Compressible Flow - TME085

Lecture 12

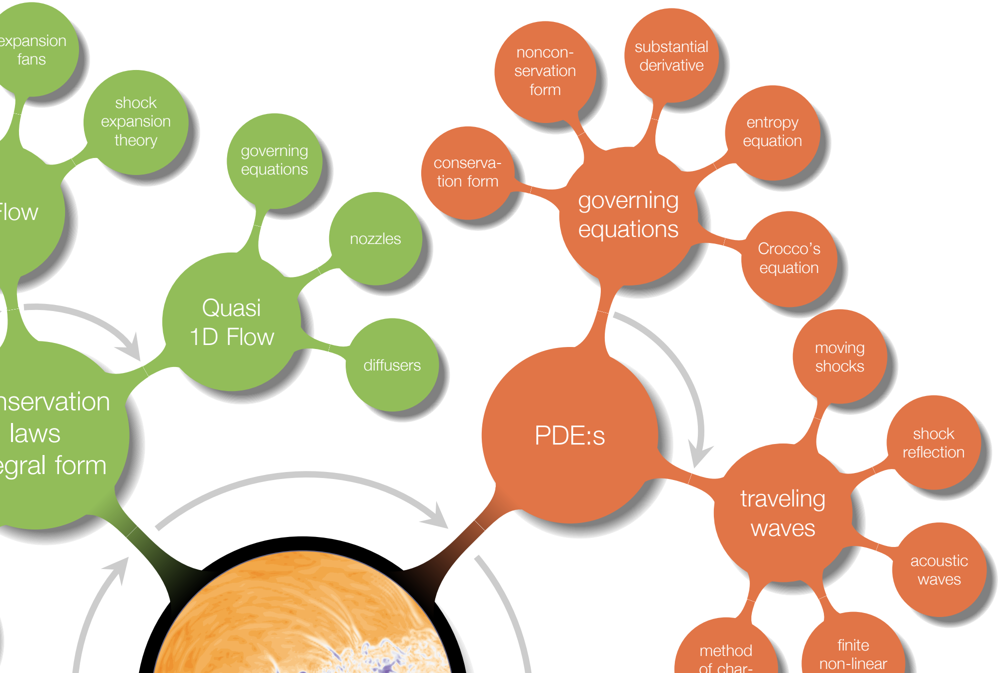
Niklas Andersson

Chalmers University of Technology
Department of Mechanics and Maritime Sciences
Division of Fluid Mechanics
Gothenburg, Sweden

niklas.andersson@chalmers.se

Chapter 7 - Unsteady Wave Motion

Overview



expansion fans
shock expansion theory
Flow
governing equations
nozzles
Quasi 1D Flow
diffusers
nservation laws egral form
noncon-servation form
substantial derivative
entropy equation
conserva-tion form
governing equations
Crocco's equation
PDE:s
moving shocks
shock reflection
traveling waves
acoustic waves
method of char-
finite non-linear

Learning Outcomes

3 **Describe** typical engineering flow situations in which compressibility effects are more or less predominant (e.g. Mach number regimes for steady-state flows)

4 **Present** at least two different formulations of the governing equations for compressible flows and **explain** what basic conservation principles they are based on

8 **Derive** (marked) and **apply** (all) of the presented mathematical formulae for classical gas dynamics

  a 1D isentropic flow*

  b normal shocks*

  j unsteady waves and discontinuities in 1D

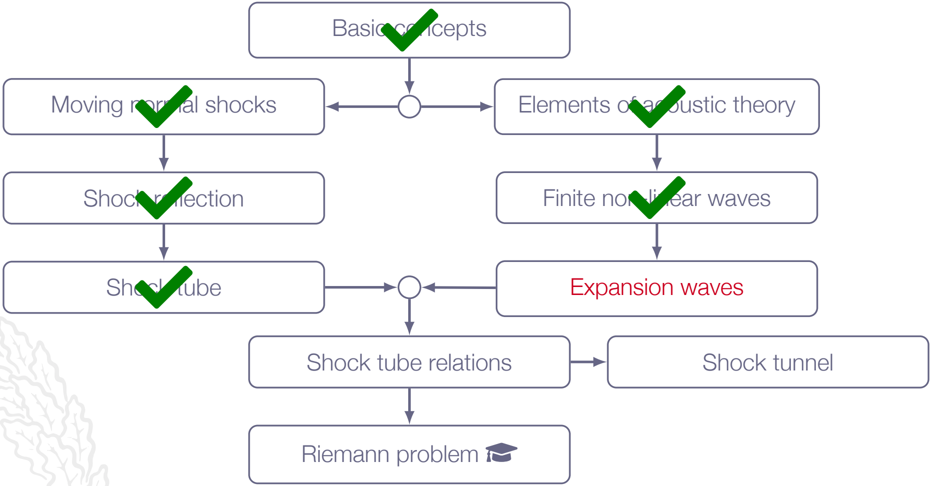
  k basic acoustics

9 **Solve** engineering problems involving the above-mentioned phenomena (8a-8k)

11 **Explain** how the equations for aero-acoustics and classical acoustics are derived as limiting cases of the compressible flow equations

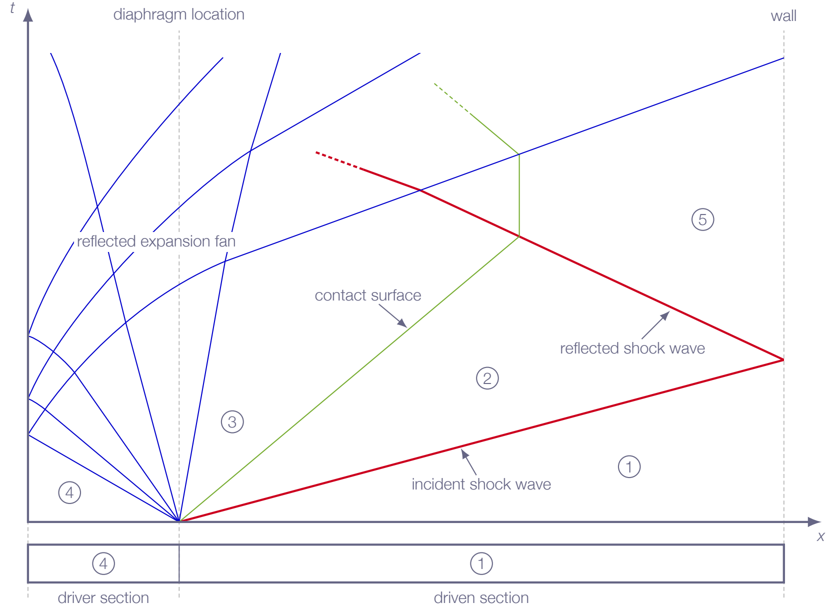
moving normal shocks - frame of reference seems to be the key here?!

