Excersise session 7

P4 exam 2011

A constant-area duct with electric heating elements added is joined with a converging-diverging nozzle (see figure below). The cross-section area of the heated duct betwen stations 1 and 2 is $0'05 m^2$. The nozzle part between stations 2 and 3 has a throat area A_{th} which is smaller than both A_2 and A_3. The flow of air entering at station 1 has a total pressure $p_{01} = 4$ bar, total temperature $T_{01} = 300K$, and a Mach number $M_1 = 0'2$. The flow is heated by the electric heating elements between stations 1 and 2, with a total heating power of 9'716 MW. The nozzle throat area A_{th} is chosen such that supersonic flow is achieved and that pressure matching occurs, i.e. $p_3 = p_{amb}$. The ambient pressure is assumed to be 1 bar.

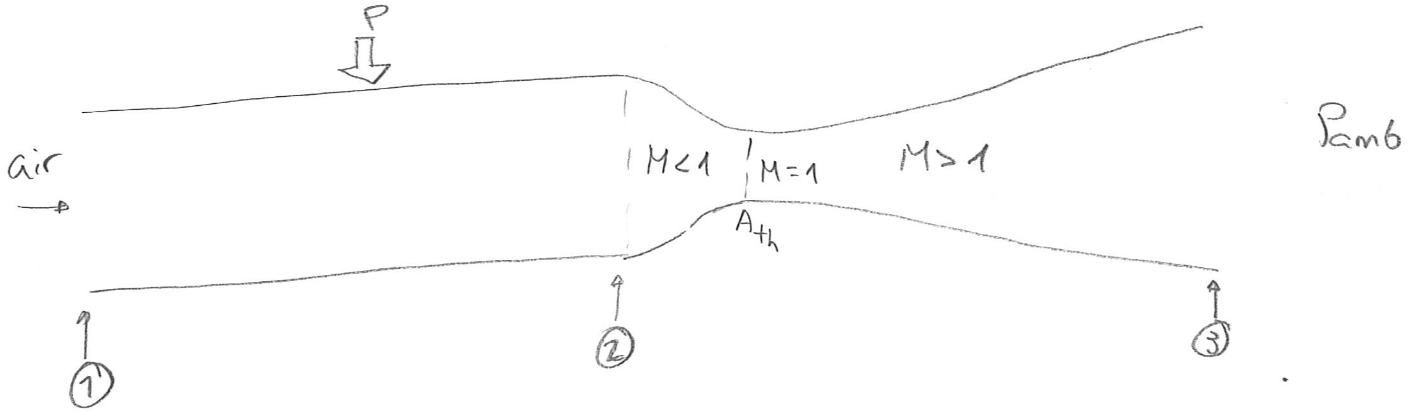
The flow may be regarded an 1-dimensional and steady-state. All viscous effects may be neglected. In the heated duct part all electric power may be assumed to be transferred to the air, and in the nozzle part the flow may be regarded as adiabatic. The air may be treated as calorically perfect gas with $R = 287\ J/kgK$ and $\gamma = 1'4$.

a) what is the nozzle throat area A_{th}?

b) what is the nozzle exit area A_3?

c) Compute the nozzle exit Mach number M_3 and static temperature T_3.

Supersonic flow at the nozzle exit is assumed.



Data: $A_1 = A_2 = 0'05\,m^2$ $\quad R = 287\ J/kg\,K$

$P_{01} = 4\,bar$ $\quad \gamma = 1'4$

$T_{01} = 300K$

$M_1 = 0'2$

$P = 9'716\,MW$

$P_3 = P_{amb} = 1\,bar$

a)

First is to calculate the mass flow through the system

$$P_1 = \frac{P_{01}}{\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/\gamma-1}} = 3'89\ bar$$

$$T_1 = \frac{T_{01}}{1+\frac{\gamma-1}{2}M_1^2} = 297'62\,K$$

$$\rho_1 = \frac{P_1}{RT_1} = 4'554\ \frac{kg}{m^3}$$

$$u_1 = M_1 a_1 = M_1\sqrt{\gamma R T_1} = 69'162\,m/s$$

$$\dot m = \rho_1 u_1 A_1 = 15'75\ kg/s$$

q: added heat per unit mass of air

$$q = \frac{P}{\dot m} = 616945\ J/kg$$

$$T_{02} = T_{01} + \frac{q}{c_p} = 300 + \frac{616945}{1004'5} = 914'18\,K$$

Table A.3 $\left\{M_1 = 0'2\right\} \Rightarrow \frac{T_{01}}{T_0^*} = 0'1736$

$$T_0^* = 1728'11\,K$$

$\frac{T_{02}}{T_0^*} = 0'529 \Rightarrow$ Table A.3 $\left\{\frac{T_{02}}{T_0^*} = 0'529\right\} \Rightarrow M_2 = 0'4$

