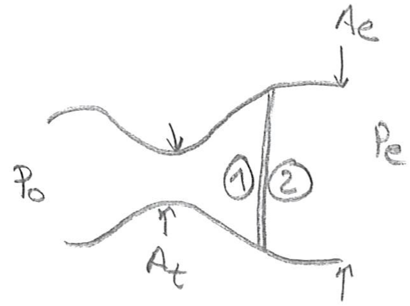
Exercise session 5:

5.11



Data: $A_e = 0'5\, m^2$

$A_t = 0'25\, m^2$

$P_o = 1\, atm$

$P_e = 0'6\, atm$

$\frac{A}{A^*}$ where the shock is located

Assume air $\rightarrow \gamma = 1'4$

Before the shock the flow is isentropic $\left\{ \begin{array}{l} A_t = A_1^* \\ P_{o1} = P_o \end{array} \right.$

(5.24) $\frac{P_e}{P_{o1}} \frac{A_e}{A_t} = \frac{P_e A_e}{P_{oe} A_e^*} = \frac{0'6}{1} \frac{0'5}{0'25} = 1'2$

After shock isentropic flow again $\left\{ \begin{array}{l} P_{oe} = P_{o2} \\ A_e^* = A_2^* \end{array} \right.$

(5.28) $\frac{-1}{\gamma - 1} + \sqrt{\frac{1}{(\gamma-1)^2} + \left(\frac{2}{\gamma-1}\right)\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(\frac{P_{oe} A_e^*}{P_e A_e}\right)^2} = M_e^2$

$M_e^2 = 0'2227 \rightarrow M_e = 0'4719$

(3.30) $\frac{P_{oe}}{P_e} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow P_{oe} = 0'6988\, atm$

Total pressure ratio over the shock is $\frac{P_{o2}}{P_{o1}} = \frac{P_{oe}}{P_o} = 0'6988$

From Table A.2 $\left\{ \frac{P_{o2}}{P_{o1}} = 0'6988 \right\} \Rightarrow M_1 = 2'047$

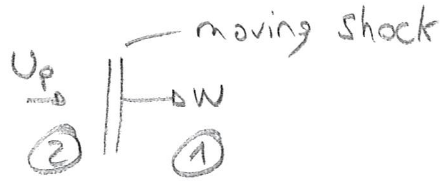
From table A.1 $\left\{ M_1 = 2'047 \right\} \Rightarrow \boxed{\frac{A_1}{A^*} = 1'7556}$

7.2

Data: $W = 680\, m/s$ air $\left\{ \begin{array}{l} R = 287 \frac{J}{kg K} \\ \gamma = 1'4 \end{array} \right.$

$P_1 = 1\, atm$

$T_1 = 288\, K$



T_2, P_2, U_p using equations?

T_2, P_2, U_p using table A.2?

$$a_1 = \sqrt{\gamma R T_1} = \sqrt{1'4 \cdot 287 \cdot 288} = 340'2\, m/s$$

(7.13) $$M_s = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{P_2}{P_1} - 1\right) + 1} = \frac{W}{a_1} \Rightarrow \frac{P_2}{P_1} = 4'4952 \Rightarrow \boxed{P_2 = 4'4952\, atm}$$

(7.10) $$\frac{T_2}{T_1} = \frac{P_2}{P_1}\left[\frac{\frac{\gamma+1}{\gamma-1} + \frac{P_2}{P_1}}{1 + \frac{\gamma+1}{\gamma-1}\left(\frac{P_2}{P_1}\right)}\right] = 1'6867 \Rightarrow \boxed{T_2 = 485'76\, K}$$

(7.16) $$\boxed{U_p = \frac{a_1}{\gamma}\left(\frac{P_2}{P_1} - 1\right)\left[\frac{\frac{2\gamma}{\gamma+1}}{\frac{P_2}{P_1} + \frac{\gamma-1}{\gamma+1}}\right]^{1/2} = 424'9\, m/s}$$

Now we solve the same problem but using the Tables instead

T.A.2 for $M_1' = \frac{W}{a_1} \approx 2$

$$\frac{P_2}{P_1} = 4'5 \; ; \; \frac{T_2}{T_1} = 1'687 \; ; \; M_2' = 0'5774$$

$\boxed{P_2' = 4'5\, atm}$ $\boxed{T_2' = 485'86\, K}$

$$a_2' = \sqrt{\gamma R T_2'} = 441'83\, m/s$$

$$u_2' = M_2' a_2' = 255'11\, m/s$$

$$\boxed{U_p = W - u_2' = 424'9\, m/s}$$

7.3 Data $\begin{cases} W = 680\, m/s \\ P_1 = 1\, atm \\ T_1 = 288\, K \end{cases}$ $\begin{matrix} U_p = 425\, m/s \\ P_2 = 4'5\, atm \\ T_2 = 486\, K \end{matrix}$ P_{02}, T_{02}?

