Exercise session 4:

(4.10) Data: $M_1 = 2$; $P_1 = 3\,atm$; $T_1 = 400K$; $\theta = 30°$

Calculate $M_2, P_2, T_2, T_{02}, P_{02}$

Assuming air $\left\{ \begin{array}{l} \gamma = 1'4 \\ R = 287\ J/kg\,K \end{array} \right.$

From table A.5 with $M_1 = 2 \Rightarrow \nu(M_1) = 26'38°$

(4.45) $\theta = \nu(M_2) - \nu(M_1) \Rightarrow \nu(M_2) = \theta + \nu(M_1) = 56'38°$

From table A.5 with $\nu(M_2) = 56'38° \Rightarrow \boxed{M_2 \simeq 3'35}$

(3.30) $P_{01} = P_1\left(1 + \frac{\gamma-1}{2}M_1^2\right)^{\gamma/\gamma-1} = 23'47\,atm$

Isentropic expansion process $\Rightarrow P_{01} = P_{02} \Rightarrow \boxed{P_{02} = 23'47\,atm}$

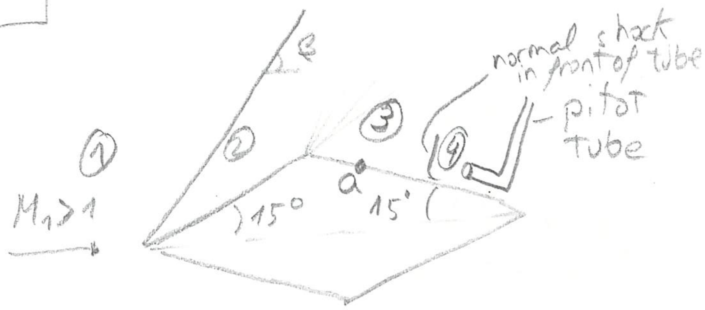
$$\boxed{P_2 = P_{02}\left(1 + \frac{\gamma-1}{2}M_2^2\right)^{-\frac{\gamma}{\gamma-1}} = 0'3816\,atm}$$

(3.28) $T_{01} = T_1\left(1 + \frac{\gamma-1}{2}M_1^2\right) = 720K$

Isentropic expansion process $\Rightarrow T_{01} = T_{02} \Rightarrow \boxed{T_{02} = 720K}$

$$\boxed{T_2 = T_{02}\left(1 + \frac{\gamma-1}{2}M_2^2\right)^{-1} = 221'9\,K}$$

(E.4.14) Data: $P_{04} = 2'956\,atm$
$P_a = P_3 = 0'1\,atm$



Calculate M_1

First lets calculate around the normal shock in front of the tube

$\frac{P_{04}}{P_3} = \frac{2'956}{0'1} = 29'56$

in the table index 2 and 1 are used

From table A.2 for normal shocks with $\frac{P_{04}}{P_3} \left(\frac{P_{02}}{P_1} \right) = 29'56$

we get Mach number before the normal shock, $M_3 = 4'45$

Now lets calculate the expansion corner. {2→3}

From table A.5 ⇒ $\nu(M_3) = \nu(4'45) = 71'27°$

θ in this case is 30°

(4.45) $\nu(M_2) = \nu(M_3) - \theta = 41'27°$

From table A.5 with $\nu(M_2) = 41'27° \Rightarrow M_2 = 2'6$

In region 2 we have then:

$M_{n2} = M_2 \sin(\beta - \theta) = 2'6 \sin(\beta - 15°)$ (*)

Both M_{n2} and β are unknown, solve by Trial and error

Assume $M_1 = 4 \Rightarrow \theta$-$\beta$-$M \Rightarrow \beta = 27° \Rightarrow M_{n1} = M_1 \sin(\beta) = 1'816$

Table A.2 for normal shocks with $M_{n1} = 1'816 \Rightarrow M_{n2} = 0'612$

Putting This into (* to check)

$M_{n2} \stackrel{?}{=} M_2 \sin(\beta - \theta) = 0'54 \neq M_{n2} = 0'612$

Another iteration:

Assume $M_1 = 3'5 \Rightarrow \theta$-$\beta$-$M \Rightarrow \beta = 29'2° \Rightarrow M_{n1} = 1'71$

