Rhode Island wind tunnel:

| $y[\mathrm{~mm}]$ | 0.64 | 0.89 | 1.19 | 1.40 | 1.65 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $u[\mathrm{~m} / \mathrm{s}]$ | 15.6 | 16.5 | 17.3 | 17.6 | 18.0 |

Estimate a) the wall shear stress and b) the velocity $u$ at $y=5.6 \mathrm{~mm}$.
6.14 Air at 20 degrees Celsius flows along a plane surface developing a turbulent boundary layer. A pitot-static tube is placed 4 mm from the wall measuring a gauge pressure of 36 Pa . Calculate the wall shear stress.
6.15 Suppose in Fig. 6.15 that $h=3 \mathrm{~cm}$, the fluid in water at $20^{\circ} \mathrm{C}$, and the flow is turbulent, so that the logarithmic law is valid. If the shear stress in the fluid is 15 Pa , what is $V$ in $\mathrm{m} / \mathrm{s}$ ?


Fig. 6.15
f 6.16 Water at $20^{\circ} \mathrm{C}$ flows in a 9 -cm-diameter pipe under fully developed conditions. The centerline velocity is $10 \mathrm{~m} / \mathrm{s}$. Compute a) $Q$, b) $V$, c) $\tau_{w}$ and d) $\Delta p$ for a 100 m long pipe.

## Chapter 7

7.1 A gas at $20^{\circ} \mathrm{C}$ and 1 atm flows at $2 \mathrm{~m} / \mathrm{s}$ past a thin flat plate. At $x=1 \mathrm{~m}$, the boundary layer thickness is 0.016 m . Assuming laminar flow, which of the gases in Table A. 4 is this likely to be?
7.2 Equation (7.1b) assumes that the boundary layer on the plate is turbulent from the leading edge onward. Devise a scheme for determining the boundary layer thickness more accurately when the flow is laminar up to a point $\operatorname{Re}_{x, \text { crit }}$ and turbulent thereafter. Apply this scheme to computation of the boundary layer thickness at $x=$ 1.5 m in $40 \mathrm{~m} / \mathrm{s}$ flow of air at $20^{\circ} \mathrm{C}$ and 1 atm past a flat plate. Compare your result with Eq. (7.1b). Assume $\operatorname{Re}_{x, \text { crit }} \approx 1.2 \mathrm{E} 6$.
7.3 For the laminar parabolic boundary layer profile

$$
\frac{u}{U} \approx\left(\frac{2 y}{\delta}-\frac{y^{2}}{\delta^{2}}\right) \text { for } 0 \leq y \leq \delta
$$

compute the shape factor $H$ and compare with the exact Blasius result, Eq. (7.31).
7.4 A thin flat plate 55 by 110 cm is immersed in a $6-\mathrm{m} / \mathrm{s}$ stream of SAE 10 oil at $20^{\circ} \mathrm{C}$. Compute the total friction drag if the stream is parallel to (a) the long side and (b) the short side.
7.5 Air at $20^{\circ} \mathrm{C}$ and 1 atm flows at $20 \mathrm{~m} / \mathrm{s}$ past the flat plate in Fig. 7.5. A pitot stagnation tube, placed 2 mm from the wall, develops a manometer head $h=16 \mathrm{~mm}$ of Meriam red oil, $\mathrm{SG}=0.827$. Use this information to estimate the downstream position $x$ of the pitot tube. Assume laminar flow.


Fig. 7.5
7.6* Air at $20^{\circ} \mathrm{C}$ and 1 atm flows past the flat plate in Fig. 7.6 under laminar conditions. There are two equally spaced pitot stagnation tubes, each placed 2 mm from the wall. The manometer fluid is water at $20^{\circ} \mathrm{C}$. If $U=15 \mathrm{~m} / \mathrm{s}$ and $L=50 \mathrm{~cm}$, determine the values of the manometer readings $h_{1}$ and $h_{2}$, in mm.


Fig. 7.6
7.7 Flow straighteners are arrays of narrow ducts placed in wind tunnels to remove swirl and other in-plane secondary velocities. They can be idealized as square boxes constructed by vertical and horizontal plates, as in Fig. 7.7.The cross section is $\alpha$ by $\alpha$, and the box length is $L$. Assuming laminar flat-plate flow and an array of $N \times N$ boxes, derive a formula for a) the total drag on the bundle of boxes and b) the effective pressure drop across the bundle.