$$M_2 = \frac{U_p}{a_2} = \frac{U_p}{\sqrt{\gamma R T_2}} = 0'962$$

$$\frac{T_{02}}{T_2} = 1 + \frac{\gamma-1}{2} M_2^2 \Rightarrow \boxed{T_{02} = 576\, K}$$

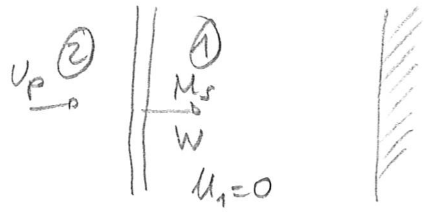
$$\frac{P_{02}}{P_2} = \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\gamma/\gamma-1} \Rightarrow \boxed{P_{02} = 8'15\, atm}$$

(7.5)

Known: $P_1 = 0'01$ atm

$T_1 = 300K$

$\frac{P_2}{P_1} = 1050$



a) W_R

Air $\begin{cases} \gamma = 1'4 \\ R = 287 \frac{J}{kg K} \end{cases}$

$W_R = \left\{ \begin{array}{l} \text{velocity of the gas} \\ \text{behind the shock wave} \\ \text{relative to the wave} \end{array} \right\} = M_R a_2 - U_p$

$M_R = \left\{ \begin{array}{l} \text{Mach number of the reflected} \\ \text{shock wave relative to} \\ \text{the gas ahead of it} \end{array} \right\} = \frac{W_R + u_p}{a_2}$

From table A.2 for $\frac{P_2}{P_1} = 1050$

$M_s = 30$; $\frac{T_2}{T_1} = 175'9$; $M_2' = 0'379$

$M_s = \frac{W}{a_1} \Rightarrow W = M_s \sqrt{\gamma R T_1} = 10416$ m/s

$T_2 = 175'9 \cdot 300 = 52770K$ ¿calorically perfect?

(7.23) $$\frac{M_R}{M_R^2 - 1} = \frac{M_s}{M_s^2 - 1} \sqrt{1 + \frac{2(\gamma - 1)}{(\gamma + 1)^2} (M_s^2 - 1)\left(\gamma + \frac{1}{M_s^2}\right)} = 0'4426$$

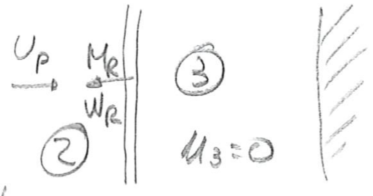
$M_R^2 - 1 - \frac{1}{0'4426} M_R = 0 \xRightarrow{\text{positive solution}} M_R = 2'6382$

$u_2' = M_2' a_2 = 0'379 \sqrt{1'4 \cdot 287 \cdot 52770} = 1745'2$ m/s

$u_p = W - u_2' = 10416 - 1745'2 = 8670'8$ m/s

$$\boxed{W_R = a_2 M_R - u_p = 2'6382 \cdot \sqrt{1'4 \cdot 287 \cdot 52770} - 8670'8 = 3477'2 \text{ m/s}}$$

b) T_3, P_3?



From table A.2 $\{M = M_R = 2'6382\}$

$$\frac{P_3}{P_2} = 7'95 \quad ; \quad \frac{T_3}{T_2} = 2'2777 \quad ; \quad M'_3 = 0'5006$$

$$\boxed{P_3 = \frac{P_3}{P_2}\frac{P_2}{P_1}P_4 = 7'95 \cdot 1050 \cdot 0'01 = 83'5 \text{ atm}}$$

$$\boxed{T_3 = \frac{T_3}{T_2}\frac{T_2}{T_1}T_1 = 2'2777 \cdot 175'9 \cdot 300 = 120190 \text{ K}}$$

$$\left(u'_3 = M'_3 a_3 = 0'5006\sqrt{1'4287 \cdot 120190} = 3478'9 \text{ m/s} \right.$$

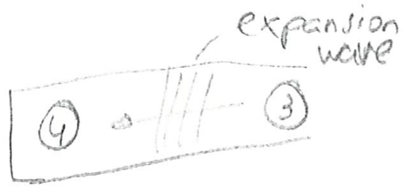
$$\left. u'_3 \approx W_R \quad \text{ok!} \right)$$

7.8

Data $\quad P_4 = 10 \text{ atm} \qquad \frac{P_3}{P_4} = 0'4$

$\quad T_4 = 2500 \text{ K} \qquad \text{air} \begin{cases} \gamma = 1'4 \\ R = 287 \text{ J/kgK} \end{cases}$