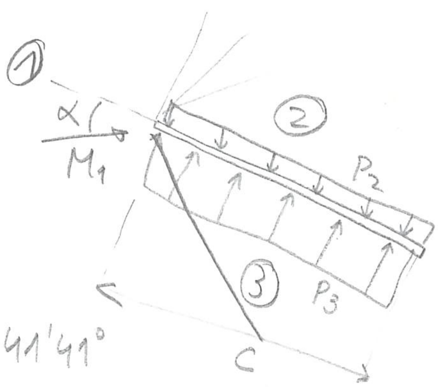
Table A.2 ⇒ $M_{n2} = 0'638$

$M_{n2} \stackrel{?}{=} M_2 \sin(\beta - \theta) = 2'6 \cdot \sin(14'2) = 0'638 = M_2$

The guen is right ⇒ $\boxed{M_1 = 3'5}$

E.4.15

Infinitely Thin plate
angle of attack: $\alpha = 5°$
$M_1 = 2'6$
Calculate lift and drag coefficients



Lets calculate The suction side first

From table A.5 with $M = 2'6 \rightarrow \nu(M_1) = 41'41°$

(4.45) $\nu_2(M_2) = \nu_1(M_1) + \alpha = 46'41°$

From table A.5 for $\nu(M_2) = 46'41^\circ \Rightarrow M_2 = 2'85$

From table A.1 for $M_1 = 2'6 \Rightarrow \frac{p_{01}}{p_1} = 19'95$

From table A.1 for $M_2 = 2'85 \Rightarrow \frac{p_{02}}{p_2} = 29'29$

We can now derive the ratio of static pressures $\frac{p_2}{p_1}$ as

$$\frac{p_2}{p_1} = \frac{p_2}{p_{02}} \frac{p_{02}}{p_{01}} \frac{p_{01}}{p_1} = \left\{ \begin{array}{l} \text{isentropic process} \\ p_{01} = p_{02} \Rightarrow \frac{p_{02}}{p_{01}} = 1 \end{array} \right\} = \frac{1}{29'29} \cdot 1 \cdot 19'95 = 0'681$$

Now lets go to the pressure side:

From θ-β-M diagram $\left\{ \begin{array}{l} M_1 = 2'6 \\ \theta = \alpha = 5^\circ \end{array} \right. \Rightarrow \beta = 26'5$

$$M_{n1} = M_1 \sin\beta = 1'16$$

From table A.2 with $M_{n1} = 1'16 \Rightarrow \frac{p_3}{p_1} = 1'403$

Since we are working with inviscid flow, the pressure force is the only one acting on the plate, the lift per unit span, L', and the drag per unit span, D', can be written as:

$$L' = (p_3 - p_2)\cos\alpha \cdot c$$

$$D' = (p_3 - p_2)\sin\alpha \cdot c$$

We define lift and drag coefficients as

$$C_L = \frac{L' \cdot s}{\frac{1}{2}\rho_1 V_1^2 \cdot c \cdot s}$$

$$C_D = \frac{L' \cdot s}{\frac{1}{2}\rho_1 V_1^2 c \cdot s}$$

Lets put $\frac{1}{2}\rho_1 V_1^2$ in terms of what we know:

$$\frac{1}{2}\rho_1 V_1^2 = \frac{1}{2}\rho_1 a_1^2 M_1^2 = \frac{1}{2}\rho_1 \left(\sqrt{\frac{\gamma p_1}{\rho_1}}\right)^2 M_1 = \frac{1}{2}\rho_1 \frac{\gamma p_1}{\rho_1} M_1^2 = \frac{\gamma}{2} p_1 M_1^2$$

Now we can compute the coefficients

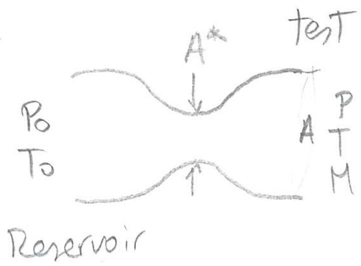
$$\boxed{C_L = \frac{L'\cdot s}{\frac{\gamma}{2}p_1 M_1^2\cdot c\cdot s} = \frac{2}{\gamma M_1^2}\left(\frac{p_3}{p_1}-\frac{p_2}{p_1}\right)\cos\alpha = 0'152}$$

$$\boxed{C_D = \frac{D'\cdot s}{\frac{\gamma}{2}p_1 M_1^2\cdot c\cdot s} = \frac{2}{\gamma M_1^2}\left(\frac{p_3}{p_1}-\frac{p_2}{p_1}\right)\sin\alpha = 0'0133}$$

