Exercise session 2:

(3.8) Data:

$P_1 = 1\,atm$ air $\left\{ \begin{array}{l} \gamma = 1'4 \\ R = 287 \frac{J}{kg K} \end{array} \right\} \Rightarrow C_p = 1004'5 \frac{J}{kg K}$

$T_1 = 288 K$

no friction

Amount of heat per unit mass $\left(\frac{J}{kg}\right)$ needed to choke the flow at the exit of the duct for both $\underset{a)}{M_1 = 2}$ and $\underset{b)}{M_1 = 0'2}$ cases.

Also P_2 and T_2 for both cases

(3.28) $\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$

$$T_0 = T\left(1 + \frac{\gamma - 1}{2} M_1^2\right)$$

$T_{01,a} = 518'4 K$

$T_{01,b} = 290'3 K$

(3.84) $$\frac{T_{02}}{T_{01}} = \left(\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}\right)^2 \left(\frac{M_2}{M_1}\right)^2 \left(\frac{1 + \frac{\gamma - 1}{2} M_2^2}{1 + \frac{\gamma - 1}{2} M_1^2}\right)$$

$T_{02,a} = 653'4 K$

$T_{02,b} = 1672'7 K$

(3.77) $q = C_p (T_{02} - T_{01})$

$$\boxed{q_a = C_p (T_{02,a} - T_{01,a}) = 135607'5 \ J/kg}$$

$$\boxed{q_b = C_p (T_{02,b} - T_{01,b}) = 1388620'8 \ J/kg}$$

(3.78) $\frac{P_2}{P_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}$

$$\boxed{P_{2,a} = P_1 \left(\frac{1 + \gamma M_{1,a}^2}{1 + \gamma M_2^2}\right) = 278644 \, Pa}$$

$$\boxed{P_{2,b} = 44583 \, Pa}$$

(3.81) $$\frac{T_2}{T_1} = \left(\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}\right)^2 \left(\frac{M_2}{M_1}\right)^2$$

$$\boxed{T_{2,a} = 544'5 K}$$

$$\boxed{T_{2,b} = 1393 K}$$

3.9 Data:

$P_1 = 10 \text{ atm} = 1013250 \text{ Pa}$

$T_1 = 1000°R = 555'5 K$

$M_1 = 0'2$

$\alpha_{fuel} = 0'06 \left[\frac{\text{kg fuel}}{\text{kg air}}\right]$

$h_{fuel} = 4'5 \cdot 10^8 \frac{ft \cdot lb}{\text{slug of fuel}} \frac{1 \text{ slug of fuel}}{14'5939 \text{ kg fuel}} \frac{0'3048 m}{1 ft} \frac{4'44822 N}{1 lb} = 4'1806 \cdot 10^7 \frac{J}{\text{kg fuel}}$

$\gamma = 1'4$

Calculate M_2, P_2 and T_2:

$q = h_{fuel} \cdot \alpha_{fuel} = 4'1806 \cdot 10^7 \cdot 0'06 = 2'508 \cdot 10^6 \ J/kg$

(3.28) $T_{01} = T_1(1 + 0'2 M_1^2) = 560 K$

(3.77) $T_{02} = \frac{q}{C_p} + T_{01}$

(1.22) $C_p = \frac{\gamma R}{\gamma - 1} = \frac{1'4 \cdot 287}{0'4} = 1004'5 \frac{J}{kg K}$

(3.77) $T_{02} = \frac{q}{C_p} + T_{01} = \frac{2'508 \cdot 10^6}{1004'5} + 560 = 3056'8 K$

To find M_2 we can iterate on eq. 3.81 or go to table A.3

Table A.3 $\{M = 0'2\} \Rightarrow \frac{T_{01}}{T_0^*} = 0'1736$

$\frac{T_{02}}{T_0^*} = \frac{T_{02}}{T_{01}} \frac{T_{01}}{T_0^*} = \frac{3056'8}{560} \cdot 0'1736 = 0'9476$

Table A.3 $\{\frac{T_{02}}{T_0^*} = 0'9476\} \Rightarrow$ interpolation $\Rightarrow \boxed{M_2 = 0'7643}$

(3.78) $\frac{P_2}{P_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \Rightarrow \boxed{P_2 = 5'809 \text{ atm}}$

(3.28) $\frac{T_{02}}{T_2} = 1 + \frac{\gamma - 1}{2} M_2^2 \Rightarrow \boxed{T_2 = 2737 K}$

3.10 Data:

$P_1 = 10 atm$

$T_1 = 555'55 K$

$M_1 = 0'2$

$h_{fuel} = 4'1806 \cdot 10^7 \frac{J}{kg\ fuel}$

$\gamma = 1'4$

Calculate $\alpha_{fuel\,max}$ - fuel air ratio that chokes the flow at the exit

From 3.9 we know: $\begin{cases} T_{01} = 560 K \\ \frac{T_{01}}{T_0^*} = 0'1736 \end{cases}$

