# Fluid Mechanics MTF053

One-Dimensional Steady Compressible Flow

Hugoniot Equation

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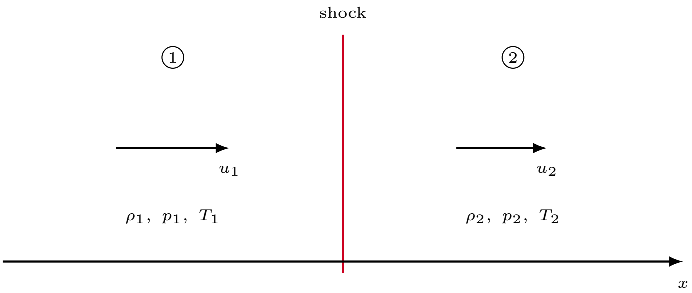

Figure 1: Stationary normal shock

## Governing Equations

The starting point is to set up the governing equations for one-dimensional steady compressible flow over a control volume enclosing the normal shock (Fig. 1).

continuity:

$$\rho_1 u_1 = \rho_2 u_2 \tag{1}$$

momentum:

$$\rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2 \tag{2}$$

energy:

$$h_1 + \frac{1}{2}u_1^2 = h_2 + \frac{1}{2}u_2^2 \tag{3}$$

## Derivation of the Hugoniot Equation

The continuity equation is rewritten and inserted into the momentum equation

$$u_1 = \left(\frac{\rho_2}{\rho_1}\right) u_2 \tag{4}$$

Replace $u_1$ in Eqn. 2 using Eqn. 4

$$\rho_1 \left(\frac{\rho_2}{\rho_1}\right)^2 u_2^2 + p_1 = \rho_2 u_2^2 + p_2$$

$$u_2^2 \left(\rho_1 \left(\frac{\rho_2}{\rho_1}\right)^2 - \rho_2\right) = (p_2 - p_1)$$

$$u_2^2\left(\left(\frac{\rho_2}{\rho_1}\right)(\rho_2-\rho_1)\right)=(p_2-p_1)$$

$$u_2^2=\left(\frac{\rho_1}{\rho_2}\right)\frac{p_2-p_1}{\rho_2-\rho_1} \tag{5}$$

Eqn. 4 and 5 gives

$$u_1^2=\left(\frac{\rho_2}{\rho_1}\right)\frac{p_2-p_1}{\rho_2-\rho_1} \tag{6}$$

Eqn. 5 and Eqn. 6 inserted in the energy equation (Eqn. 3) gives

$$h_1+\frac{1}{2}\left(\frac{\rho_2}{\rho_1}\right)\left(\frac{p_2-p_1}{\rho_2-\rho_1}\right)=h_2+\frac{1}{2}\left(\frac{\rho_1}{\rho_2}\right)\left(\frac{p_2-p_1}{\rho_2-\rho_1}\right) \tag{7}$$

$$h_2-h_1=\frac{p_2-p_1}{2}\left[\left(\frac{\rho_2}{\rho_1}\right)\left(\frac{1}{\rho_2-\rho_1}\right)-\left(\frac{\rho_1}{\rho_2}\right)\left(\frac{1}{\rho_2-\rho_1}\right)\right]$$

$$h_2-h_1=\frac{p_2-p_1}{2}\left[\frac{\rho_2^2-\rho_1^2}{\rho_1\rho_2(\rho_2-\rho_1)}\right]=\frac{p_2-p_1}{2}\left[\frac{\rho_2+\rho_1}{\rho_1\rho_2}\right]$$

$$h_2-h_1=\frac{p_2-p_1}{2}\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right) \tag{8}$$

Now, replacing the enthalpies with internal energies using $h=e+p/\rho$ gives

$$e_2-e_1=\frac{p_1}{\rho_1}-\frac{p_2}{\rho_2}+\frac{p_2-p_1}{2}\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right)$$

which after some rewriting becomes the Hugoniot equation

$$e_2-e_1=\frac{p_2+p_1}{2}\left(\frac{1}{\rho_1}-\frac{1}{\rho_2}\right) \tag{9}$$

The Hugoniot equation relates thermodynamic properties over the normal shock