Table A.1 $\Rightarrow \{M_2 = 0'4\} \Rightarrow \frac{A_2}{A^*} = 1'59 = \frac{A_2}{A_{th}}$

$$\boxed{A_{th} = 0'03145 \, m^2}$$

b & c)

$$P_{02} = \frac{P_{02}}{P_0^*} \frac{P_0^*}{P_{01}} P_{01} = 1'157 \cdot \frac{1}{1'235} \cdot 4 = 3'75 \text{ bar}$$

T.A.1 (M_2) T.A.1 (M_1)

If the exit of the nozzle is supersonic, no shocks and thus isentropic $\Rightarrow P_{02} = P_{03}$ & $T_{02} = T_{03}$ & $A_2^* = A_3^* = A_{th}$

$$\frac{P_{03}}{P_3} = \left(1 + \frac{\gamma - 1}{2} M_3^2\right)^{\frac{\gamma}{\gamma-1}} = 3'75 \Rightarrow \boxed{M_3 = 1'514} \text{ c)}$$

Table A.1 $\Rightarrow \{M_3 = 1'514\} \Rightarrow \frac{A_3}{A^*} = 1'186 \Rightarrow \boxed{A_3 = 0'0373 \, m^2}$ b)

$$\boxed{T_3 = \frac{T_{03}}{1 + \frac{\gamma - 1}{2} M_3^2} = 626'82 \, K} \text{ c)}$$

P3 exam 2012

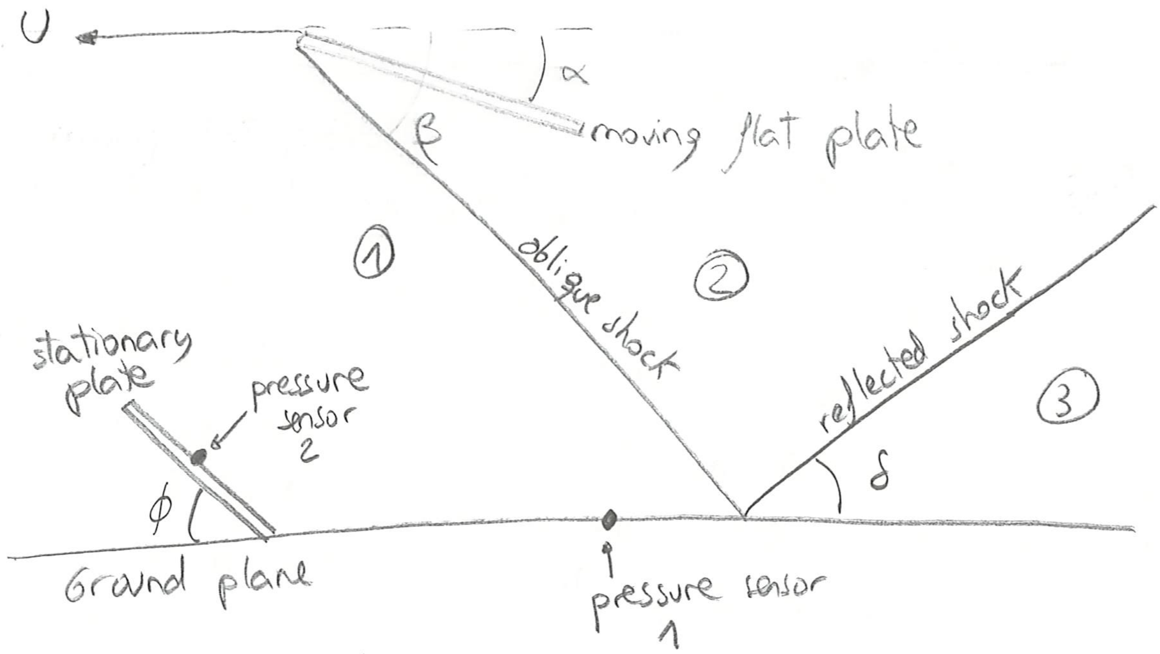
A flat plate is moving above the groud plate at a constant speed $U = 1735'94$ m/s and in a direction which is parallel to the ground (see figure below). The flat plate has an angle of attack $\alpha = 20°$. The span of the flat plate (the extent in the direction out of the paper plane) is assumed to be so large that the flow may be considered two dimensional. The oblique shock which is generated by the flat plate as it moves through the air at supersonic speed is reflected by the ground plane. Two pressure sensors are mounted, one on the ground plane and one on a stationary flat plate that has an angle ϕ that is equal the shock angle β. As the shock system sweeps to the left (following the moving flat plate) it will first hit the ground-mounted sensor. This sensor will then register a sudden pressure increase. Next the shock system will hit the stationary flat plate. The pressure sensor mounted there will then increase a sudden pressure increase.

