Exercise session 6:

P2 exam 2009

A convergent/divergent nozzle with circular cross section is connected to a large reservoir with pressurized air at pressure of 10.0 bar and a temperature of 600K. The throat diameter is 1cm and the nozzle exit diameter is 3'044 cm. The air flowing inside the tube may be regarded as calorically perfect gas with $\gamma = 1'4$ and $R = 287\ J/kgK$. We assume inviscid steady-state flow.

Case 1: the nozzle pressure ratio (p_o/p_{exit}) is slowly raised from an initial value of 1'0 by lowering the exit pressure. At some critical value sonic conditions are obtained at the throat.

Compute:

a) The critical pressure ratio

b) The Mach number at the exit (subsonic branch) at the critical pressure ratio

c) The mass flow through the nozzle at the critical pressure ratio

Data: $P_{o_1} = 10\,bar$ $\quad\gamma = 1'4$
$T_{o_1} = 600K$ $\quad R = 287\ J/kgK$
$D_t = 0'01m$
$D_e = 0'03044m$

a) Critical flow → subsonic all the way, except throat ($M_t = 1$)
No shock → isentropic → constant $\begin{cases} A^* \\ P_o \\ T_o \end{cases}$

$$\frac{A_e}{A_t} = \left(\frac{D_e}{D_t}\right)^2 = 9'266$$

Table A.1 → $\boxed{\dfrac{P_o}{P_{exit}} \approx 1'003}$

b)

From T A1 $\rightarrow$ $\boxed{Me \approx 0'06}$

c)

(5.21) $\boxed{\dot{m} = \frac{P_0 A^*}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} = 0'1296 \, kg/s}$

Other altarative:

From T.A1 $\Rightarrow$ $\frac{T_0}{T_e} \approx 1'009 \Rightarrow T_e = 594'64 \, K$

$\frac{P_0}{P_e} \approx 1'003 \Rightarrow P_e = 997008'97 \, Pa$

$\rho_e = \frac{P_e}{R T_e} = 5'842 \, \frac{kg}{m^3}$

$\boxed{\dot{m}_e = \dot{m} = \rho_e M_e \sqrt{R \gamma T_e} \cdot A_e = 0'1246 \, kg/s}$

Case 2: For a nozzle pressure ratio (P_0/P_{exit}) of 1'6033 we obtain choked flow and a normal shock between the throat and the exit. Compute

a) The location of the normal shock (D/D_t)

b) The Mach number upstream and downstream of the normal shock

c) The exit Mach number, temperature and velocity

d) The mass flow through the nozzle

(5.28) $Me^2 = \frac{-1}{\gamma-1} + \sqrt{\frac{1}{(\gamma-1)^2} + \left(\frac{2}{\gamma-1}\right)\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(\frac{P_{01} A_t}{P_e A_e}\right)^2}$

$\boxed{Me = 0'10}$ c)

$P_e = \frac{P_e}{P_{01}} P_{01} = 6'237 \, bar$

$P_{0e} = P_e \left(1 + \frac{\gamma-1}{2} Me^2\right)^{\gamma/\gamma+1} = 6'2807 \, bar$ (since we have a shock between the throat and the outlet P_0 is not constant in the entire nozzle)

① condition upstream the shock (same P_0, T_0 as inlet)

② condition downstream the shock (same P_0, T_0 as outlet)

$$\frac{P_{02}}{P_{01}} = \frac{P_{0e}}{P_{01}} = 0'62807$$

$$\text{Table A.2} \left\{ \frac{P_{02}}{P_{01}} = 0'62807 \right\} \Rightarrow \left\{ \boxed{\begin{array}{l} M_1 = 2'2 \\ M_2 = 0'5471 \end{array}} \right. \quad b)$$

Isentropic flow from the inlet until righ before the shock ($A_1^* = A_t$)

$$T.A.1 \left\{ M = 2'2 \right\} \Rightarrow \frac{A_1}{A^*} = \frac{A_1}{A_t} = 2'00497$$

$$\frac{A_1}{A_t} = 2'00497 = \frac{\pi\left(\frac{D_1}{2}\right)^2}{\pi\left(\frac{D_t}{2}\right)^2} = \left(\frac{D_1}{D_t}\right)^2 \Rightarrow \boxed{\frac{D_1}{D_t} = \sqrt{2'00497} = 1'416} \quad a)$$