Roadmap - Unsteady Wave Motion



Chapter 7.7
Incident and Reflected Expansion Waves

Expansion Waves



Expansion Waves

Properties of a left-running expansion wave

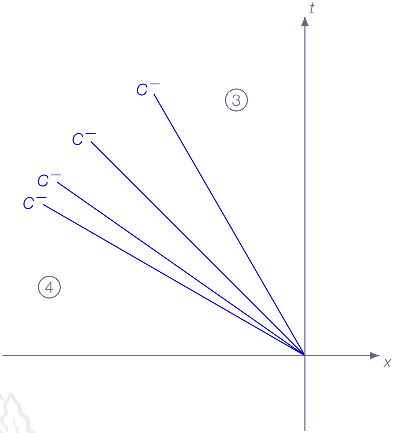
1. All flow properties are constant along C^- characteristics
2. The wave **head** is propagating **into region 4** (high pressure)
3. The wave **tail** defines the **limit of region 3** (lower pressure)
4. Regions 3 and 4 are assumed to be **constant states**

For calorically perfect gas:

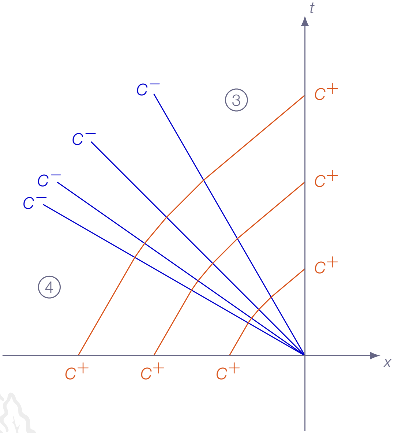
$$J^+ = u + \frac{2a}{\gamma - 1} \quad \text{is constant along } C^+ \text{ lines}$$

$$J^- = u - \frac{2a}{\gamma - 1} \quad \text{is constant along } C^- \text{ lines}$$

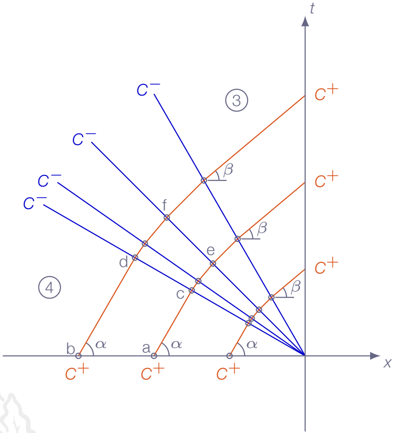
Expansion Waves



Expansion Waves



Expansion Waves



constant flow properties in region 4: $J_a^+ = J_b^+$

J^+ invariants constant along C^+ characteristics:

$$J_a^+ = J_c^+ = J_e^+$$

$$J_b^+ = J_d^+ = J_f^+$$

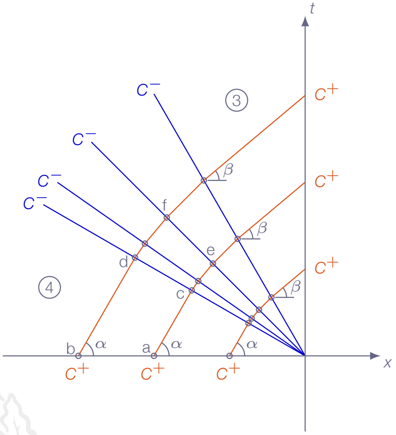
since $J_a^+ = J_b^+$ this also implies $J_e^+ = J_f^+$

J^- invariants constant along C^- characteristics:

$$J_c^- = J_d^-$$

$$J_e^- = J_f^-$$

Expansion Waves



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$$J_b^+ = J_d^+ = J_f^+$$

since $J_a^+ = J_b^+$ this also implies $J_e^+ = J_f^+$

J^- invariants constant along C^- characteristics:

$$J_c^- = J_d^-$$

$$J_e^- = J_f^-$$

$$u_e = \frac{1}{2}(J_e^+ + J_e^-),\ u_f = \frac{1}{2}(J_f^+ + J_f^-),\ \Rightarrow u_e = u_f$$

$$a_e = \frac{\gamma - 1}{4}(J_e^+ - J_e^-),\ a_f = \frac{\gamma - 1}{4}(J_f^+ - J_f^-),\ \Rightarrow a_e = a_f$$

Expansion Waves

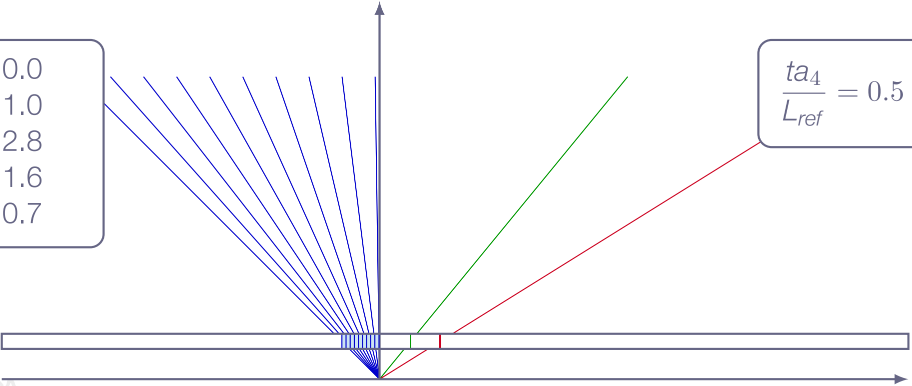
Along each C^- line u and a are **constants** which means that

$$\frac{dx}{dt} = u - a = const$$

C^- characteristics are **straight lines** in xt-space

Expansion Waves - Shock Tube

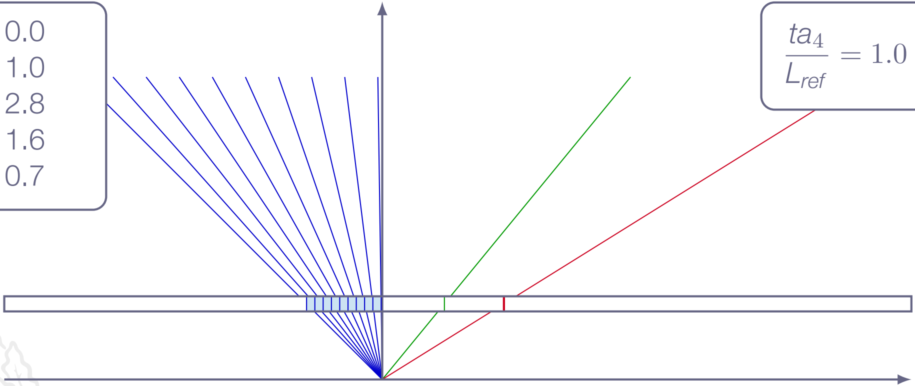
p_4/p_1	10.0
T_4/T_1	1.0
p_2/p_1	2.8
M_s	1.6
M_p	0.7



