Exercise session 3:

3.12 Data:

$D = 0'02\,m$; $L = 40\,m$;

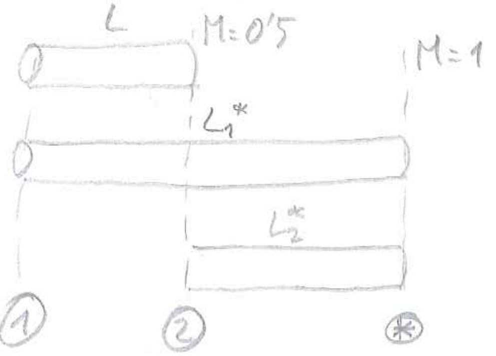
Exit conditions $\begin{cases} M_2 = 0'5 \\ P_2 = 1\,atm \\ T_2 = 290\,K \end{cases}$

air $\rightarrow \gamma = 1'4$

$f = 0'005$

Calculate M_1, P_1 and T_1:

Since the exit of the pipe, 2, is subsonic, we know that 1 has to be subsonic as the flow goes to sonic condition inside a pipe with friction. We can draw the following scenario



$L = L_1^* - L_2^*$

We start by assuming the inlet is condition 2 so that we can find L_2^*. Using table A.4 with $M_2 = 0'5$

$$\frac{4fL_2^*}{D} = 1'06906 \rightarrow L_2^* = \frac{1'06906 \cdot D}{4 \cdot f} = 1'06906\,m$$

From the original inlet to sonic:

$$L_1^* = L + L_2^* = 40 + 1'06906 = 41'06906\,m$$

We compute now

$$\frac{4fL_1^*}{D} = \frac{4 \cdot 0'005 \cdot 41'06906}{0'02} = 41'06906$$

We use table A.4 with $\frac{4fL}{D} = 41'06906$ to find inlet 1

$$\left.\begin{array}{l} \frac{4fL^*}{D} = 45'4 \rightarrow M = 0'12 \\ \frac{4fL^*}{D} = 32'51 \rightarrow M = 0'14 \end{array}\right\} \text{interpolating} \rightarrow \boxed{M_1 = 0'1267}$$

For flow with friction we can use

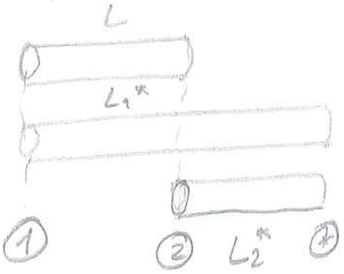
$$(3.100)\quad \frac{P_2}{P_1} = \frac{M_1}{M_2}\left[\frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2}\right]^{1/2} \Rightarrow \boxed{P_1 = 4'037\,atm}$$

(3.98) $\frac{T_2}{T_1} = \frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2} \Rightarrow \boxed{T_1 = 282'59\,K}$

3.13

Data: $D = 0'2\,ft$; $L = 3\,ft$; $M_1 = 2'5$; $p_1 = 0'5\,atm$; $T_1 = 520\,°R$;
$f = 0'005$; air $\rightarrow \gamma = 1'4$

Calculate: M_2, p_2, T_2 and p_{o2}



Similar approach as before:

$L = L_1^* - L_2^*$

Using table A.4 for $M_1 = 2'5$

$\frac{4fL_1^*}{D} = 0'43197 \Rightarrow L_1^* = 4'3197\,ft$

$L_2^* = L_1^* - L = 1'3197\,ft$

$\frac{4fL_2^*}{D} = 0'13197$

Using table A.4 for $\frac{4fL^*}{D} = 0'13197 \simeq 0'132$

$\frac{4fL^*}{D} = 1'36 \rightarrow M = 1'5$
$\frac{4fL^*}{D} = 1'287 \rightarrow M = 1'48$
} interpolating $\rightarrow \boxed{M_2 = 1'488}$

For flow with friction we can use:

(3.100) $\frac{p_2}{p_1} = \frac{M_1}{M_2}\left[\frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2}\right]^{1/2} \rightarrow \boxed{p_2 = 1'048\,atm}$

(3.98) $\frac{T_2}{T_1} = \frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2} \rightarrow \boxed{T_2 = 810'9\,°R = 450'5\,K}$

(3.30) $\frac{p_o}{p} = \left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/(\gamma-1)} \Rightarrow \boxed{p_{o2} = 3'786\,atm}$

(4.1) Oblique shock data:

$\beta = 35°$

$P_1 = 2000 \frac{lb}{ft^2}$

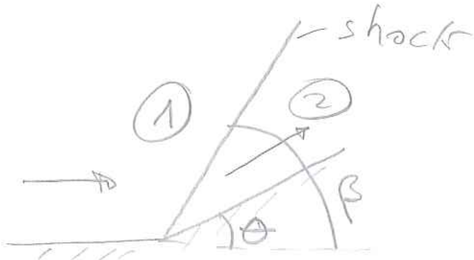
$T_1 = 520 °R$

$V_1 = 3355 \, ft/s$

assume air

$\gamma = 1'4$

$R = 287 \frac{J}{kg K}$



Calculate p_2, T_2, V_2 and θ

$$P_1 = 2000 \frac{lb}{ft^2} \cdot \frac{4'4482N}{1 lb} \left(\frac{1 ft}{0'3048 m}\right)^2 = 95761 \, Pa$$

$$T_1 = 520 °R \, \frac{5K}{9°R} = 288'89 K$$

$$V_1 = 3355 \frac{ft}{s} \cdot \frac{0'3048 m}{1 ft} = 1022'604 \, m/s$$

Lets calculate speed of sound and Mach number before the shock

$$a_1 = \sqrt{\gamma R T_1} = 340'7 \, m/s$$

$$M_1 = \frac{V_1}{a_1} = 3'00$$

Lets calculate M_{n1}

(4.7) $M_{n1} = M_1 \sin(\beta) = 3 \cdot \sin(35) = 1'722$

Using equation for normal shocks for M_{n2} calculation

(3.51) $$M_{n2} = \sqrt{\frac{1 + \frac{\gamma-1}{2} M_{n1}^2}{\gamma M_{n1}^2 - \frac{\gamma-1}{2}}} = 0'635$$ (We can also use eq. 4.10)

Lets calculate the deflection angle θ

(4.17) $$\tan\theta = 2 \cot\beta \left[\frac{M_1^2 \sin^2\beta - 1}{M_1^2(\gamma + \cos(2\beta)) + 2}\right]$$

$$\left[\theta = \tan^{-1}\left(\frac{2}{\tan(\beta)} \cdot \left[\frac{M_1^2 \sin^2\beta - 1}{M_1^2(\gamma + \cos(2\beta)) + 2}\right]\right)\right] =$$

$$= \tan^{-1}\left[\frac{2}{\tan(35)} \cdot \left[\frac{3^2 \sin^2(35) - 1}{3^2(1'4 + \cos(70)) + 2}\right]\right] = \boxed{17'58°}$$

Now that we know shock angle, deflection angle and M_{n2} we can calculate M_2 as:

$$(4.12) \quad M_2 = \frac{M_{n2}}{\sin(\beta-\theta)} = \frac{0'635}{\sin(17'4)} = 2'121$$

Using again equations for normal shocks:

$$(3.57) \quad \frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{n1}^2 - 1\right) \Rightarrow \boxed{P_2 = 315324'82 \text{ Pa}}$$

$$(3.59) \quad \frac{T_2}{T_1} = \left[1 + \frac{2\gamma}{\gamma+1}\left(M_{n1}^2 - 1\right)\right]\left[\frac{2+(\gamma-1)M_{n1}^2}{(\gamma+1)M_{n1}^2}\right]$$

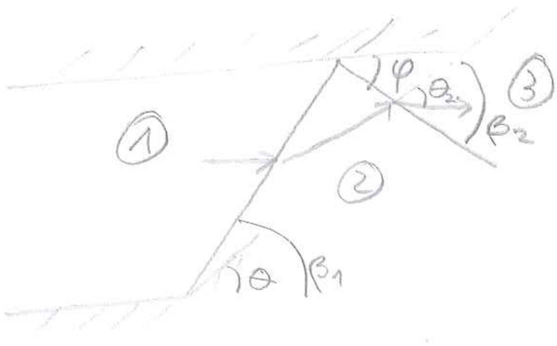
$$\boxed{T_2 = 425'9 \text{ K}}$$

Since we have M_2, if we calculate a_2 we can calculate V_2

$$a_2 = \sqrt{\gamma R T_2} = 413'66 \text{ m/s}$$

$$\boxed{V_2 = M_2 a_2 = 2'12 \cdot 413'66 = 877'4 \text{ m/s}}$$

4.6



Data: $M_1 = 3'6$; $\theta = 20°$

Calculate angle of deflected shock:

Asume air $\begin{cases} \gamma = 1'4 \\ R = 287 \text{ J/kgK} \end{cases}$

θ-β-M chart with $\begin{cases} M_1 = 3'6 \\ \theta = 20° \end{cases} \Rightarrow \beta_1 \simeq 34°$

Also can get by solving iteratively

$$(4.17) \quad \tan\theta = 2\cot\beta\left[\frac{M_1^2 \sin^2(\beta) - 1}{M_1^2(\gamma + \cos(2\beta)) + 2}\right] \Rightarrow \beta_1 \simeq 34'11°$$