Adiabatic steady-state flow → constant T_0 through the nozzle

$$T_{0e} = T_{01} = 600k$$

$$\boxed{T_e = \frac{T_{0e}}{1 + \frac{\gamma - 1}{2} M_e^2} = 598'8k} \quad c)$$

$$\boxed{u_e = M_e a_e = M_e \sqrt{\gamma R T_e} = 49'05 m/s} \quad c)$$

$$\rho_e = \frac{P_e}{R T_e} = \frac{6'237 \cdot 10^5}{287 \cdot 598'8} = 3'629 \frac{kg}{m^3}$$

$$\boxed{\dot{m}_e = \dot{m} = \rho_e u_e A_e = 0'1295 \frac{kg}{s}} \quad d) \quad \text{or use (eq. 5.21)}$$

P 1 exam 2011

A steady-state supersonic flow of air first passes a convex corner and then a concave corner, bringing the flow direction back to the original one. The flow deflection θ is 20° for both corners. The inflow Mach number is such that the angle ϕ for the corresponding Mach wave is 138'1897°

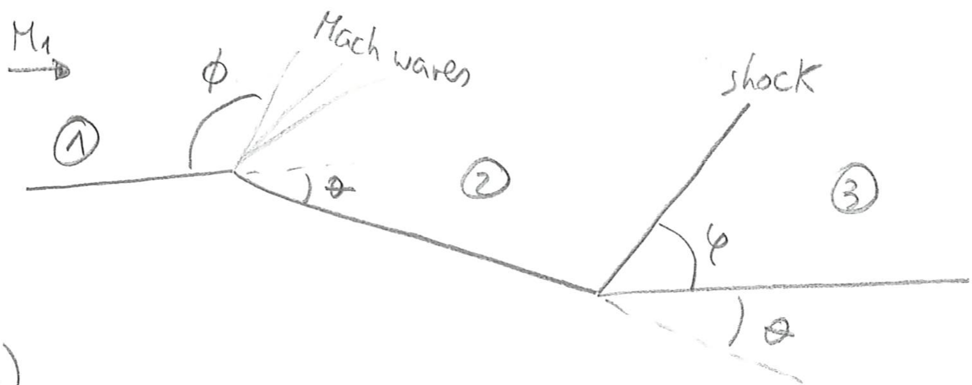
a) M_1

b) M_2

c) M_3 and φ

d) Find the minimum M_1 for which there is an attached shock solution at the concave corner

The air may be treated as calorically perfect gas with $\gamma = 1'4$ and $R = 287 \frac{J}{kg\,K}$. All viscous effects may be neglected



a)

Angle μ of incoming flow

$$\mu = 180° - \phi = 41'8103°$$

$$(4.1)\quad \sin(\mu) = \frac{1}{M_1} \Rightarrow \boxed{M_1 = \frac{1}{\sin(\mu)} = 1'5}$$

b) Table A.5 $\{M_1 = 1'5\} \Rightarrow \nu(M_1) = 11'91$

$$(4.45)\quad \theta = \nu(M_2) - \nu(M_1) \Rightarrow \nu(M_2) = 31'91$$

Table A.5 $\{\nu(M_2) = 31'91\} \Rightarrow \boxed{M_2 = 2'207}$

c)

$\theta-\beta-M$ relation $\left\{ \begin{array}{l} \theta=20^\circ \\ M_2=2'207 \end{array} \right\} \Rightarrow \beta=48^\circ$

$\boxed{\rho=\beta-\theta=28^\circ}$

$M_{n2}=M_2 \sin\beta = 1'6401$

(4.10) $M_{n3}^2 = \dfrac{M_{n2}^2+\frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_{n2}^2-1} \Rightarrow M_{n3}=0'6567$

Other alternative is to use the tables or equation for normal shocks

$\boxed{M_3=\dfrac{M_{n3}}{\sin(\beta-\theta)}=\dfrac{0'6567}{\sin(28^\circ)}=1'4}$

d) $\theta-\beta-M$ relation $\{\theta=20^\circ\} \Rightarrow M_2 \simeq 1'84$ is the smallest value of M_2 for which an attached shock occurs

Prandtl-Mayer expansion

TA5 $M_2=1'84 \Rightarrow \nu(M_2)=21'88$

$\nu(M_1)=\nu(M_2)-20^\circ=1'88^\circ$

TA.5 $\nu(M_1)=1'88^\circ \Rightarrow \boxed{M_1=1'127}$

P2 exam 2011

A pipe with circular cross-section has a constant diameter $D=1cm$. The pipe is insulated to prevent any heat transfer to the flow of air inside. At one end the pipe is connected to a reservoir with pressure $P_0=10$ bar and $T_0=300K$. The other end of the pipe is open to ambient air, with pressure $P_{amb}=1$ bar. The flow of air in the pipe is assumed to be chocked with $M_2=1$ and $P_2=P_{amb}$