$$\frac{ta_4}{L_{ref}} = 0.5$$

Expansion Waves - Shock Tube

p_4/p_1	10.0
T_4/T_1	1.0
p_2/p_1	2.8
M_s	1.6
M_p	0.7

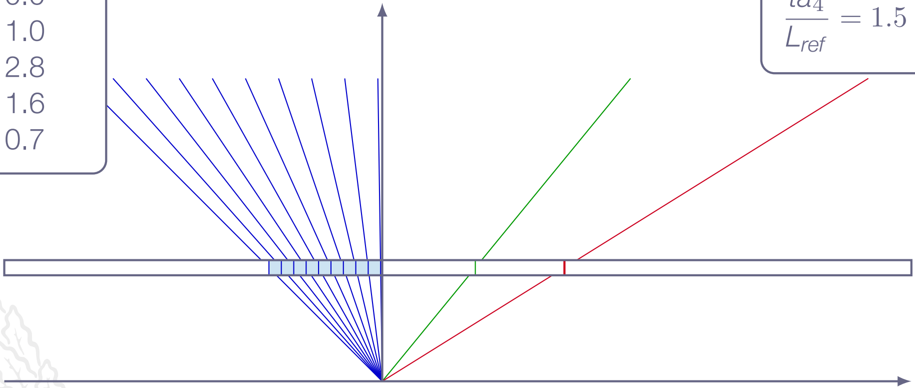


$$\frac{ta_4}{L_{ref}} = 1.0$$

Expansion Waves - Shock Tube

p_4/p_1	10.0
T_4/T_1	1.0
p_2/p_1	2.8
M_s	1.6
M_p	0.7

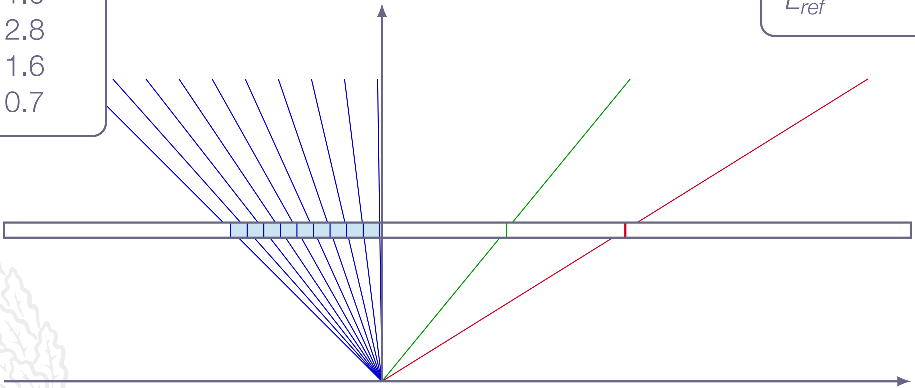
$$\frac{ta_4}{L_{ref}} = 1.5$$



Expansion Waves - Shock Tube

p_4/p_1	10.0
T_4/T_1	1.0
p_2/p_1	2.8
M_s	1.6
M_p	0.7

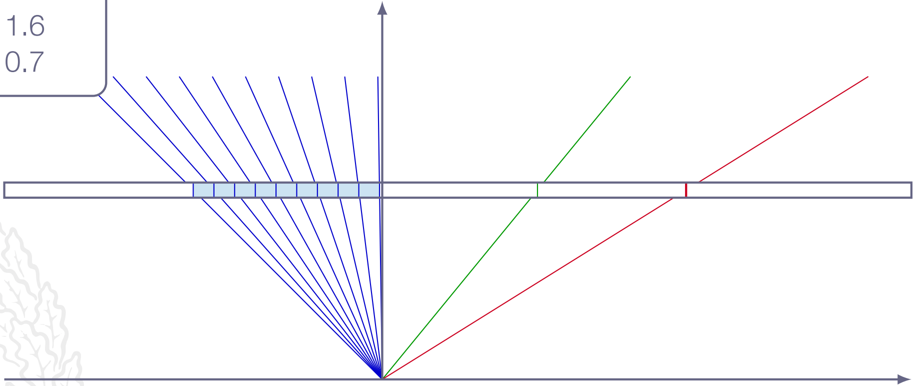
$$\frac{ta_4}{L_{ref}} = 2.0$$



Expansion Waves - Shock Tube

p_4/p_1	10.0
T_4/T_1	1.0
p_2/p_1	2.8
M_s	1.6
M_p	0.7

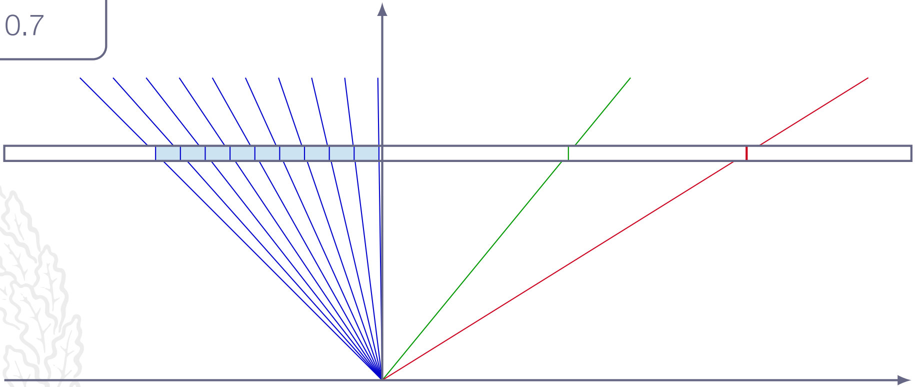
$$\frac{ta_4}{L_{ref}} = 2.5$$



Expansion Waves - Shock Tube

p_4/p_1	10.0
T_4/T_1	1.0
p_2/p_1	2.8
M_s	1.6
M_p	0.7

$$\frac{ta_4}{L_{ref}} = 3.0$$



Shock Tube Expansion Waves - Summary

The start and end conditions are the same for all C^+ lines

J^+ invariants have the same value for all C^+ characteristics

C^- characteristics are straight lines in xt-space

Simple expansion waves centered at $(x,t)=(0,0)$

Expansion Waves

In a left-running expansion fan:

J^+ is constant throughout expansion fan, which implies:

$$u + \frac{2a}{\gamma - 1} = u_4 + \frac{2a_4}{\gamma - 1} = u_3 + \frac{2a_3}{\gamma - 1}$$

J^- is constant along C^- lines, but varies from one line to the next, which means that

$$u - \frac{2a}{\gamma - 1}$$

is constant along each C^- line

Expansion Waves

Since $u_4 = 0$ we obtain:

$$u + \frac{2a}{\gamma - 1} = u_4 + \frac{2a_4}{\gamma - 1} = \frac{2a_4}{\gamma - 1} \Rightarrow$$

$$\frac{a}{a_4} = 1 - \frac{1}{2}(\gamma - 1)\frac{u}{a_4}$$

with $a = \sqrt{\gamma R T}$ we get

$$\frac{T}{T_4} = \left[1 - \frac{1}{2}(\gamma - 1)\frac{u}{a_4}\right]^2$$

Expansion Wave Relations

Isentropic flow $\Rightarrow$ we can use the isentropic relations

complete description in terms of u/a_4

$$\frac{T}{T_4} = \left[1 - \frac{1}{2}(\gamma - 1)\frac{u}{a_4}\right]^2$$

$$\frac{p}{p_4} = \left[1 - \frac{1}{2}(\gamma - 1)\frac{u}{a_4}\right]^{\frac{2\gamma}{\gamma-1}}$$

$$\frac{\rho}{\rho_4} = \left[1 - \frac{1}{2}(\gamma - 1)\frac{u}{a_4}\right]^{\frac{2}{\gamma-1}}$$

Expansion Wave Relations

Since C^- characteristics are straight lines, we have:

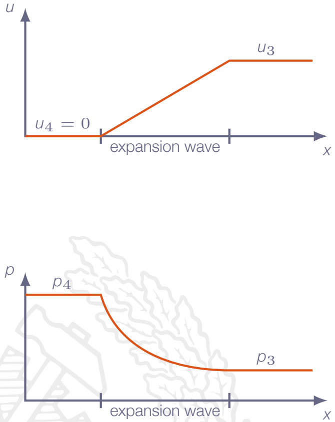
$$\frac{dx}{dt} = u - a \Rightarrow x = (u - a)t$$

$$\frac{a}{a_4} = 1 - \frac{1}{2}(\gamma - 1)\frac{u}{a_4} \Rightarrow a = a_4 - \frac{1}{2}(\gamma - 1)u \Rightarrow$$

$$x = \left[u - a_4 + \frac{1}{2}(\gamma - 1)u\right]t = \left[\frac{1}{2}(\gamma - 1)u - a_4\right]t \Rightarrow$$

$$u = \frac{2}{\gamma + 1}\left[a_4 + \frac{x}{t}\right]$$

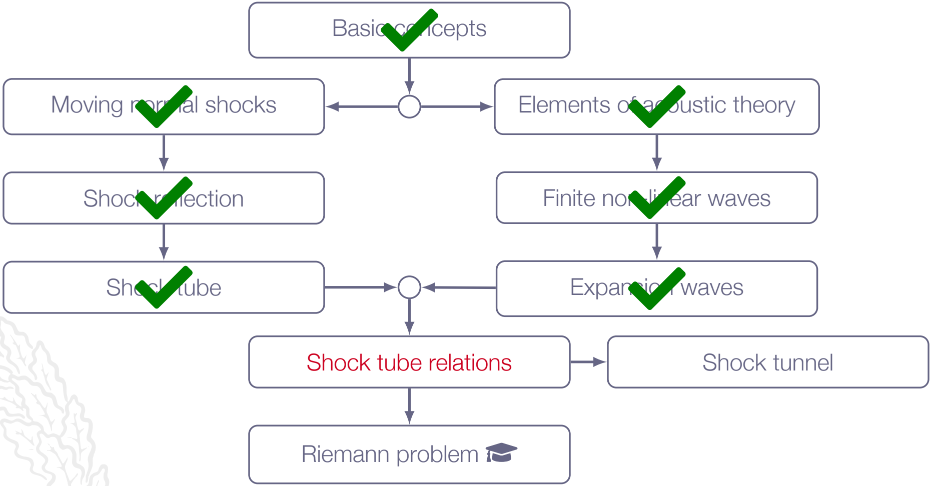
Expansion Wave Relations



Expansion wave head is advancing to the left with speed a_4 into the stagnant gas

Expansion wave tail is advancing with speed $u_3 - a_3$, which may be positive or negative, depending on the initial states

Roadmap - Unsteady Wave Motion



Chapter 7.8
Shock Tube Relations

Shock Tube Relations

$$u_p = u_2 = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left[\frac{\dfrac{2\gamma_1}{\gamma_1 + 1}}{\dfrac{p_2}{p_1} + \dfrac{\gamma_1 - 1}{\gamma_1 + 1}}\right]^{1/2}$$

$$\frac{p_3}{p_4} = \left[1 - \frac{\gamma_4 - 1}{2}\left(\frac{u_3}{a_4}\right)\right]^{2\gamma_4/(\gamma_4 - 1)}$$

solving for u_3 gives

$$u_3 = \frac{2a_4}{\gamma_4 - 1}\left[1 - \left(\frac{p_3}{p_4}\right)^{(\gamma_4 - 1)/(2\gamma_4)}\right]$$

Shock Tube Relations

But, $p_3 = p_2$ and $u_3 = u_2$ (no change in velocity and pressure over contact discontinuity)

$$\Rightarrow u_2 = \frac{2a_4}{\gamma_4 - 1}\left[1 - \left(\frac{p_2}{p_4}\right)^{(\gamma_4-1)/(2\gamma_4)}\right]$$

We have now two expressions for u_2 which gives us

$$\frac{a_1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\left[\frac{\frac{2\gamma_1}{\gamma_1+1}}{\frac{p_2}{p_1} + \frac{\gamma_1-1}{\gamma_1+1}}\right]^{1/2} = \frac{2a_4}{\gamma_4 - 1}\left[1 - \left(\frac{p_2}{p_4}\right)^{(\gamma_4-1)/(2\gamma_4)}\right]$$