$\quad u_4 = 0$



u_3, M_3?

$$a_4 = \sqrt{\gamma R T_4} = 1002'2 \text{ m/s}$$

$$(7.86) \quad \frac{P}{P_4} = \left(1 - \frac{\gamma - 1}{2}\left(\frac{u}{a_4}\right)\right)^{2\gamma/\gamma-1}$$

$$\frac{P_3}{P_4} = \left(1 - 0'2\left(\frac{u_3}{a_4}\right)\right)^{2'8/0'4} = 0'4 \Rightarrow \boxed{u_3 = 614'8 \text{ m/s}}$$

$$(1.43) \quad \frac{P_3}{P_4} = \left(\frac{T_3}{T_4}\right)^{\gamma/\gamma-1} \Rightarrow T_3 = 1924'2 \text{ K} \quad \text{(expansion waves are isentropic)}$$

$$\boxed{M_3 = \frac{u_3}{a_3} = \frac{u_3}{\sqrt{\gamma R T_3}} = 0'6992}$$

P3 exam 2009

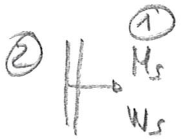
A moving normal shock is travelling at a speed of 500 m/s into quiescent (not moving) air at conditions $p = 1$ bar and $T = 300K$. The air may be treated as a calorically perfect gas with $\gamma = 1'4$ and $R = 287 \frac{J}{kgK}$. Compute:

a) The Mach number of the moving shock

b) the conditions behind the moving shock (p, T, ρ)

c) The air velocity behind the shock (u_p)

d) assuming that the moving shock hits a solid wall (aligned with the shock), compute the Mach number, M_R, of the reflected shock and pressure behind it.



Data
- Moving shock
- $W_s = 500\ m/s$
- $P_1 = 1$ bar
- $T_1 = 300K$
- Calorically perfect gas, $\gamma = 1'4$, $R = 287\ J/kgK$

a) $\boxed{M_s = \frac{W_s}{a_1} = \frac{W_s}{\sqrt{\gamma R T_1}} = 1'44}$

b)
(7.13) $M_s = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{P_2}{P_1} - 1\right) + 1}$

$$\left(M_s^2 - 1\right)\frac{2\gamma}{\gamma+1} + 1 = \frac{P_2}{P_1} = 2'253 \Rightarrow \boxed{P_2 = 2'253 \text{ bar}}$$

(7.11) $$\frac{\rho_2}{\rho_1} = \frac{1 + \frac{\gamma+1}{\gamma-1}\left(\frac{P_2}{P_1}\right)}{\frac{\gamma+1}{\gamma-1} + \frac{P_2}{P_1}} = 1'7591$$

$$\rho_1 = \left\{\text{Ideal gas law}\right\} = \frac{P_1}{RT_1} = 1'161 \frac{kg}{m^3}$$

$$\boxed{\rho_2 = 1'7591 \rho_1 = 2'043 \frac{kg}{m^3}}$$

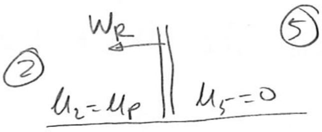
$$\boxed{T_2 = \left\{\text{Ideal gas law}\right\} = \frac{P_2}{R\rho_2} = 384'2K}$$

c)

$$(7.16)\quad \boxed{u_p = \frac{a_1}{\gamma}\left(\frac{P_2}{P_1}-1\right)\left[\frac{\frac{2\gamma}{\gamma+1}}{\frac{P_2}{P_1}+\frac{\gamma-1}{\gamma+1}}\right]^{1/2} = \left\{a_1 = \sqrt{\gamma R T_1}\right\} = 215'8\,m/s}$$

(Alternative, move to a frame of reference moving with the shock so the shock becomes stationary and $u_1 = 500\,m/s$, then $u_p = W_s - u_2$)

d) shock reflection



$$(7.23)\quad \frac{M_R}{M_R^2-1} = \frac{M_s}{M_s^2-1}\sqrt{1+\frac{2(\gamma-1)}{(\gamma+1)^2}(M_s^2-1)\left(\gamma+\frac{1}{M_s^2}\right)} = 1'51788$$

$$M_R^2 - 1 - \frac{M_R}{1'51788} = 0 \Rightarrow \boxed{M_R = 1'382}$$

positive solution

$$(7.13)\quad M_R = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{P_5}{P_2}-1\right)+1}$$

$$\frac{P_5}{P_2} = (M_R^2-1)\frac{2\gamma}{\gamma+1}+1 = 2'062 \Rightarrow \boxed{P_5 = 2'062 \cdot P_2 = 4'644\,bar}$$