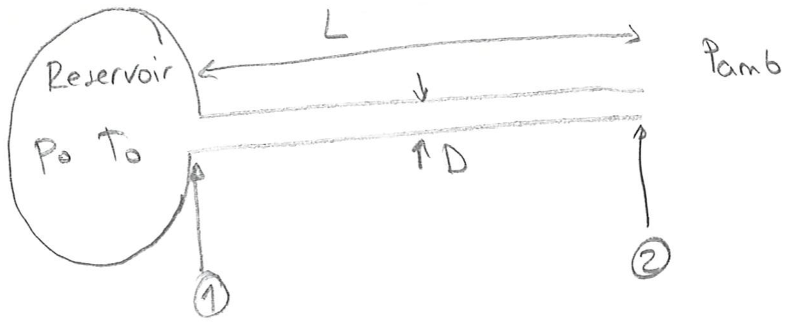
The air may be treated as calorically perfect gas with $\gamma = 1'4$ and $R = 287 \frac{J}{kg\,K}$. The friction coefficient of the pipe is found to be $f = 0'005$

a) What is the length of the pipe?

b) What is the mass flow of air in the pipe?

c) A small hole is drilled through the pipe at a position 1'154 m from the outflow end and a pressure probe is inserted. What static pressure do you expect to measure?

Data:
$D = 1\,cm$
$P_{0_1} = 10\,bar$
$T_{0_1} = 300\,K$
$M_2 = 1$
$P_2 = P_{amb} = 1\,bar$
$f = 0'005$
$\gamma = 1'4$
$R = 287 \frac{J}{kg\,K}$



a) Since the flow is chocked $(M_2 = 1) \rightarrow L = L^*$; $P_{02} = P_0^*$; $P_2 = P^*$

$$P_{02} = P_2 \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}} = 1'8929\,bar$$

$$\frac{P_{01}}{P_0^*} = \frac{10}{1'8929} = 5'283$$

Table A.4 $\left\{ \frac{P_0}{P_0^*} = 5'283 \right\} \Rightarrow M_1 = 0'111$

Table A.4 $\{ M = 0'111 \} \Rightarrow \frac{4fL^*}{D} = 54'81 \Rightarrow \boxed{L^* = L = 27'4\,m}$

b) Adiabatic flow: T_0 = constant through the pipe

$$\left.\begin{matrix} M_2 \\ T_{02} = T_{01} \end{matrix}\right\} \quad T_2 = \frac{T_{02}}{1 + \frac{\gamma-1}{2} M_2^2} = 250\,K$$

$$\rho_2 = \frac{P_2}{R T_2} = 1'394 \frac{kg}{m^3}$$

$u_2 = M_2 a_2 = M_2 \sqrt{\gamma R T_2} = 316'9\ m/s$

$$\boxed{\dot{m}_2 = \dot{m} = \rho_2 u_2 A_2 = \rho_2 u_2 \pi \left(\frac{D}{2}\right)^2 = 0'0347\ kg/s}$$

c) At 1'154 m from The end of The Tube, we have a new

$L^* = 1'154\ m$

$\frac{4fL^*}{D} = 2'308$

Table A.4 $\frac{4fL^*}{D} = 2'308 \Rightarrow \frac{p}{p^*} \approx 2'696$

$$\boxed{p_{measured} = 2'696 \cdot P_2 = 2'696\ bar}$$

P3 exam 2011

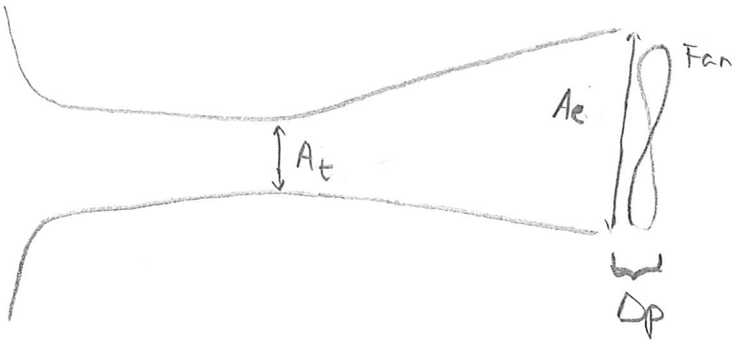
A so-called "bell mouth" inlet is connected To a converging-divergin nozzle. The ambient air outside The inleT is at resT with $p = 1$ bar and $T = 300K$. The Throat area of The nozzle $A_{throat} = 0'01 m^2$. At the nozzle exit a fan "sucks" air out of the nozzle by creating a pressure difference Δp across The fan blades. The ambient pressure downstream of The fan is 1 bar. The ducT area at The position of The fan is $A_e = 0'015 m^2$. All viscous effects might be neglected, adiabatic flow may be assumed. The air may be Treated as calorically perfect gass with $R = 287 \frac{J}{kg K}$ and $\gamma = 1'4$.