$T_0^* = \frac{T_{01}}{0'1736} = 3225'8 K = T_{02}$

since we are in the new scenario where the flow is choked at the exit

(3.77) $q_{max} = C_p (T_{02} - T_{01}) = 1004'5 (3225'8 - 560) = 2'678 \cdot 10^6 J/kg$

$$\boxed{\alpha_{fuel,max} = \frac{q_{max}}{h_{fuel}} = \frac{2'678 \cdot 10^6}{4'1806 \cdot 10^7} = 0'064 \left[\frac{kg\ fuel}{kg\ air}\right]}$$

P.1 exam 2009

A straight Tube with constant circular cross-section has an inner diameter of 1 cm and length of 1m. The outside of The Tube is evenly covered by electric heating wires with an overal power output of 15 kW. The heater wires are isolated from The exterior air so that all heat generated by the wires may be assumed to enter through The Tube wall and into The air flowing on the inside. AT The inflow end of The Tube The conditions of The air are $T_0 = 295K$, $P_0 = 2 bar$, $M = 0'2$

The air flowing inside The Tube might be regarded as calorically perfect gas with $\gamma = 1'4$ and $R = 287 J/kgK$. Friction inside The tube might be neglected.

Compute:

a) The mass flow of air through the tube (mass per unit time)

b) The total temperature at the outflow end of the tube (tube exit)

c) Mach number at the exit

d) The velocity of the air at the tube exit

e) if the heater power is increased, at what power level will the flow at the tube exit become sonic?

At inflow: $\begin{cases} T_{01} = 295K \\ P_{01} = 2 bar \\ M_1 = 0'2 \end{cases}$

Calorically perfect gas $\Rightarrow \frac{T_{01}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2 \Rightarrow T_1 = 292'65 K$

$\gamma = 1'4$

$R = 287 \frac{J}{kg K}$

Similarly $\frac{P_{01}}{P_1} = \left[1 + \frac{\gamma - 1}{2} M_1^2\right]^{\frac{\gamma}{\gamma - 1}} \Rightarrow P_1 = 1'944$ bar

Using now equation of state:

$P_1 = \rho_1 R T_1 \Rightarrow \rho_1 = \frac{P_1}{R T_1} = 2'3156 \frac{kg}{m^3}$

$M_1 = \frac{u_1}{a_1} \Rightarrow u_1 = a_1 M_1 = \sqrt{\gamma R T_1} \cdot M_1 = 68'582$ m/s

Mass flow: $\boxed{\dot{m}_1 = u_1 A_1 \rho_1 = u_1 \rho_1 \frac{\pi D^2}{4} = 0'01247 \, kg/s}$

Added heat per unit mass $q = \frac{\text{Total heat per unit time}}{\text{mass flow}}$

$q = \frac{15 \cdot 10^3}{0'01247} = 1'203 \cdot 10^6 \frac{J}{kg}$

$q = c_p (T_{02} - T_{01}) \Longrightarrow \boxed{T_{02} = 1492'6 K}$

$c_p = \frac{\gamma R}{\gamma - 1} = 1004'5 \frac{J}{kg K}$

Between inflow and outflow: table A.3

$\frac{T_{02}}{T_0^*} = \frac{T_{01}}{T_0^*} \frac{T_{02}}{T_{01}} = 0'1736 \cdot \frac{1492'6}{295} = 0'878 \Rightarrow \boxed{M_2 = 0'66}$

Also : $P_2 = \frac{P_2}{P^*} \frac{P^*}{P_1} \cdot P_1 = 1'491 \frac{1}{2'273} 1'9449 \text{ bar} = 1'276 \text{ bar}$

$$T_2 = \frac{T_2}{T^*} \frac{T^*}{T_1} T_1 = 0'9682 \frac{1}{0'2066} 292'65 = 1371'5 \text{ K}$$

$$\rho_2 = \frac{\rho_2}{\rho^*} \frac{\rho^*}{\rho_1} \rho_1 = 1'54 \cdot \frac{1}{11} \cdot 2'3156 = 0'324 \text{ kg/m}^3$$

Mass flow is conserved $\rightarrow \dot{m}_1 = \dot{m}_2$

$\dot{m}_2 = u_2 \rho_2 A_2 \Rightarrow \boxed{u_2 = 490'2 \text{ m/s}}$

other alternative: $u_2 = M_2 a_2 = M_2 \sqrt{\gamma R T_2} = 489'9 \text{ m/s}$

To obtain sonic conditions at exit:

Table A.3 $\{M = 1\} \Rightarrow \frac{T_0}{T_0^*} = 1$

$$T_{02} = \frac{T_{02}}{T_0^*} \frac{T_0^*}{T_{01}} T_{01} = 1699'3 \text{ K}$$

$$q = c_p (T_{02} - T_{01}) = 1'4106 \cdot 10^6 \text{ J/kg}$$

$\boxed{\text{heat power} = q \cdot \dot{m} = 17'595 \text{ kW}}$