Fig. 7.7
7.8 Let the flow straighteners in Fig. 7.7 form an array of $20 \times 20$ boxes of size $\alpha=4 \mathrm{~cm}$ and $L=$ 25 cm . If the approach velocity is $U_{0}=12 \mathrm{~m} / \mathrm{s}$ and the fluid is sea-level standard air, estimate a) the total array drag and b) the pressure drop
across the array. Compare with Sec. 6.8(flow in non circular ducts).
7.9 Repeat the flat-plate momentum analysis of Sec. 7.2 (momentum integral estimates) by using the trigonometric profile approximation:

$$
\frac{u}{U} \approx \sin \left(\frac{y \pi}{2 \delta}\right) \text { for } 0 \leq y \leq \delta
$$

Compute estimates of $\theta, \delta^{*}, \delta / x$ and $c_{f}$. This is a clarified version of the book exercise.
7.10 A ship is 125 m long and has a wetted area of $3500 \mathrm{~m}^{2}$. Its propellers can deliver a maximum power of 1.1 MW to seawater at $20^{\circ} \mathrm{C}$. If all drag is due to friction, estimate the maximum ship speed, in kn.
7.11 A hydrofoil 50 cm long and 4 m wide moves at 28 kn in seawater at $20^{\circ} \mathrm{C}$. Using flat-plate theory with $\operatorname{Re}_{\text {trans }}=5 \mathrm{E} 5$, estimate its drag, in N , for a) a smooth wall and b ) a rough wall, $\varepsilon=0.3 \mathrm{~mm}$.
7.12 Atmospheric boundary layers are very thick but follow formulas very similar to those of flat-plate theory. Consider wind blowing at $10 \mathrm{~m} / \mathrm{s}$ at a height of 80 m above a smooth beach. Estimate the wind shear stress, in Pa , on the beach if the air is standard sea-level conditions. What will the wind velocity striking your nose be if a) you are standing up and your nose is 170 cm off the ground and b) you are lying on the beach and your nose is 17 cm off the ground?
7.13 A ship is 150 m long and has a wetted area of $5000 \mathrm{~m}^{2}$. If it is encrusted with barnacles, the ship requires 7000 hp to overcome friction drag when moving in seawater at 15 kn and $20^{\circ} \mathrm{C}$. What is the average roughness of the barnacles? How fast would the ship move with the same power if the surface were smooth? Neglect wave drag.
7.14 The main cross-cable between towers of a coastal suspension bridge is 60 cm in diameter and 90 m long. Estimate the total drag force on this cable in crosswinds of $80 \mathrm{~km} / \mathrm{h}$. Are these laminar flow conditions?
7.15 A sea-level smokestack is 52 m high and has a square cross section. Its supports can withstand a maximum side force of 90 kN . If the stack is to survive $40-\mathrm{m} / \mathrm{s}$ hurricane winds, what is its maximum possible width?
7.16 A parachutist jumps from a plane, using an 8.5m -diameter chute in the standard atmosphere. The total mass of the chutist and the chute is 90 kg. Assuming an open chute and quasi-steady motion, estimate the time to fall from 2000- to 1000-m altitude.
7.17 A sphere of density $\rho_{s}$ and diameter $D$ is dropped from rest in a fluid of density $\rho$ and viscosity $\mu$. Assuming a constant drag coefficient $C_{d_{0}}$, derive a differential equation for the fall velocity $V(t)$ and show that the solution is

$$
\begin{aligned}
& V=\left[\frac{4 g D(S-1)}{3 C_{d_{0}}}\right]^{1 / 2} \tanh C t \\
& C=\left[\frac{3 g C_{d_{0}}(S-1)}{4 S^{2} D}\right]^{1 / 2}
\end{aligned}
$$

where $S=\rho_{s} / \rho$ is the specific gravity of the sphere material.
7.18 Two baseballs of diameter 7.35 cm are connected to a rod 7 mm in diameter and 56 cm long, as in Fig. 7.18. What power, in $W$, is required to keep the system spinning at $400 \mathrm{r} / \mathrm{min}$ ? Include the drag of the rod, and assume sea-level standard air.