The ambient air conditions are $p = 1\,bar$ and $T = 300K$. The calorically perfect gas assumption with $R = 287 \frac{J}{kgK}$ and $\gamma = 1'4$ may be used. All viscous effects may be neglected.

a) Compute the primary shock angle β and the reflected shock angle δ.

b) Compute the value of the sudden increase at sensor 1

c) Compute the value of the sudden increase at sensor 2



Data: $u = 1735'94\,m/s$

$\alpha = 20°$

$P_1 = 1\,bar$

$T_1 = 300K$

$R = 287 \frac{J}{kgK}$

$\gamma = 1'4$

a) First we change the frame of reference so that it follow the flat plate

$$M_1 = \frac{U}{a_1} = \frac{U}{\sqrt{\gamma R T_1}} = \frac{1735'94}{\sqrt{1'4 \cdot 287 \cdot 300}} = 5$$

Main shock:

$$\theta\text{-}\beta\text{-}M \left\{ \begin{array}{l} M = 5 \\ \theta = 20° \end{array} \right\} \Rightarrow \boxed{\beta_1 \simeq 29'8°}$$

$$M_{n1} = M_1 \sin(\beta_1) = 2'485$$

$$M_{n2} = \left[\frac{M_{n1}^2 + 2/(\gamma-1)}{[2\gamma/(\gamma-1)] M_{n1}^2 - 1} \right]^{1/2} = 0'514$$

$$M_2 = \frac{M_{n2}}{\sin(\beta_1 - \theta)} = 3'022$$

$$\text{Table A.2} \left\{ M_1 = 2'485 \right\} \Rightarrow \frac{P_2}{P_1} = 7'037 \Rightarrow P_2 = 7'037 \text{ bar}$$

Reflected shock:

$$\theta\text{-}\beta\text{-}M \left\{ \begin{array}{l} M = 3'022 \\ \theta = 20° \end{array} \right\} \Rightarrow \beta_2 = 37'6° \Rightarrow \boxed{\delta = \beta_2 - \theta = 17'6°}$$

b) Sudden increase in pressure measured by sensor 1 $\Rightarrow P_3 - P_1 = \Delta P_1$

$$M_{n2} = M_2 \sin(\beta_2) = 1'844$$

$$M_{n3} = \left[\frac{M_{n2}^2 + 2/(\gamma-1)}{[2\gamma/(\gamma-1)] M_{n2}^2 - 1} \right]^{1/2} = 0'607$$

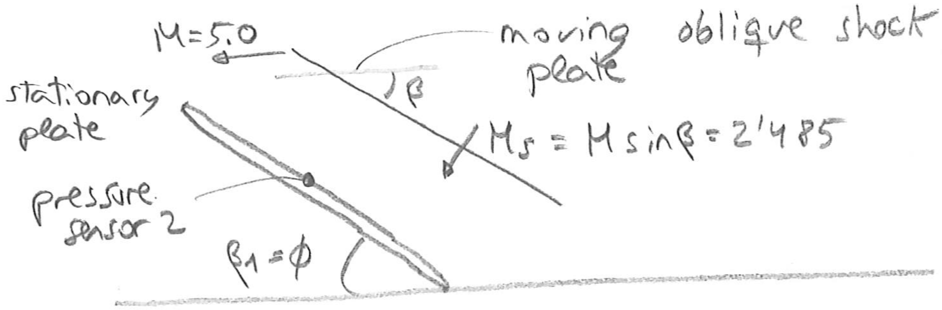
$$M_3 = \frac{M_{n3}}{\sin(\beta_2 - \theta)} = 2'007$$

$$\text{T.A.2} \left\{ M_2 = 1'844 \right\} \Rightarrow \frac{P_3}{P_2} = 3'801 \Rightarrow P_3 = 26'75 \text{ bar}$$