(4.7) $M_{n1} = M_1 \sin(\beta) = 3'6 \sin(34'11) = 2'019$

Lets calculate M_{n2}

(3.51) $M_{n2} = \sqrt{\dfrac{1+\frac{\gamma-1}{2}M_{n1}^2}{\gamma M_{n1}^2 - \frac{\gamma-1}{2}}} = \sqrt{\dfrac{1+0'2\cdot 2'019^2}{1'4\cdot 2'019^2 - 0'2}} = 0'5742$

(4.12) $M_2 = \dfrac{M_{n2}}{\sin(\beta-\theta)} = \dfrac{0'5742}{\sin(14'11)} = 2'355$

We know that $\theta_2 = \theta$

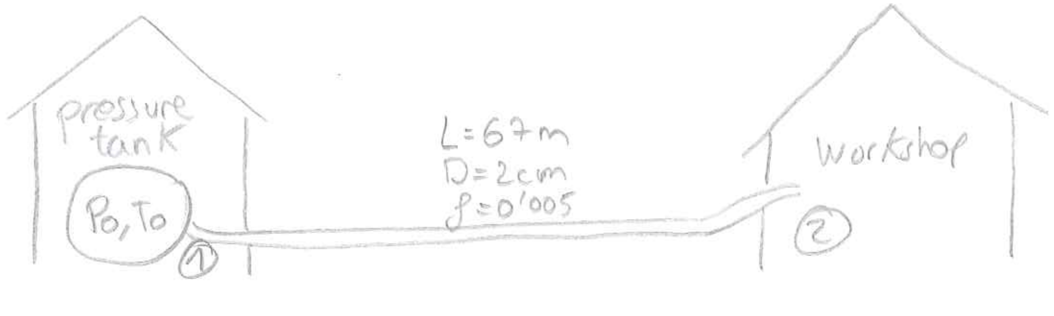
θ-β-M relation with $\begin{cases} M = 2'355 \\ \theta = 20 \end{cases} \Rightarrow \beta_2 \approx 45$

It can also be obtaine by iterating Eq. 4.17

$\boxed{\varphi = \beta_2 - \theta_2 = 45 - 20 = 25^\circ}$

P.1 exam 2014-03-10

A manufacturing company has just finished a new workshop. The workshop needs pressurised air, and a Tube is connected To another building where They have a pressure tank. The pressure and the temperature inside The Tank is 10 bar and 288 K, and The Tube length anddiameter are 67m and 2cm respectively. At some point during The Testing of The new facility sonic flow was obtained at The end of The Tube ($M_2=1$). Compute M_1, P_1, T_1, P_2, T_2, P_{02} and The mass flow of air Through The Tube, assuming adiabatic flow with constant friction coefficient $f = 0'005$.



Data: $M_2 = 1$; $L = 67m$; $D = 2cm$; $\underbrace{T_0 = 288K; P_0 = 10 \text{ bar}}_{\text{in a tank}}$; $f = 0'005$

adiabatic

Calculate: $M_1, p_1, T_1, P_2, T_2, P_{02}$ and $\dot{m}$

Air $\begin{cases} R = 287 \text{ J/kg K} \\ \gamma = 1'4 \end{cases}$

Since we are told $M_2 = 1 \rightarrow L^* = L = 67 m$

$$\frac{4fL^*}{D} = 67$$

Looking at table A.4

$\frac{4fL^*}{D} = 66'92 \Rightarrow \boxed{M_1 = 0'1}$, such a small difference, it is not worth interpolating

The pressure and temperature at the Tank are actually equal to total pressure and total temperature. So using the isentropic relations (calorically perfect gas)

(3.30) $\frac{P_{01}}{P_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\gamma/(\gamma-1)} \Rightarrow \boxed{P_1 = 9'93 \text{ bar}}$

P_{01} is The pressure in The Tank

(3.28) $\frac{T_{01}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2 \Rightarrow \boxed{T_1 = 287'4 K}$

T_{01} is The temperature in The Tank

Now lets calculate point 2, from table A.4 we can get

$$\frac{T_1}{T^*} = \frac{T_1}{T_2} = 1'1976 \rightarrow \boxed{T_2 = 239'97 K}$$

$$\frac{P_1}{P^*} = \frac{P_1}{P_2} = 10'9435 \rightarrow \boxed{P_2 = 0'9073 \text{ bar}}$$

$$\frac{P_{01}}{P_0^*} = \frac{P_{01}}{P_{02}} = 5'821 \rightarrow \boxed{P_{02} = 1'7176 \text{ bar}}$$

$$\boxed{\dot{m} = \rho_2 A_2 V_2 = \frac{P_2}{RT_2} \pi \left(\frac{D}{2}\right)^2 M_2 \cdot \sqrt{\gamma R T_2} = 0'1285 \frac{kg}{s}}$$