Fig. 7.18
7.19 A 1500-kg automobile uses its drag area $C_{\mathrm{D}} A=$ $0.4 \mathrm{~m}^{2}$, plus brakes and a parachute, to slow down from $50 \mathrm{~m} / \mathrm{s}$. Its brakes apply 5000 N of resistance. Assume sea-level standard air. If the automobile must stop in 8 s , what diameter parachute is appropriate?
7.20 An automobile has a mass of 1000 kg and a drag area $C_{\mathrm{D}} A=0.7 \mathrm{~m}^{2}$. The rolling resistance of 70 N is approximately constant. The car is coasting
without brakes at $90 \mathrm{~km} / \mathrm{h}$ as it begins to climb a hill of 10 percent grade (slope $=\tan ^{-1}(0.1)=$ $5.71^{\circ}$ ). How far up the hill will the car come to a stop?
7.21 An airplane weighs 180 kN and has a wing area of $160 \mathrm{~m}^{2}$ and a mean chord of 4 m . The airfoil properties are given by Fig. 7.25 (see FS). What force is needed to fly the plane forward at constant speed if, $V=112 \mathrm{~m} / \mathrm{s}$ and $h=3000 \mathrm{~m}$. This is a modification of the book exercise.
7.22 Suppose that the airplane of Prob. 7.21 is designed to land at $V_{0}=1.2 V_{\text {stall }}$, using a split flap set at $60^{\circ}$, a) what is the proper landing speed in $\mathrm{km} / \mathrm{h}$ ? b)What power is required for takeoff at the same speed?

## Chapter 9

9.1 Steam enters a nozzle at $377^{\circ} \mathrm{C}, 1.6 \mathrm{MPa}$, and a steady speed of $200 \mathrm{~m} / \mathrm{s}$ and accelerates isentropically until it exits at saturation conditions. Estimate the exit velocity and temperature.
9.2 Carbon dioxide $\left(\mathrm{CO}_{2}\right)$ enters a constant-area duct at $477.6 \mathrm{~K}, 689.49 \mathrm{kPa}$ absolute and $152.4 \mathrm{~m} / \mathrm{s}$. Farther downstream the properties are $V_{2}=304.8 \mathrm{~m} / \mathrm{s}$ and $T_{2}=755.4 \mathrm{~K}$. Compute a) $p_{2}, \mathrm{~b}$ ) the heat added between sections, c) the entropy change between sections, and d) the mass flow per unit area. Hint: This problem requires the continuity equation. This is a modification of the book exercise.
$9.3 \mathrm{CO}_{2}$ expands isentropically through a duct from $p_{1}=125 \mathrm{kPa}$ and $T_{1}=100^{\circ} \mathrm{C}$ to a downstream section where $p_{2}=80 \mathrm{kPa}$ and $V_{2}=325 \mathrm{~m} / \mathrm{s}$. Compute a) $T_{2}$; b) $M a_{2}$; c) $T_{o}$; d) $p_{o}$; e) $V_{1}$; and f) $M a_{1}$. This is a modification of the book exercise.
9.4 The large compressed-air tank in Fig. 9.4 exhausts from a nozzle at an exit velocity of 235 $\mathrm{m} / \mathrm{s}$. The mercury manometer reads $h=30 \mathrm{~cm}$. Assuming isentropic flow, compute the pressure a) in the tank and b) in the atmosphere. c) What is the exit Mach number?
9.5 In wind tunnel testing near Mach 1, a small area decrease caused by model blockage can be important. Suppose the test section area is $1 \mathrm{~m}^{2}$, with unblocked test conditions $M a=1.10$ and


Fig. 9.4
$T=20^{\circ} \mathrm{C}$. What model area will first cause the test section to choke? If the model cross section is $0.004 \mathrm{~m}^{2}$ ( 0.4 percent blockage), what percentage change in test section velocity results?
9.6 Air flows steadily from a reservoir at $20^{\circ} \mathrm{C}$ through a nozzle of exit area $20 \mathrm{~cm}^{2}$ and strikes a vertical plate as in Fig. 9.6. The flow is subsonic throughout. A force of 135 N is required to hold the plate stationary. Compute a) $V_{e}$, b) $M a_{e}$, and c) $p_{o}$ if $p_{a}=101 \mathrm{kPa}$.