Sensor 1 pressure increase: $\boxed{\Delta p_1 = P_3 - P_1 = 25'75 \text{ bar}}$

c) We change back the frame of reference to that which is stationary.

We are left now with an unsteady 1D problem of the second sensor mounted in the flat plate



1D moving shock with $M_s = 2'485$

Reflected shock Mach number given by

(7.23) $$\frac{M_R}{M_R^2-1} = \frac{M_s}{M_s^2-1}\sqrt{1+\frac{2(\gamma-1)}{(\gamma+1)^2}(M_s^2-1)\left(\gamma+\frac{1}{M_s^2}\right)} = 0'699615$$

$$M_R^2 - 1 = \frac{M_R}{0'699615}$$

$$M_R^2 - \frac{1}{0'699615}M_R - 1 = 0 \xrightarrow[\text{solution}]{\text{positive}} M_R = 1'944$$

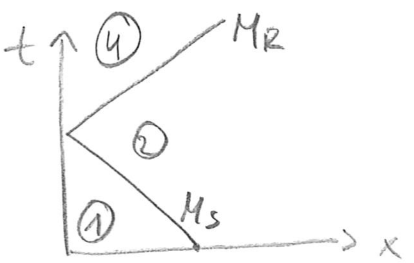


Table A.2 with $\{M_R = 1'944\} \Rightarrow \frac{P_4}{P_2} = 4'23 \Rightarrow P_4 = 29'76$ bar

The sudden increase in pressure measured by the sensor 2 is.

$$\boxed{\Delta p_2 = P_4 - P_1 = 28'76 \text{ bar}}$$

P4 exam 2012

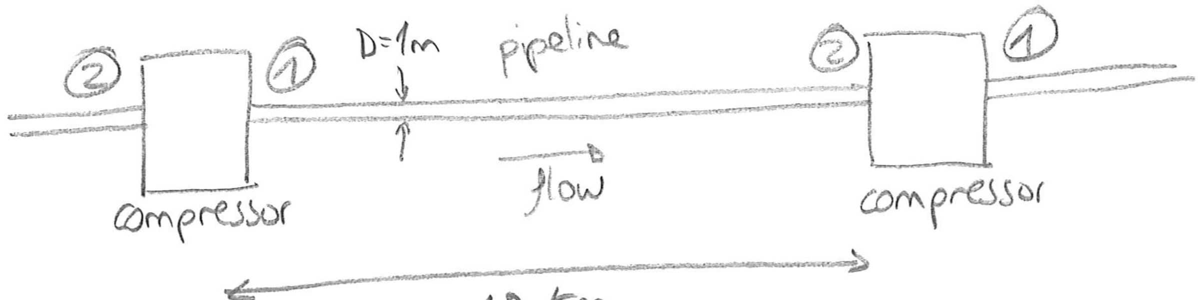
A pipeline for natural gas consists of 10 km long sections with compressor stations in between (see figure below). The Tube has a round cross-section with an inner diameter $D = 1m$. Each section has the same inflow conditions (1) and the same outflow conditions (2). All compressors give the same pressure ratio (P_1/P_2). For simplicity we assume that the pipeline is thermally insulated to prevent any heat exchange with the surroundings. Natural gas consist of mostly methane, so it is assumed that the gas behaves like a calorically perfect gas with $R = 520 \frac{J}{kg\,K}$ and $\gamma = 1'24$. The compressors were all adjusted to give $P_2 = 2$ bar and $T_2 = 300K$ and a mass flow rate of 44'286 kg/s. The friction coefficient is assumed to be $\bar{f} = 0'005$.

a) compute the pressure ratio of the compressors (P_1/P_2)

b) Assuming that the compressors are ideal isentropic devices, should the gas coming out of each compressor be cooled or heated before being led into the next tube section?

c) In practice pipelines are not thermally insulated, which means that there will be exchange of heat with the surroundings. How will this affect the answer to point b?