Shock Tube Relations

Rearranging gives:

$$\frac{p_4}{p_1} = \frac{p_2}{p_1}\left\{1 - \frac{(\gamma_4 - 1)(a_1/a_4)(p_2/p_1 - 1)}{\sqrt{2\gamma_1\left[2\gamma_1 + (\gamma_1 + 1)(p_2/p_1 - 1)\right]}}\right\}^{-2\gamma_4/(\gamma_4-1)}$$

p_2/p_1 as implicit function of p_4/p_1

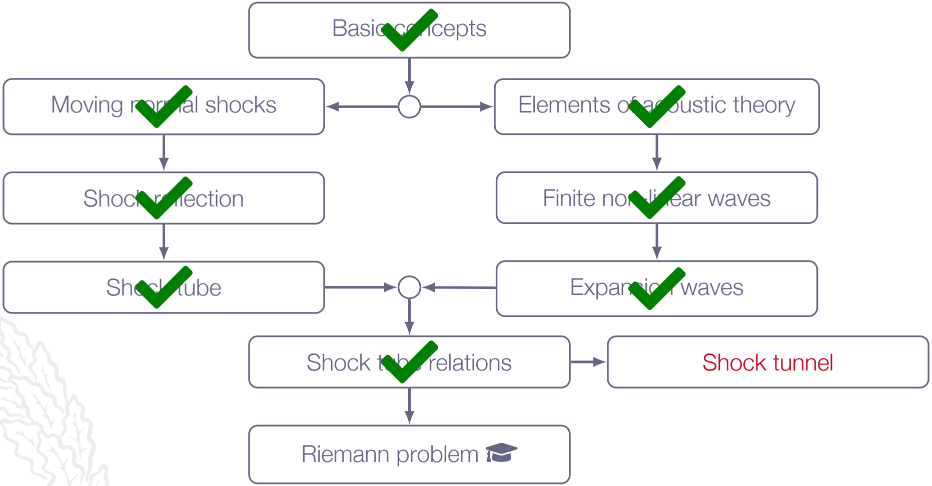
for a given p_4/p_1, p_2/p_1 will increase with decreased a_1/a_4

$$a = \sqrt{\gamma RT} = \sqrt{\gamma(R_u/M)T}$$

the speed of sound in a light gas is higher than in a heavy gas

- driver gas: low molecular weight, high temperature
- driven gas: high molecular weight, low temperature

Roadmap - Unsteady Wave Motion



Shock Tunnel

Addition of a convergent-divergent nozzle to a shock tube configuration

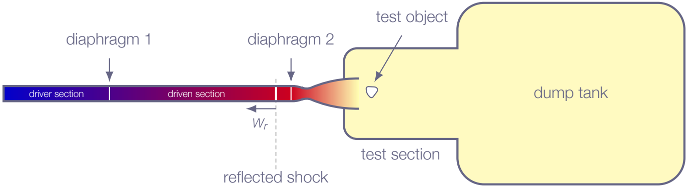
Capable of producing flow conditions which are close to those during the reentry of a space vehicles into the earth's atmosphere

- high-enthalpy, hypersonic flows (short time)
- real gas effects

Example - Aachen TH2:

- velocities up to 4 km/s
- stagnation temperatures of several thousand degrees

Shock Tunnel



1. High pressure in region 4 (driver section)

   diaphragm 1 burst
   primary shock generated

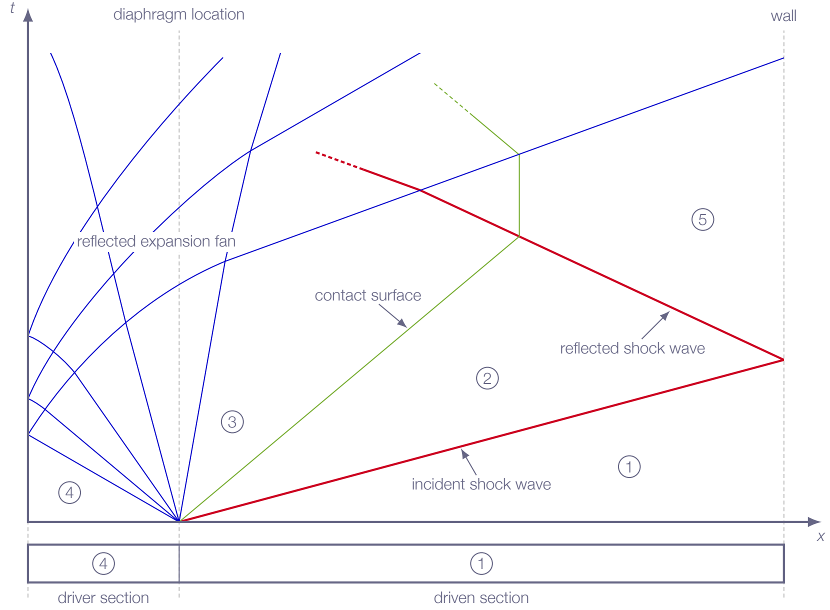
2. Primary shock reaches end of shock tube

   shock reflection

3. High pressure in region 5

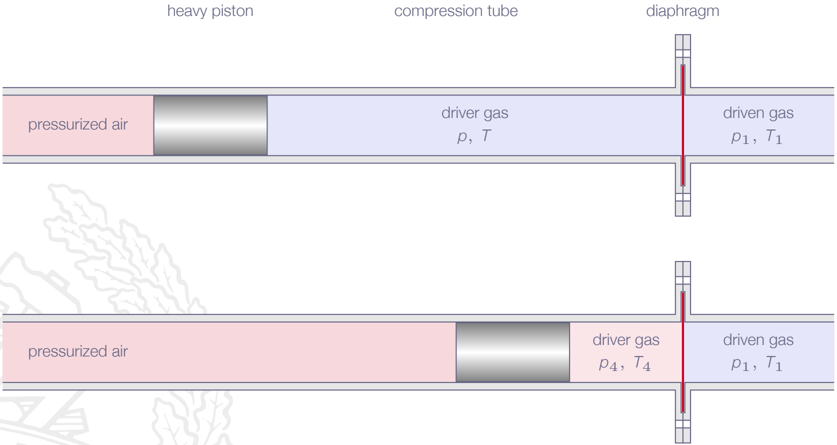
   diaphragm 2 burst
   nozzle flow initiated
   hypersonic flow in test section