Fig. 9.6
9.7 Air, supplied by a reservoir at 450 kPa , flows through a converging-diverging nozzle whose throat area is $12 \mathrm{~cm}^{2}$. A normal shock stands where $A_{1}=20 \mathrm{~cm}^{2}$. a) Compute the pressure just downstream of this shock. Still farther downstream, at $A_{3}=30 \mathrm{~cm}^{2}$, estimate b) $p_{3}$, c) $A_{3}^{*}$, and d) $M a_{3}$.
9.8 Air from a reservoir at $20^{\circ} \mathrm{C}$ and 500 kPa flows through a duct and forms a normal shock downstream of a throat of area $10 \mathrm{~cm}^{2}$. By an odd coincidence it is found that the stagnation pressure downstream of this shock exactly equals the throat pressure. What is the area where the shock wave stands?
9.9 A supply tank at 500 kPa and 400 K feeds air to a converging-diverging nozzle whose throat area is $9 \mathrm{~cm}^{2}$. The exit area is $46 \mathrm{~cm}^{2}$. State the conditions in the nozzle if the pressure outside the exit plane is a) 400 kPa, b) 120 kPa , and c) 9 kPa . d) In each of these cases, find the mass flow.
9.10 Air in a tank at 120 kPa and 300 K exhausts to the atmosphere through a $5-\mathrm{cm}^{2}$-throat converging nozzle at a rate of $0.12 \mathrm{~kg} / \mathrm{s}$. What is the atmospheric pressure? What is the maximum mass flow possible at low atmospheric pressure?
9.11 Air, with $p_{o}=500 \mathrm{kPa}$ and $T_{o}=600 \mathrm{~K}$, flows through a converging-diverging nozzle. The exit area is $51.2 \mathrm{~cm}^{2}$, and mass flow is $0.825 \mathrm{~kg} / \mathrm{s}$. What is the highest possible back pressure that will still maintain supersonic flow inside the diverging section?
9.12 Air flows through a duct as in Fig. 9.12, where $A_{1}=24 \mathrm{~cm}^{2}, A_{2}=18 \mathrm{~cm}^{2}$, and $A_{3}=32 \mathrm{~cm}^{2}$. A normal shock stands at section 2. Compute a) the mass flow, b) the Mach number, and c) the stagnation pressure at section 3 .


Fig. 9.12
9.13 Air flows at supersonic speed toward a compression ramp, as in Fig. 9.13. A scratch on the wall at point a creates a wave of $30^{\circ}$ angle, while the oblique shock created has a $50^{\circ}$ angle. What is a) the ramp angle $\theta$ and b ) the wave angle $\phi$ caused by a scratch at b?
9.14 Air flows at $M a=3$ and $p=70 \mathrm{kPa}$ absolute toward a wedge of $16^{\circ}$ angle at zero incidence in Fig. 9.14. If the pointed edge is forward, what will be the pressure at point A? If the blunt edge $\mathbf{9 . 1 7}$ is forward, what will be the pressure at point B?


Fig. 9.13


Fig. 9.14
9.15 When an oblique shock strikes a solid wall, it reflects as a shock of sufficient strength to cause the exit flow $M a_{3}$ to be parallel to the wall, as in Fig. 9.15. For airflow with $M a_{1}=2.5$ and $p_{1}$ $=100 \mathrm{kPa}$, compute $M a_{3}, p_{3}$, and the angle $\phi$.


Fig. 9.15
9.16 A bend in the bottom of a supersonic duct flow induces a shock wave that reflects from the upper wall, as in Fig. 9.16. Compute the Mach number and pressure in region 3 .
9.17 The supersonic nozzle of Fig. 9.17 is overexpanded (case G of Fig. 9.12b-see book) with


Fig. 9.16
$A_{e} / A_{t}=3.0$ and a stagnation pressure of 350 kPa . If the jet edge makes a $4^{\circ}$ angle with the nozzle centerline, what is the back pressure pr in kPa ?


Fig. 9.17
9.18 A supersonic airflow at $M a_{1}=3.2$ and $p_{1}=50$ kPa undergoes a compression shock followed by an isentropic expansion turn. The flow deflection is $30^{\circ}$ for each turn. Compute $M a_{2}$ and $p_{2}$ if a) the shock is followed by the expansion and $b$ ) the expansion is followed by the shock.
9.19 Air at $M a_{1}=2.0$ and $p_{1}=100 \mathrm{kPa}$ undergoes an isentropic expansion to a downstream pressure of 50 kPa . What is the desired turn angle in degrees?
9.20 A converging-diverging nozzle with a $4: 1$ exitarea ratio and $p_{o}=500 \mathrm{kPa}$ operates in an underexpanded condition (case I of Fig. 9.12b-see book) as in Fig. 9.20. The receiver pressure is $p_{a}$ $=10 \mathrm{kPa}$, which is less than the exit pressure, so that expansion waves form outside the exit. For
the given conditions, what will the Mach number $M a_{2}$ and the angle $\phi$ of the edge of the jet be? Assume $k=1.4$ as usual.


Fig. 9.20