NOTE THAT γ IS NOT 1'4, TABLES CAN NOT BE USED!



Data: $L = 10\ km$ $\quad P_2 = 2.0\ bar$

$D = 1m$ $\quad T_2 = 300\ K$

$R = 520\ J/kg\,K$ $\quad \dot{m} = 44'286\ kg/s$

$\gamma = 1'24$ $\quad \bar{f} = 0'005$

a) $\rho_2 = \dfrac{P_2}{RT_2} = 1'282\ kg/m^3$

$\dot{m}_2 = \rho_2 u_2 \cdot \pi \dfrac{1}{4} \Rightarrow u_2 = 43'98 m/s$

$a_2 = \sqrt{\gamma R T_2} = 439'82 m/s$

$M_2 = \dfrac{u_2}{a_2} = 0'1$

(3.103) $\dfrac{T_2}{T^*} = \dfrac{\gamma+1}{2+(\gamma-1)M_2^2} = 1'118658 \Rightarrow T^* = 268'18\ K$

(3.104) $\dfrac{P_2}{P^*} = \dfrac{1}{M_2}\left(\dfrac{\gamma+1}{2+(\gamma-1)M_2^2}\right)^{1/2} = 10'57666 \Rightarrow P^* = 0'1891\ bar$

(3.107) $\dfrac{4\bar{f}L_2^*}{D} = \dfrac{1-M_2^2}{\gamma M_2^2} + \dfrac{\gamma+1}{2\gamma}\ln\left(\dfrac{(\gamma+1)M_2^2}{2+(\gamma-1)M_2^2}\right) = 75'78$

$L_2^* = \dfrac{75'78 D}{4f} = 3789\ m$

$L_1^* = L_2^* + L = 13789 m$

$\dfrac{4\bar{f}L_1^*}{D} = 275'78$

Now from (3.107) we can get M_1

$\dfrac{4\bar{f}L_1^*}{D} = 275'78 = \dfrac{1-M_1^2}{\gamma M_1^2} + \dfrac{\gamma+1}{2\gamma}\ln\left(\dfrac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right)$

Trying some values

M_1	$\frac{4\bar{f}L_1^*}{D}$
0'03	889
0'05	316'5
0'06	218'23
0'055	260'65

Linear interpolation between $M_1 = 0'55$ and $M_1 = 0'5$

$$\frac{M_1 - 0'05}{275'78 - 316'5} = \frac{0'055 - 0'05}{260'65 - 316'5} \Rightarrow M_1 = 0'05371$$

we check and for $M_1 = 0'05371 \Rightarrow \frac{4\bar{f}L_1^*}{D} = 273'51$ which we consider to be sufficiently close to 275'78.

$$(3.104) \quad \frac{P_1}{P^*} = \frac{1}{M_1}\left(\frac{\gamma+1}{2+(\gamma-1)M_1^2}\right)^{1/2} = 19'698$$

$$\boxed{\frac{P_1}{P_2} = \frac{P_1}{P^*}\frac{P^*}{P_2} = 19'698 \cdot \frac{1}{10'5766} = 1'8624}$$

b) Lets calculate the temperature at 1

$$(3.103) \quad \frac{T_1}{T^*} = \frac{\gamma+1}{2+(\gamma-1)M_1^2} = 1'119 \Rightarrow T_1 = 300'25\,K$$

If the compressors where ideal (isentropic flow) the $T_{1\,ideal}$ that we will obtain is (with the same pressure ratio)

$$\frac{P_1}{P_2} = \left(\frac{T_{1\,ideal}}{T_2}\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow T_{1\,ideal} = 338'37\,K$$

If the compressor were ideal, at the exit of the compressor (1) the flow temperature would be higher than desired ($T_{1\,ideal} > T_1$) so it would have to be cooled down, otherwise the temperature would increase in each succesive section.

c) If the problem wouldn't be thermally insulated, then it is not necessary to cool down the fluid (or in case it needs cooling, it will require less) since it would exchange heat with the surrounding atmosphere which very likely is under 65°C.