Shock Tunnel



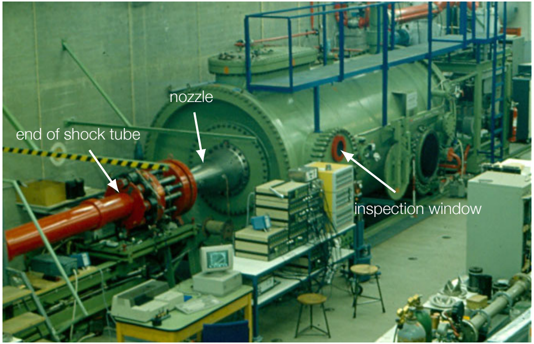
Shock Tunnel

By adding a compression tube to the shock tube a very high p_4 and T_4 may be achieved for any gas in a fairly simple manner



The Aachen Shock Tunnel - TH2

Shock tunnel built 1975



The Aachen Shock Tunnel - TH2

Shock tube specifications:

diameter	140 mm
driver section	6.0 m
driven section	15.4 m
diaphragm 1	10 mm stainless steel
diaphragm 2	copper/brass sheet
max operating (steady) pressure	1500 bar

The Aachen Shock Tunnel - TH2

Driver gas (usually helium):

100 bar $< p_4 <$ 1500 bar

electrical preheating (optional) to 600 K

Driven gas:

0.1 bar $< p_1 <$ 10 bar

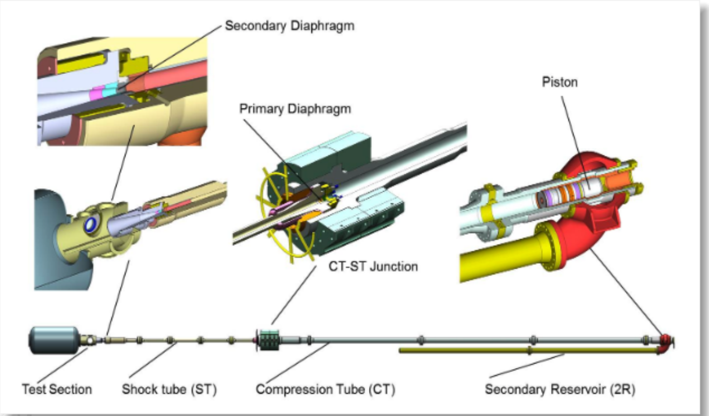
Dump tank evacuated before test

The Aachen Shock Tunnel - TH2

initial conditions			shock		reservoir		free stream			
p_4 [bar]	T_4 [K]	p_1 [bar]	M_s	p_2 [bar]	p_5 [bar]	T_5 [K]	M_∞	T_∞ [K]	u_∞ [m/s]	p_∞ [mbar]
100	293	1.0	3.3	12	65	1500	7.7	125	1740	7.6
370	500	1.0	4.6	26	175	2500	7.4	250	2350	20.0
720	500	0.7	5.6	50	325	3650	6.8	460	3910	42.0
1200	500	0.6	6.8	50	560	4600	6.5	700	3400	73.0
100	293	0.9	3.4	12	65	1500	11.3	60	1780	0.6
450	500	1.2	4.9	29	225	2700	11.3	120	2480	1.5
1300	520	0.7	6.4	46	630	4600	12.1	220	3560	1.2
26	293	0.2	3.4	12	15	1500	11.4	60	1780	0.1
480	500	0.2	6.6	50	210	4600	11.0	270	3630	0.7
100	293	1.0	3.4	12	65	1500	7.7	130	1750	7.3
370	500	1.0	5.1	27	220	2700	7.3	280	2440	26.3

The Caltech Shock Tunnel - T5

Free-piston shock tunnel



The Caltech Shock Tunnel - T5

Compression tube (CT):

length 30 m, diameter 300 mm
free piston (120 kg)
max piston velocity: 300 m/s
driven by compressed air (80 bar - 150 bar)

Shock tube (ST):

length 12 m, diameter 90 mm
driver gas: helium + argon
driven gas: air
diaphragm 1: 7 mm stainless steel
p_4 max 1300 bar

The Caltech Shock Tunnel - T5

Reservoir conditions:

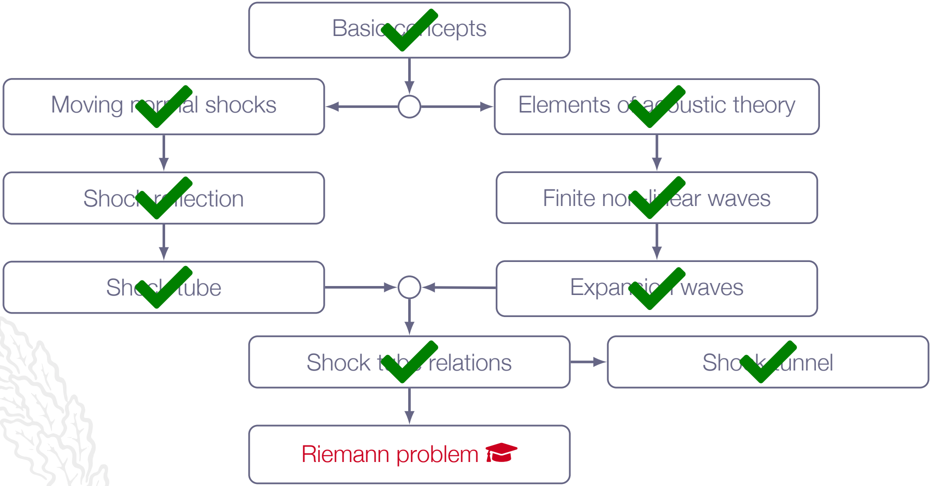
p_5 1000 bar
T_5 10000 K

Freestream conditions (design conditions):

M_∞ 5.2
T_∞ 2000 K
p_∞ 0.3 bar
typical test time 1 ms

Other Examples of Shock Tunnels

Roadmap - Unsteady Wave Motion



Riemann Problem

The shock tube problem is a special case of the general **Riemann Problem**

> *"... A Riemann problem, named after Bernhard Riemann, consists of an initial value problem composed by a conservation equation together with piece-wise constant data having a single discontinuity ..."*

Wikipedia

Riemann Problem

May show that solutions to the shock tube problem have the general form:

$$p = p(x/t)$$
$$\rho = \rho(x/t)$$
$$u = u(x/t)$$
$$T = T(x/t)$$
$$a = a(x/t)$$

where $x = 0$ denotes the position of the initial jump between states 1 and 4

Riemann Problem - Shock Tube Simulation

Numerical method:

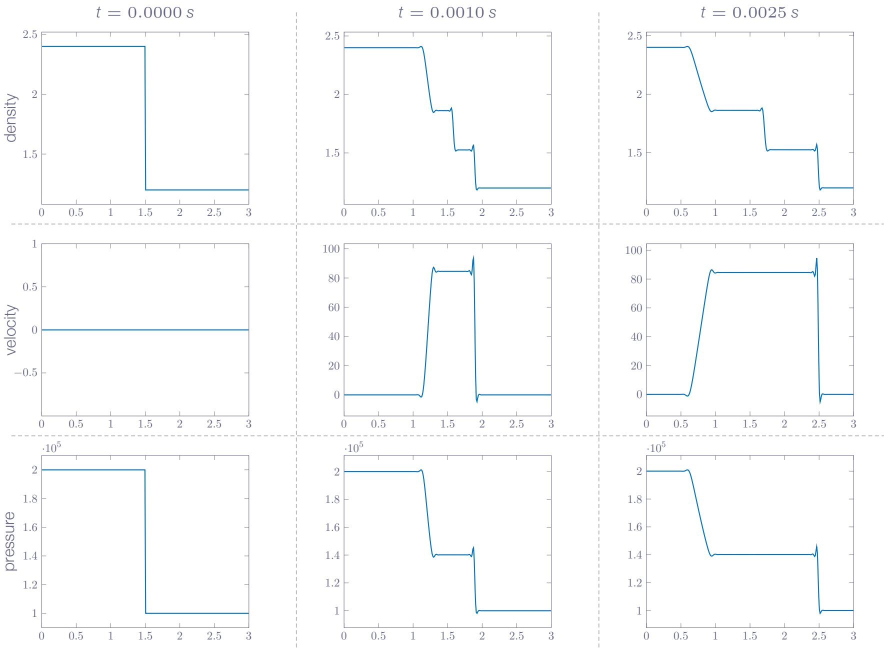
- Finite-Volume Method (FVM) solver
- three-stage Runge-Kutta time stepping
- third-order characteristic upwinding scheme
- local artificial damping

Left side conditions (state 4):

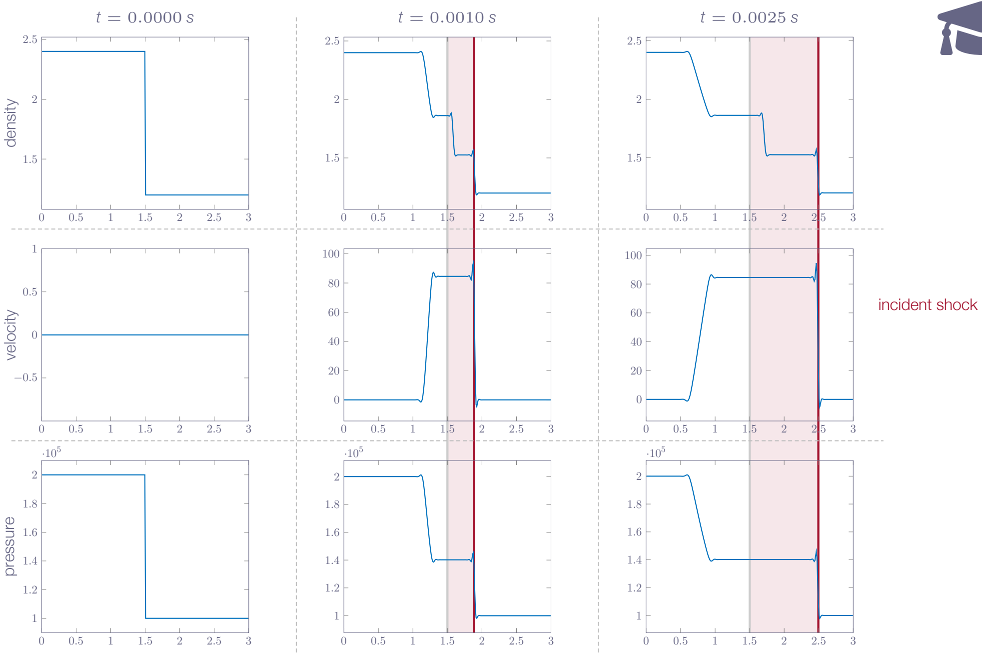
$$\rho = 2.4\ kg/m^3$$
$$u = 0.0\ m/s$$
$$p = 2.0\ bar$$

Right side conditions (state 1):

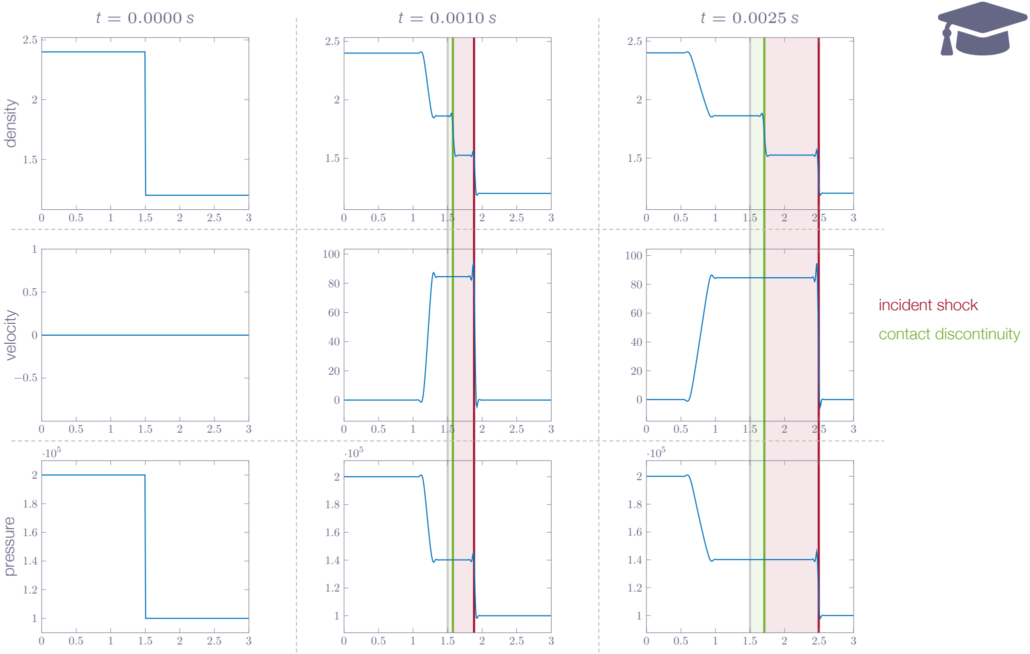
$$\rho = 1.2\ kg/m^3$$
$$u = 0.0\ m/s$$
$$p = 1.0\ bar$$