a) Compute The minimum Δp for which chocked flow is achieved Through The nozzle.

b) Compute the mass flow Through The nozzle when it is chocked

c) Compute The position of The normal shock between The Throat and exit for $\Delta p = 1'5\, \Delta p_{min}$

Data: $P_{amb} = 1\,bar$
$T_{amb} = 300K$
$A_t = 0'01 m^2$
$A_e = 0'015 m^2$
$R = 287\ J/kgK$
$\gamma = 1'4$



a)

$Po_{amb} = P_{amb} = 1\,bar$

$To_{amb} = T_{amb} = 300K$

At nozzle exit (just before the fan)

$$P_e = P_{amb} - \Delta p$$

Since we are asked for the minimum Δp we know we are in the subsonic solution after the sonic flow in the throat

$$T.A.1 \left\{ \frac{A_e}{A^*} = 1'5 \right\} \Rightarrow \frac{Poe}{P_e} \approx 1'136 \quad \& \quad M_e \approx 0'43$$

Since we are in the subsonic solution, the are no shocks and the flow is isentropic ($Poe = Po_{amb} = 1\,bar$)

$$P_e = \frac{Poe}{1'136} = 0'88\ bar$$

$$\boxed{\Delta p = P_{amb} - P_e = 0'12\ bar}$$

b) Alternative a):

$$(5.21)\quad \boxed{\dot{m} = \frac{P_0 A^*}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} = 2'334\ kg/s}$$

Alternative b):

$T_{oe} = T_{oamb} \rightarrow$ isentropic relations with M_e $\rightarrow T_e$

With T_e and P_e

$$\rho_e = \frac{P_e}{R T_e}$$

$\dot{m}_e = \dot{m} = \rho_e M_e \sqrt{R T_e \gamma}\, A_e$

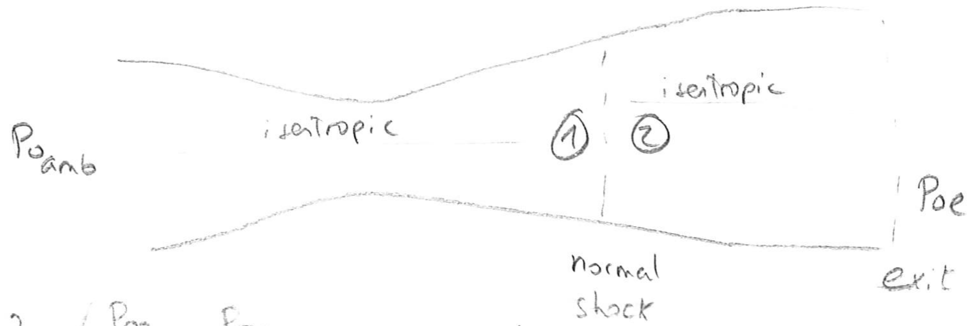
c) $\Delta p = 1'5\, \Delta p_{min} = 0'18$ bar

$P_e = P_{amb} - \Delta p = 0'82$ bar

(5.28)

$$M_e^2 = \frac{-1}{\gamma-1} + \sqrt{\frac{1}{(\gamma-1)^2} + \left(\frac{2}{\gamma-1}\right)\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \left(\frac{P_{01} A_t}{P_e A_e}\right)^2} = 0'5088$$

$$P_{oe} = P_e \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\gamma/\gamma-1} = 0'9785 \text{ bar}$$



$P_{01} = P_{o_{amb}}$

$P_{02} = P_{oe}$

T.A.2 $\left\{ \frac{P_{02}}{P_{01}} = \frac{P_{oe}}{P_{o_{amb}}} = 0'9785 \right\} \Rightarrow M_1 = 1'3$

T.A.1 $\left\{ M = 1'3 \right\} \Rightarrow \frac{A_1}{A_1^*} = 1'066 = \frac{A_1}{A_t}$

$$\boxed{A_1 = A_t \cdot 1'066 = 0'01066 m^2}$$