5.1 $M_{test} = 2'4$ at standard atmospheric conditions.

a) The exit-to-throat area ratio of the nozzle

b) Reservoir pressure and temperature



Atmospheric conditions: $\begin{cases} P = 1\,atm \\ T = 288\,K \end{cases}$

Air $\Rightarrow \gamma = 1'4$

From table A.1 $\{M_1 = 2'4\}$ we get:

$$\boxed{\frac{A}{A^*} = 2'4031}$$

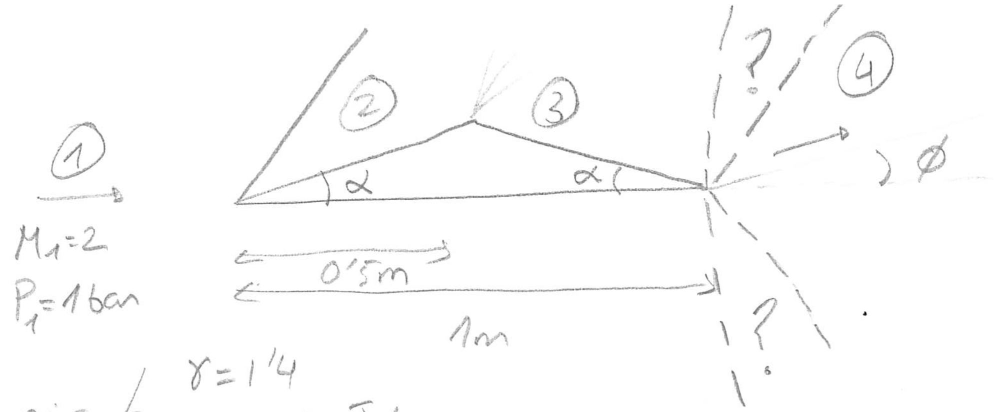
$$\left.\begin{aligned} \frac{P_0}{P} &= 14'62 \\ \frac{T_0}{T} &= 2'152 \end{aligned}\right\} \text{ isentropic flow } \begin{cases} \boxed{P_0 = 14'62\cdot P = 14'62\,atm} \\ \boxed{T_0 = 2'152\cdot T = 619'8\,K} \end{cases}$$

P.4 exam 2009

A triangle shaped wing as show in the figure below experiences a flow of air at Mach 2 and static pressure of 1 bar. The angle of attack is zero so that the bottom surface is aligned with the flow in region 1. The wing has a chord length of 1m and the angle α is 10°

a) Lift and drag of the wing

b) Explain what happens at the trailing edge, marked as ? in the figure, and then calculate the angle, ϕ, of the flow behind the wing



air $\left\{ \begin{array}{l} \gamma = 1'4 \\ R = 287 \, J/kgK \end{array} \right.$

To calculate lift and drag we need pressure in all surfaces

θ-β-M relation $\left\{ \begin{array}{l} M = 2 \\ \alpha = \theta_1 = 10^\circ \end{array} \right. \Rightarrow \beta_1^\circ = 39'31^\circ$

$M_{n1} = M_1 \sin\beta_1 = 1'26714$

(4.10) $M_{n2}^2 = \dfrac{M_{n1}^2 + [2/(\gamma-1)]}{[2\gamma/(\gamma-1)] M_{n1}^2 - 1} \Rightarrow M_{n2} = 0'8031$

$M_2 = \dfrac{M_{n2}}{\sin(\beta_1 - \theta_1)} = 1'64051$

(3.57) $\dfrac{P_2}{P_1} = 1 + \dfrac{2\gamma}{\gamma+1}(M_{n1}^2 - 1) = 1'7065 \Rightarrow P_2 = 1'7065 \text{ bar}$