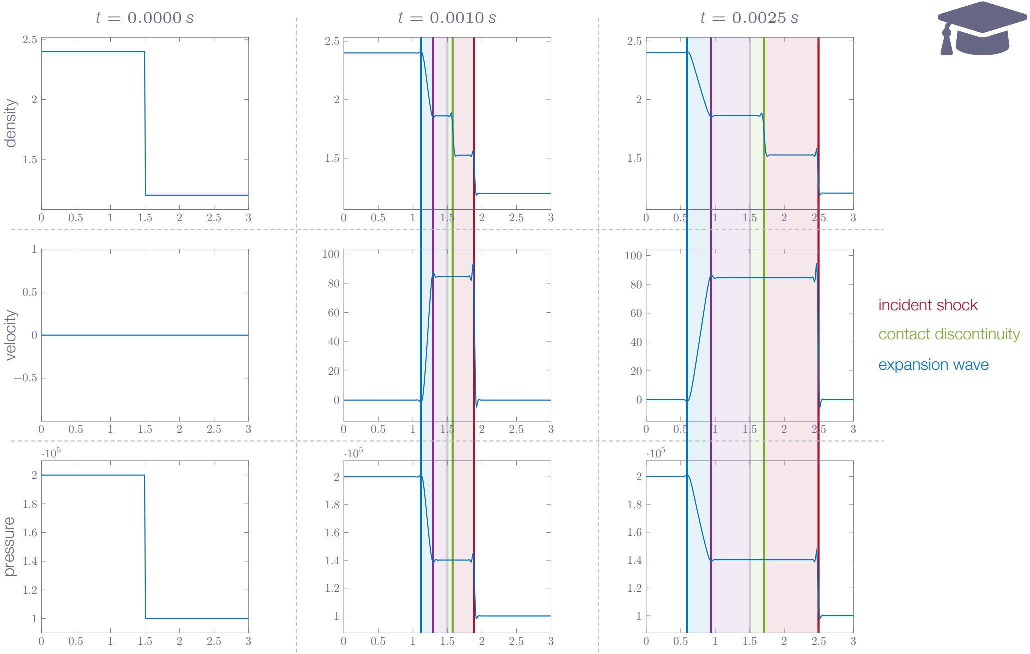
t = 0.0000 s
t = 0.0010 s
t = 0.0025 s
density
velocity
pressure
·10^5



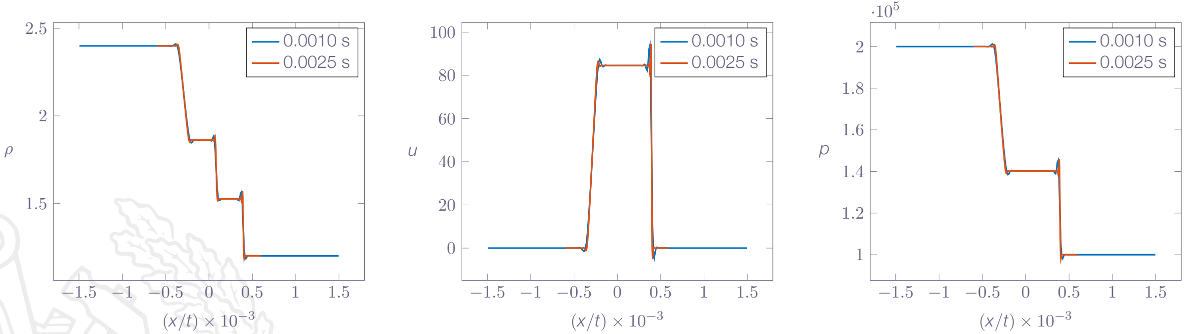
t = 0.0000 s
t = 0.0010 s
t = 0.0025 s
density
velocity
pressure
·10^5
incident shock



t = 0.0000 s
t = 0.0010 s
t = 0.0025 s
density
velocity
pressure
·10^5
incident shock
contact discontinuity

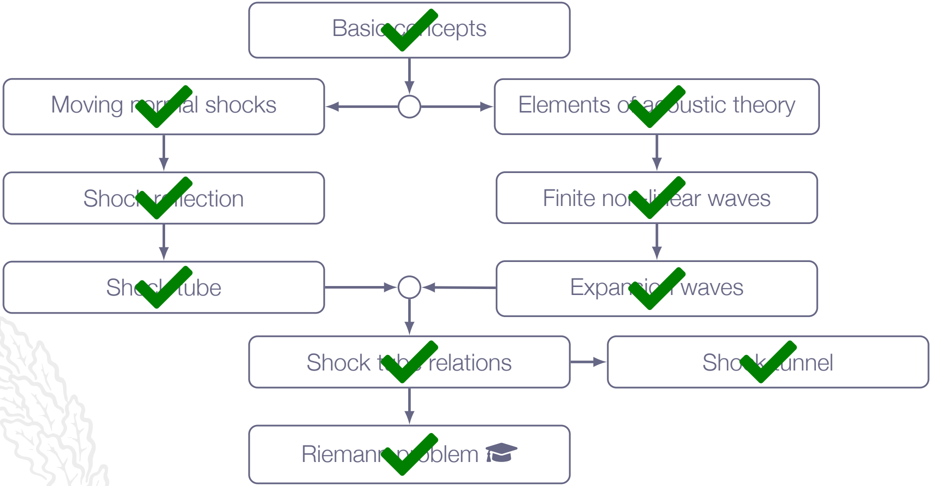

t = 0.0000 s
t = 0.0010 s
t = 0.0025 s
density
velocity
pressure
·10^5
incident shock
contact discontinuity
expansion wave

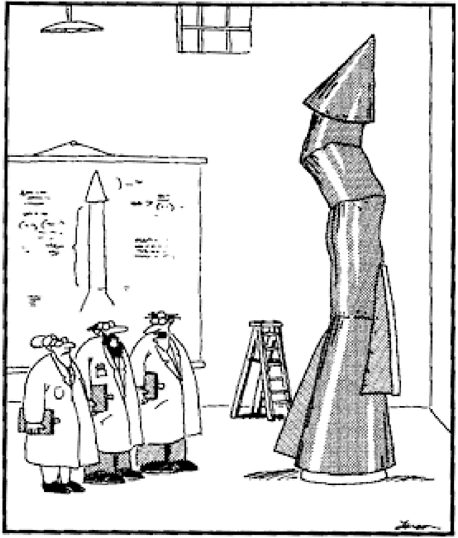
Riemann Problem - Shock Tube Simulation



The solution can be made self similar by plotting the flow field variables as function of x/t

Roadmap - Unsteady Wave Motion





"It's time we face reality, my friend. ... We're not exactly rocket scientists."