Lets calculate 3 Throug The expansion

(4.45) $\theta_2 = \nu(M_3) - \nu(M_2)$

Table A.5 $\{ M_2 = 1'64051 \} \Rightarrow \nu(M_2) = 16'057^\circ$

$\theta_2 = 2 \cdot \alpha = 20^\circ$

$\nu(M_3) = \theta_2 + \nu(M_2) = 36'057^\circ$

Table A.5 $\{ \nu(M_3) = 36'057^\circ \} \Rightarrow M_3 = 2'371$

We can calculate P_{02} from isentropic relations with P_2 and M_2

(3.30) $P_{02} = P_2 \left(1 + \dfrac{\gamma-1}{2} M_2^2\right)^{\gamma/\gamma-1} = 7'7042 \text{ bar}$

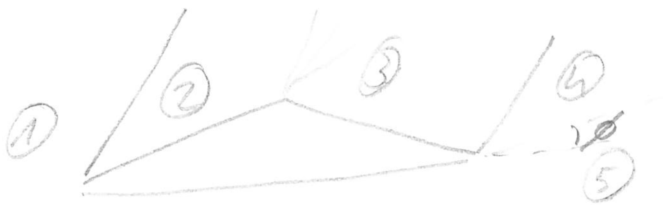
Expansion waves are isotropic processes $\Rightarrow P_{02} = P_{03}$

$$P_3 = \frac{P_{03}}{\left(1+\frac{\gamma-1}{2}M_3^2\right)^{\gamma/\gamma-1}} = 0'5507 \text{ bar}$$

$$\text{Lift} = P_1 \cdot 1 - P_2 \cdot 0'5 - P_3 \cdot 0'5 = -12'9 \text{ kN}$$

$$\text{Drag} = (P_2 - P_3)\, 0'5 \tan\alpha = 10'2 \text{ kN}$$

What happens in ?



We know that $P_4 > P_3$ and $P_4 = P_5$

Assume $\phi = 0°$

$\{M_3 = 2'371,\ \alpha + \phi = 10°\} \Rightarrow \beta_3 = 33'37°$

$M_{n3} = 1'3048$

Normal shock relations:

$\frac{P_4}{P_3} = 1'8196 \Rightarrow P_4^{\phi=0} = 1'0022 \text{ bar}$

$P_5^{\phi=0} = 1 \text{ bar}$

Pressure in 4 is too high, shock from 3 → 4 must be weaker (less flow deflection)

Assume $\phi = -1° \Rightarrow \{M_3 = 2'371,\ \alpha + \phi = 9°\}$

$\beta_3 = 32'42°$

$M_{n3} = 1'2716$

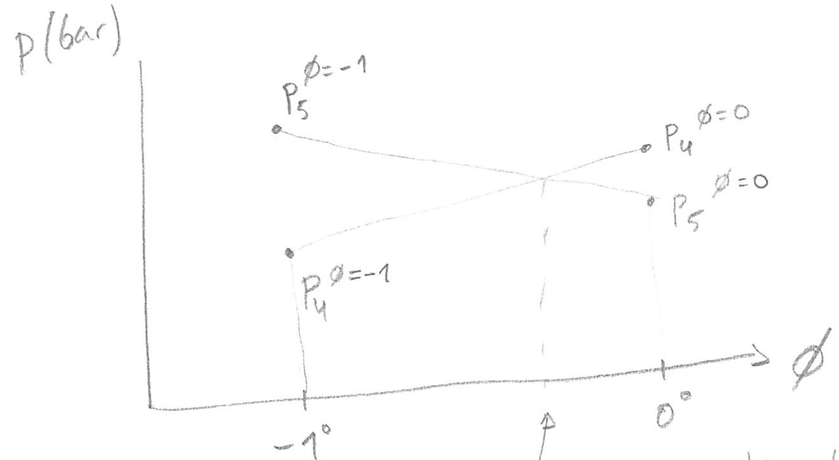
$\frac{P_4}{P_3} = 1'7198 \Rightarrow P_3^{\phi=-1} = 0'9472 \text{ bar}$

Now we get a shock wave in the lower side (1 → 5)

$\{M_1 = 2,\ \phi = -1\} \Rightarrow \beta = 30'811°$

$M_{n1} = 1'02443$

(3.57) $P_5^{\phi=-1} = P_1\left(1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right)\right) = 1'0577 \text{ bar}$



linear interpolation to get the point where P_4 and P_5 match $\Rightarrow$ $\boxed{\phi \approx -0'02°}$